# Signal Analysis – Homework 09
# (Fourier Transforms)

1) – Show that the signals $x(t)$ below have their corresponding **Fourier transforms** $X(j\omega)$, also given below.

a) $x(t) = 1, \forall t$ $\qquad X(j\omega) = 2\,\pi\, u_o(\omega)$

b) $x(t) = e^{j\omega_o t}$ $\qquad X(j\omega) = 2\,\pi\, u_o(\omega - \omega_o)$

c) $x(t) = \text{sen}\,(\omega_o t)$ $\qquad X(j\omega) = \dfrac{\pi}{j}\left[u_o(\omega-\omega_o) - u_o(\omega+\omega_o)\right]$

(**suggestion**: use the coefficients $c_k$'s of the **Fourier series** of sine to find $X(j\omega)$, or use the Euler equations to express the sine and then transform).

d) $x(t) = \cos\,(\omega_o t)$ $\qquad X(j\omega) = \pi\left[u_o(\omega-\omega_o) + u_o(\omega+\omega_o)\right]$

(**suggestion**: use the derivative property of the **Fourier transform**, since the derivative of the sine is the co-sine $\times$ its frequency).

e) $x(t) = \dfrac{\text{sen}\,(\omega_o t)}{\pi t}$ $\qquad X(j\omega) = \begin{cases} 1, & \text{se } |\omega| < \omega_o \\ 0, & \text{se } |\omega| > \omega_o \end{cases}$

(**suggestion**: use the duality property of the **Fourier transform**).

f) $x(t) = u_o(t)$ $\qquad X(j\omega) = 1$

g) $x(t) = u_1(t)$ $\qquad X(j\omega) = \dfrac{1}{(j\cdot\omega)} + \pi\, u_o(\omega)$

(**suggestion**: use the integral property of the **Fourier transform** since the integral of the impulse signal is the step signal).

h) $x(t) = u_o(t - t_o)$ $\qquad X(j\omega) = e^{j\omega t_o}$

(**suggestion**: use the duality property of the **Fourier transform**).

2) – Using the conjugate property of the **Fourier transform**:

$$\mathscr{F}\left\{x^*(t)\right\} = X^*(-j\omega),$$

Show that: If $x(t) \in \mathbb{R}$, $\forall t$ is odd, then:

$$X(j\omega) \in \text{imaginary axis} \quad \text{and} \quad X(j\omega) = -X(-j\omega)$$

(that is, $X(j\omega)$, the **Fourier transform** of $x(t)$, is itself pure imaginary and odd).

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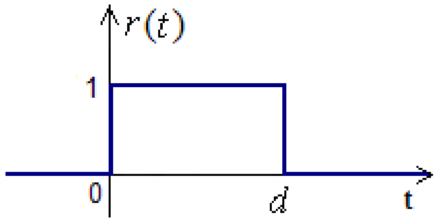
3) – The signal $x(t)$ and its **Fourier transform** $X(j\omega)$ are given below, where the symbol $\alpha^*$ means that the conjugate of $\alpha$. Find the values of the constants $\underline{a}$, $\underline{b}$, $\underline{c}$ and $\underline{d}$.

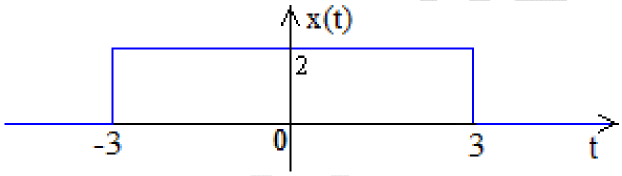
$$x(t) = a \text{ sen } (ct) + b \cos (ct) + r(t)$$

$$\alpha = 3 + 4j$$

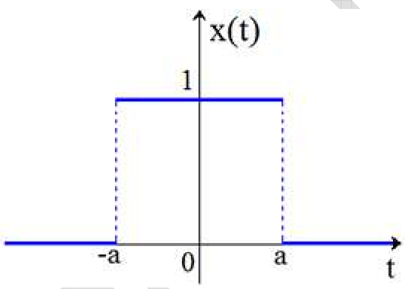



$$X(j\omega) = \frac{1}{j\omega} - \frac{e^{\omega j}}{j\omega} + \alpha \cdot \pi \cdot u_o(\omega - 5) + \alpha^* \cdot \pi \cdot u_o(\omega + 5)$$

4) – Find the **Fourier transform** of the signal $x(t)$ given below. Then, using the properties of the **Fourier transform** (linearity, reflected signal or "*time reversal*", etc.), find the **Fourier transform** of the signals $x_1(t)$, $x_2(t)$ and $x_3(t)$.

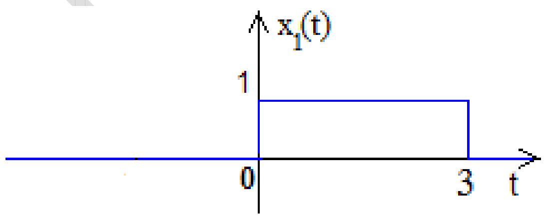


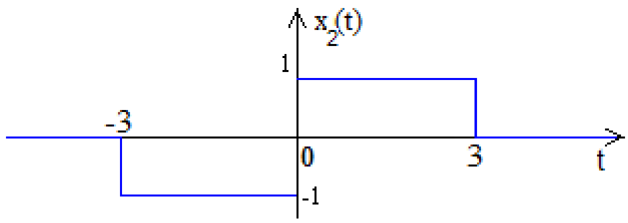

(**suggestion**: use the result of the Example 8.3, i.e., the **Fourier transforms** of $x(t)$ below is given by $X(j\omega)$ also given below).



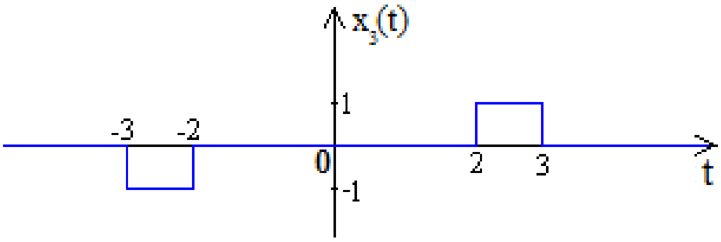

$$X(j\omega) = \frac{2 \text{ sen } (a\,\omega)}{\omega}$$

5) – Find the **Fourier transforms** of signals $x_1(t)$, $x_2(t)$ and $x_3(t)$ below.



# Signal Analysis – Homework 09 (Fourier Transforms)

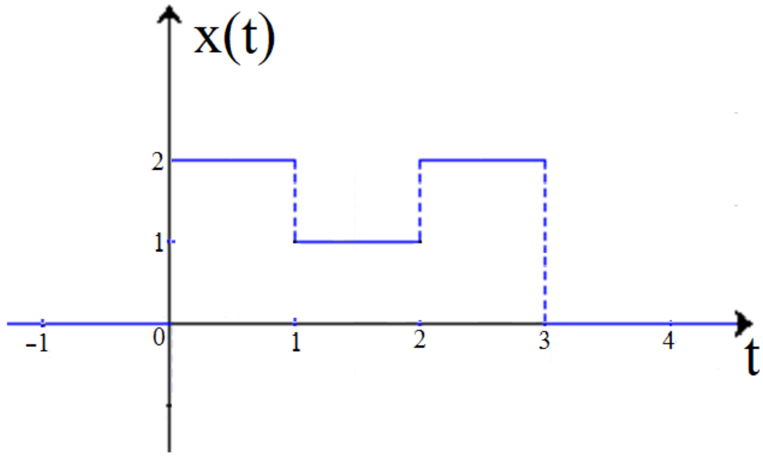



(**suggestion**: again here, as in the previous exercise, use the result of Example 8.3 and the properties of Fourier transform the linearity, translation or '*time shifting*', reflected signal or '*time reversal*', etc.).

6) – Find the Fourier transforms of the signal $x(t)$ given below.
Then, using the properties of the Fourier transforms, find the relation between $x(t)$ and the signals $x_1(t)$ and $x_2(t)$ whose Fourier transforms $X_1(j\omega)$ and $X_2(j\omega)$ are given below. That is, express $x(t)$ in terms of the signals $x_1(t)$ and $x_2(t)$.

$$X_1(j\omega) = \frac{e^{-j\frac{3\omega}{2}}}{\omega} \cdot \left( \operatorname{sen} \frac{3\omega}{2} \right)$$

$$X_2(j\omega) = -\frac{e^{-j\frac{\omega}{2}}}{\omega} \cdot \left( \operatorname{sen} \frac{\omega}{2} \right)$$




Solution : $x(t) = 2\, x_1(t) - x_2(t-1)$.

7) – Consider the signal $x(t)$ whose Fourier transform $X(j\omega)$ is given below:

$$X(j\omega) = \begin{cases} \sqrt{\pi}\,, & \text{se } 1 < |\omega| < \frac{1}{2} \\ \sqrt{\pi}/2\,, & \text{se } |\omega| \le \frac{1}{2} \\ 0\,, & \text{se } \omega \notin \{-1, 1\} \end{cases}$$

Calculate the energy $E$ of this signal

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

and $y(0)$, where $y(t) = \frac{dx}{dt}(t)$.

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8) – Calculate the **Fourier transform** $X(j\omega)$ of the periodic signal $x(t)$ given below. Observe that this **transform** it will be a '*train of impulses*'. Sketch the diagram of absolute value and phase of $X(j\omega)$.

$$x(t)=\begin{cases}-1, & \text{se } -1<t\leq 0\\ 1, & \text{se } 0<t\leq 1\end{cases}$$

9) – The derivative $y(t)$ of a periodic signal $x(t)$ has its **Fourier transform** $Y(j\omega)$ given by the "*train of impulses*" below. Find the expression of the signal $x(t)$.

$$Y(j\omega)=\frac{\pi}{2j}\cdot\left[\,u_o(\omega-1)\;-\;u_o(\omega+1)\,\right]$$

10) – The signal $x(t)$ is periodic with fundamental frequency $\omega_o = 1$ and has its coefficients of the **complex Fourier transform** given by $c_k$ below. Besides, $y(t)$ is the signal $x(t)$ shifted to the right, that is, $y(t) = x(t–a)$.
Find $Y(j\omega)$, the **Fourier transform** of $y(t)$.

$$c_o = 0{,}5 \quad , \qquad c_k = \frac{2}{3\cdot k}, \quad k=\pm1,\pm3,\pm5,\cdots \quad , \quad c_k = 0, \quad k=\pm2,\pm4,\pm6,\cdots$$

11) – $x(t)$ is a periodic signal with fundamental frequency $\omega_o = 1$ and has its coefficients of the **complex Fourier transform** given by $c_k$ below. On the other hand, $y(t)$ is a signal given by the expression below, as well as its **Fourier transform** $Y(\omega)$.

$$c_{-1}=2, \qquad c_1=-2, \qquad c_{-2}=c_2=0{,}5 \qquad \text{e} \qquad c_k=0, \forall\, k\notin\{-2,-1,1,2\}$$

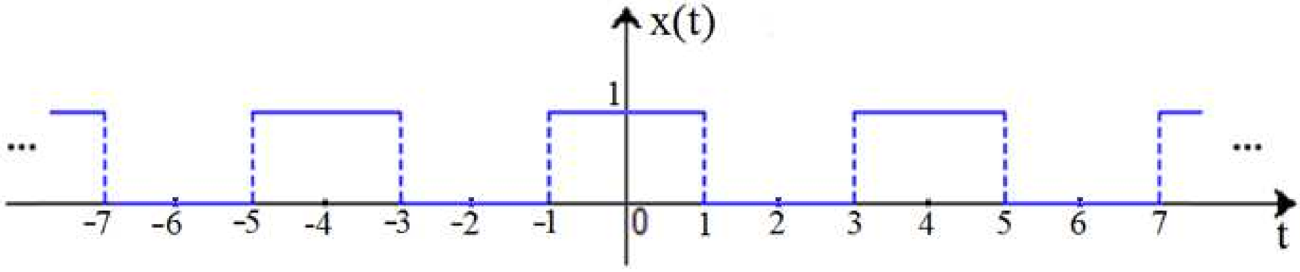
$$y(t)=\frac{x(t-\pi)}{2\pi}$$

$$Y(j\omega)=\alpha u_o(\omega+2)+\beta u_o(\omega+1)+\gamma u_o(\omega)+\lambda u_o(\omega-1)+\rho u_o(\omega-2)$$

Find the values of $\alpha$, $\beta$, $\gamma$, $\lambda$ and $\rho$.

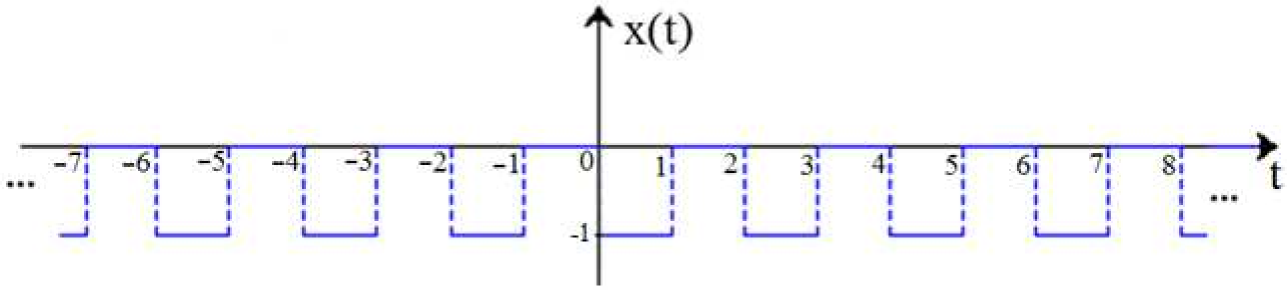
12) – Find the **Fourier transform** $X(j\omega)$ of the periodic signal $x(t)$ given by the graph below. Observe that $X(j\omega)$ is a '*train of impulses*'. Sketch the diagram of absolute value of $X(j\omega)$.
Note that $X(j\omega)\in\mathbb{R}$, so, it is not necessary two diagrams: one for absolute value and another for the phase. One is enough.

# Signal Analysis – Homework 09
# (Fourier Transforms)




**Obs.**: The Fourier series of this periodic signal $x(t)$ was calculated in exercise 7 (b) of the Homework 7 (continuous Fourier series).

13) – Find the Fourier transform $X(j\omega)$ of this periodic signal $x(t)$ given by the graph below ('*train of impulses*').
Sketch the diagram of absolute value $|X(j\omega)|$ and phase $\angle X(j\omega)$ of $X(j\omega)$.




**Obs.**: The Fourier series of this periodic signal $x(t)$ was calculated in exercise 7 (a) of the Homework 7 (continuous Fourier series).

14) – The derivative $y(t)$ of a periodic signal $x(t)$ has its Fourier transform $Y(j\omega)$ given below ('*train of impulses*').
Find the expression of the signal $x(t)$.

$$Y(j\omega) = \frac{\pi}{2j} \cdot [\, u_o(\omega - 1) - u_o(\omega + 1) \,]$$

15) – The signal $x(t)$ is the derivative of $y(t)$ which is a periodic signal and has its Fourier transform $Y(j\omega)$ given below ('*train of impulses*').
Find the expression of the signal $x(t)$.

$$Y(j\omega) = 2\pi(1+2j) \cdot u_o(\omega+3) + 2\pi(1+2j) \cdot u_o(\omega-3)$$