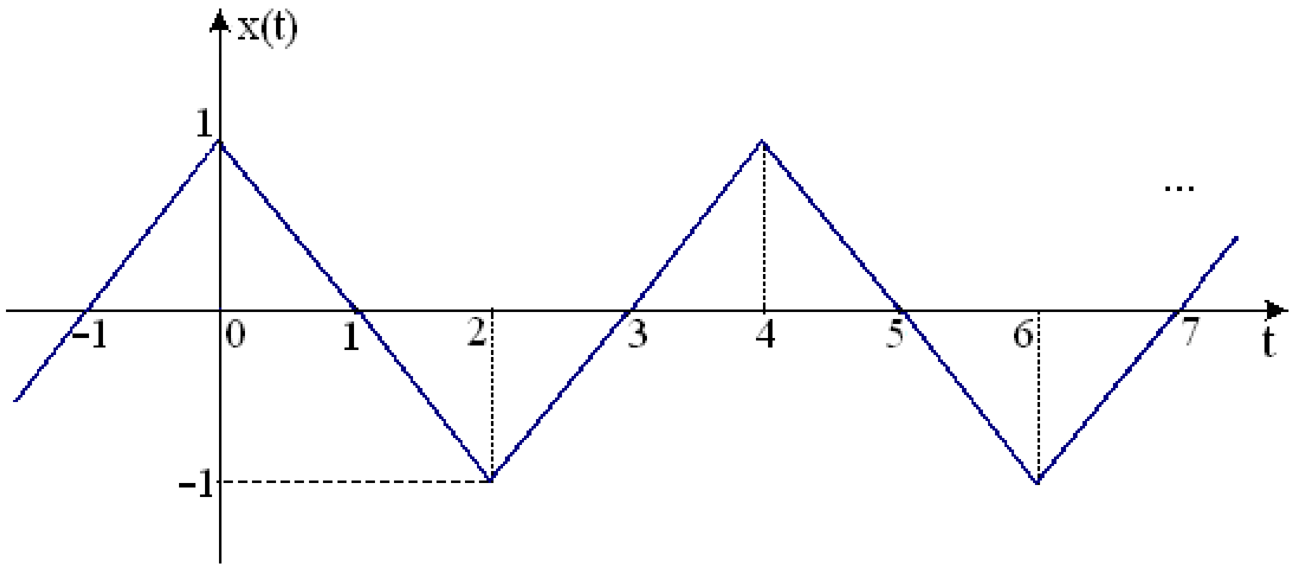
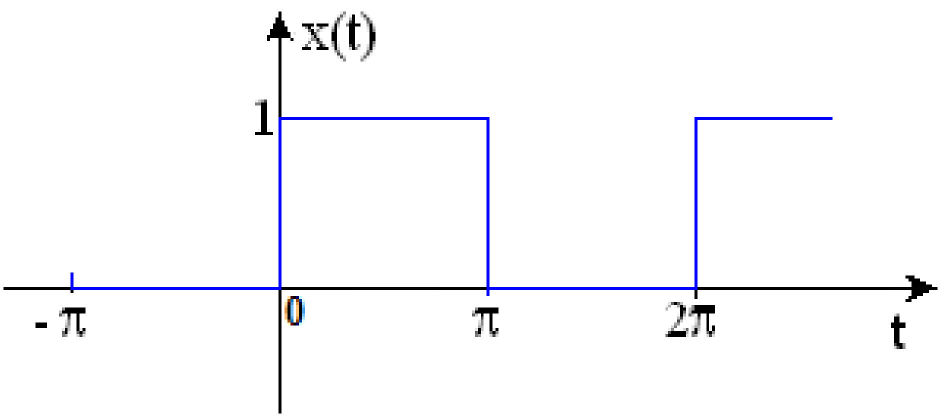
Signal Analysis – Homework 07 (Fourier Series - continuous)

1) – For the periodic signal $x(t)$ below calculate the first 9 ***coefficients*** of the Trigonometric Fourier series (i.e., a_o, a_1, b_1, a_2, b_2, a_3, b_3, a_4 and b_4).
Then write 9 first terms of the Fourier series of $x(t)$.
In Matlab, sketch some graphs using the "**plot (t, x)**" for the Fourier series of $x(t)$ with 1 term, 2 terms, 3 terms,… and see if the graph is approaching the original signal.



2) – For the periodic signal $x(t)$ below find the expression of all the ***coefficients*** of the Trigonometric Fourier series (i.e., a_k, e b_k, $\forall k$). Then calculate the value of the 9 first terms of the Trigonometric Fourier series of $x(t)$.
In Matlab, sketch some graphs using the "**plot (t, x)**", for the Fourier series of $x(t)$ with 1 term, 2 terms, 3 terms, … and see if the graph is approaching the original signal.



3) – For the signal of the previous exercise calculate $x(t)$ for $t = \pi/2$ using the Fourier series and see if it converges to 1. (i.e., verify that $x(0{,}5\pi) = 1$). The expression generated allows to calculate the value of π. This was the way that Leibniz calculated π in the XVIIth century.

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(Fourier Series - continuous)

4) – The Trigonometric Fourier series of the continuous periodic signal $x(t)$ with fundamental frequency $\omega_o = \pi$ have the following coefficients:

$$a_o = -2 \ , \qquad a_k = 0, \quad k = \pm 1, \pm 2, \pm 3, \cdots; \qquad b_k = \frac{2}{k^2}, \quad k = \pm 1, \pm 2, \pm 3, \cdots$$

a) – Find the coefficients c_k's of the Complex Fourier series of this signal $x(t)$.

b) – Find the coefficients c_k's of the Complex Fourier series of the signal

$$y(t) = x'(t) = dx/dt \qquad \text{[the derivative of } x(t)\text{]}.$$

c) – Find also the coefficients a_k's and b_k's of the Trigonometric Fourier series of the signal $y(t)$.

5) – The Trigonometric Fourier series of the continuous periodic signal $x(t)$ with fundamental frequency $\omega_o = \pi/2$ has the followings coefficients:

$$a_o = -1; \qquad a_k = \frac{2}{k}, \quad k = 1, 2, \cdots \qquad b_k = 0, \quad k = 1, 2, \cdots$$

Write the signal $y(t) = x^*(t)$ (*conjugate*) in the form of the Complex Fourier series.

6) – The Trigonometric Fourier series of the continuous periodic signal $x(t)$ with fundamental frequency $\omega_o = \pi/2$ has the following coefficients:

$$a_o = -1; \qquad a_k = \frac{2}{k}, \quad k = 1, 2, \cdots \qquad b_k = \frac{2}{3k}, \quad k = 1, 2, \cdots$$

On the other hand, continuous periodic signal $y(t)$ is given by: $y(t) = -3\sin(\pi t) + 5$.

a) – Find the coefficients c'_k of the signal $x^*(t)$, *conjugate of* $x(t)$, in the form of the Complex Fourier series.

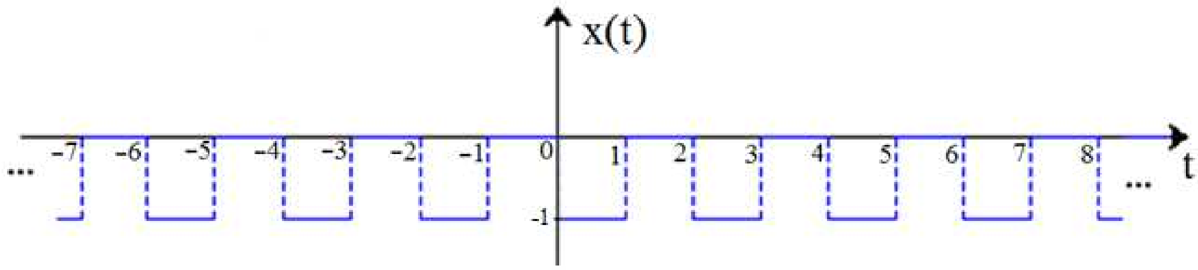
b) – Find the coefficients c''_k of the signal $y(t)$ in the form of the Complex Fourier series.

c) – Find the coefficients $\tilde{a}_k$ e $\tilde{b}_k$ of the signal $v(t) = x^*(t)\, y(t)$ in the form of the Trigonometric Fourier series.

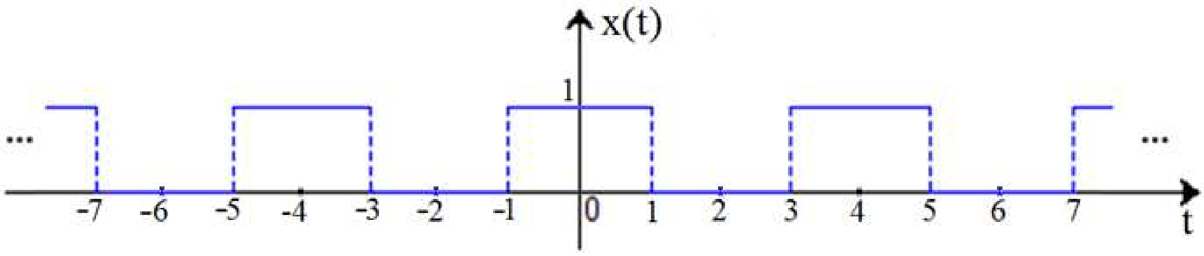
7) – Express the signals $x(t)$ of the graphs below in both forms of the Fourier series (trigonometric and complex).

In the cases where $x(t)$ is not a periodic signal, then we should imagine the expansion of $x(t)$ to both sides repeatedly in order to become periodic.

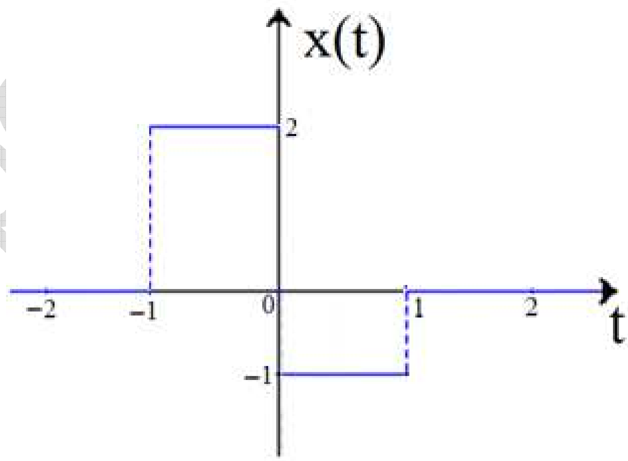
a) –


x(t)
t
-7
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5
6
7
8
-1
...

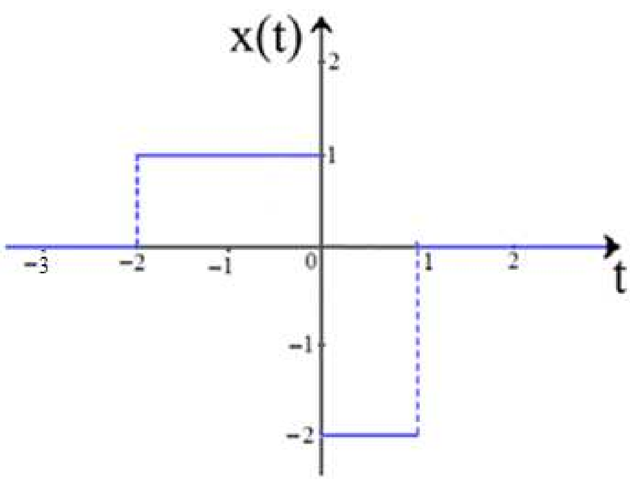
b) –


x(t)
1
t
-7
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5
6
7
...

c) –


x(t)
2
t
−2
−1
0
1
2
−1

d) –


x(t)
2
1
t
−3
−2
−1
0
1
2
−1
−2