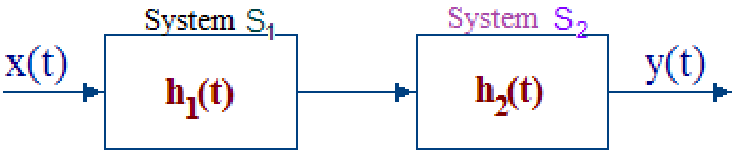
FACULDADE ENGENHARIA

# Signals Analysis – Homework 05
# (Systems & Laplace Transforms)

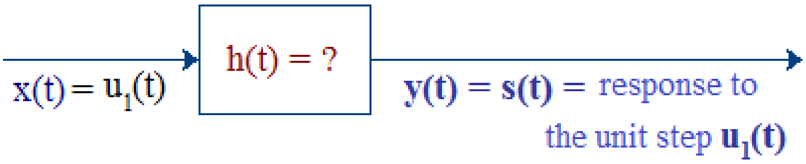
1) – Calculate the output signal $y(t)$ of 2 **continuous linear time invariant** (**LTI**) systems connected in cascade, with input signal $x(t)$ and the responses to the unit impulse $h_1(t)$ and $h_2(t)$.




a) $x(t) = e^{-t} \cdot u_1(t)$ , $h_1(t) = u_1(t) - u_1(t-1)$ e $h_2(t) = u_o(t-1)$

b) $x(t) = u_1(t)$ , $h_1(t) = t \cdot (u_1(t) - u_1(t-1))$ e $h_2(t) = u_o(t-2)$

2) – Find the response to the impulse $h(t)$ of a **continuous linear time invariant** (**LTI**) system which the response to the unit step $u_1(t)$ is $r(t)$.

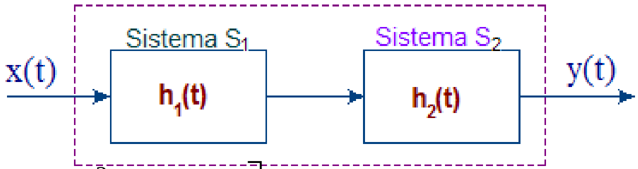



a) $r(t) = 1 - \frac{1}{2} \cdot e^{-2t} + \cos(4t)$

b) $r(t) = 1 - 2e^{-t/4} + \frac{1}{8} \cdot \cos(4t)$

3) – The systems $S_1$ and $S_2$ below are **LTI** (**linear time invariant**) system. Correct / say what is wrong in the expression of $y(t)$, the output signal (i.e., response of the system).

$$x(t) = \cos(2t) + e^{-2t}$$




$$y(t) = \left[ h_1(t) * 2\cos(2t) * h_2(t) + h_2(t) * \frac{e^{2t}}{2} * h_1(t) \right]$$

4) – Using the **theorems IVT** and **FVT** calculate $x(0^+)$ and $x(\infty)$ for each one of the signals $x(t)$ which **Laplace transform** $X(s)$ are given below:

a) $X(s) = \frac{3s+2}{s^2+s}$

b) $X(s) = \frac{s^2-s+2}{s^2+s}$

c) $X(s) = \frac{1}{s^2}$

d) $X(s) = \frac{\omega}{(s+a)^2+\omega^2}$.

e) $X(s) = \frac{(s+a)}{(s+a)^2+\omega^2}$

f) $X(s) = \frac{n!}{(s+a)^{n+1}}$

FACULDADE ENGENHARIA

# Signal Analysis – Homework 05
# (Systems & Laplace Transforms)

5) – Using the properties of the **Laplace transforms** show that:

a) $\mathcal{L}\left[t \cdot e^{-at}\right] = \dfrac{1}{(s+a)^2}$

b) $\mathcal{L}\left[t^2 \cdot e^{-at}\right] = \dfrac{2}{(s+a)^3}$

c) $\mathcal{L}\left[t^3 \cdot e^{-at}\right] = \dfrac{6}{(s+a)^4}$

d) $\mathcal{L}\left[t^n \cdot e^{-at}\right] = \dfrac{n!}{(s+a)^{n+1}}$

e) Se $x(t) = e^{-at} \cdot \sin \omega t,\ t \geq 0$, then

$$X(s) = \frac{\omega}{(s+a)^2 + \omega^2}$$

f) Se $x(t) = e^{-at} \cdot \cos \omega t,\ t \geq 0$, then

$$X(s) = \frac{(s+a)}{(s+a)^2 + \omega^2}$$

6) – Find the **inverse Laplace transforms** $x(t) = \mathcal{L}^{-1}\{X(s)\}$ for the signals which **Laplace transforms** $X(s)$ are given below:

a) $X(s) = \dfrac{(s+3)}{(s+1)(s+2)}$

b) $X(s) = \dfrac{8s^2 + 14s - 8}{(2s^3 + 10s^2 + 12s)}$

c) $X(s) = \dfrac{5s^3 + 33s^2 + 44s - 8}{(2s^3 + 10s^2 + 12s)}$

d) $X(s) = \dfrac{1}{s(s^2 + s + 1)}$

e) $X(s) = \dfrac{1}{(2s^2 + s)}$

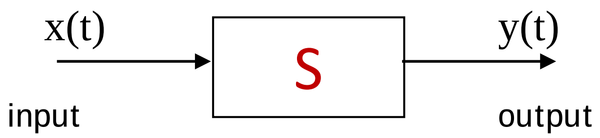
f) $X(s) = \dfrac{s^2 + s + 3}{(s+1)^3}$

g) $X(s) = \dfrac{2s^4 + 7s^3 + 10s^2 + 4s + 1}{(s^5 + 3s^4 + 3s^3 + s^2)}$

h) $X(s) = \dfrac{s^3 - 38s - 77}{(s^4 - 33s^2 - 100s - 84)}$

# Signal Analysis – Homework 05
## (Systems & Laplace Transforms)

7) – Find the solutions of the **ordinary differential equations** below:




a) $y'' + 2y(t) = x(t)$, zero initial conditions, that is, $y(0) = 0$ e $y'(0) = 0$, input $x(t)$ = unit step.

b) $y'' + 3y' + 6y(t) = 0$ (EDO homogeneous), initial conditions $y(0) = 0$ and

$y'(0) = 3$.

c) $\dfrac{d^3y}{dt^3} + 3\dfrac{d^2y}{dt^2} + 3\dfrac{dy}{dt} + y(t) = \dfrac{d^2x}{dt^2} + \dfrac{dx}{dt} + 3x(t)$, zero initial conditions, that is,

$y(0) = 0$, $y'(0) = 0$ e $y''(0) = 0$, input $x(t)$ = unit step.

d) $\dfrac{d^3y}{dt^3} + 10\dfrac{d^2y}{dt^2} + 33\dfrac{dy}{dt} + 36y(t) = -\dfrac{d^2x}{dt^2} - 3\dfrac{dx}{dt} + x(t)$, zero initial conditions,

that is,

$y(0) = 0$, $y'(0) = 0$ e $y''(0) = 0$, input $x(t)$ = unit step.