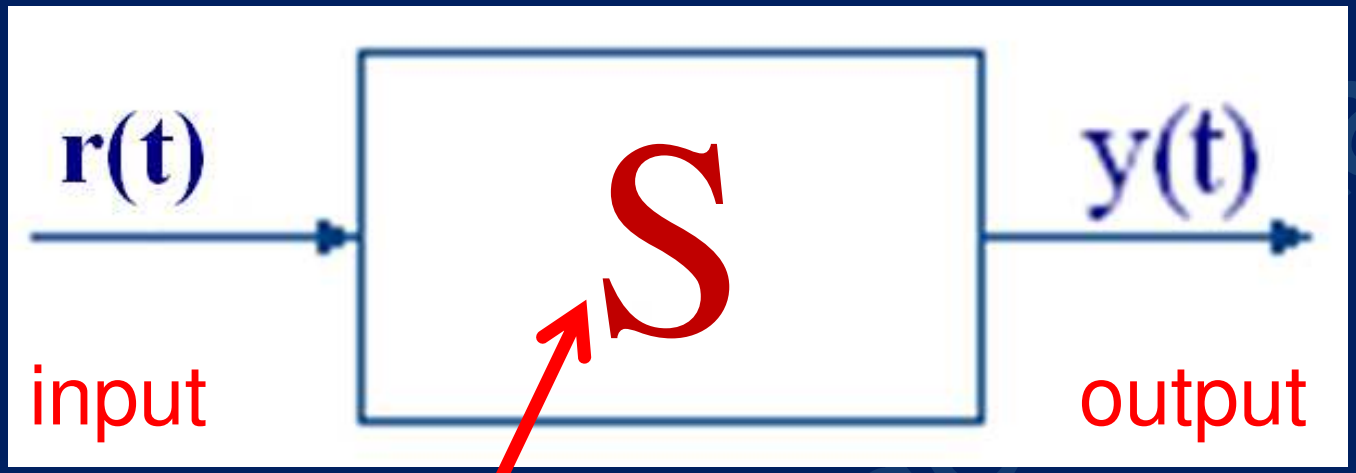
Controlo de Sistemas

6
"Análise no domínio do tempo"
(*Time domain analysis*)

parte I – Sistemas de 1ª ordem

J. A. M. Felippe de Souza

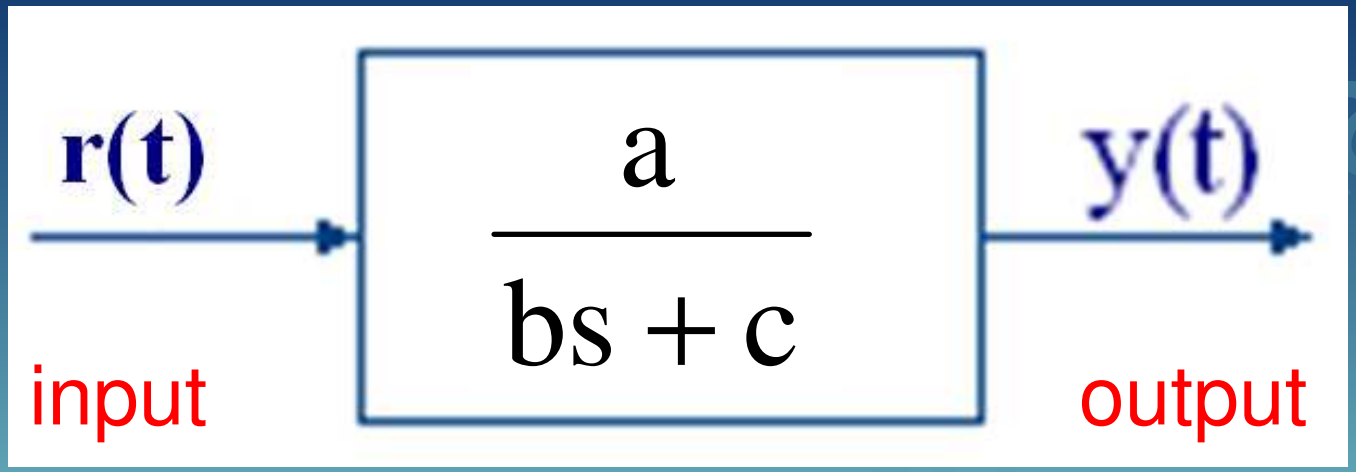


Sistemas de primeira ordem

Sistema de primeira ordem

do tipo

$$G(s) = \frac{a}{bs + c}$$



Sistemas de primeira ordem

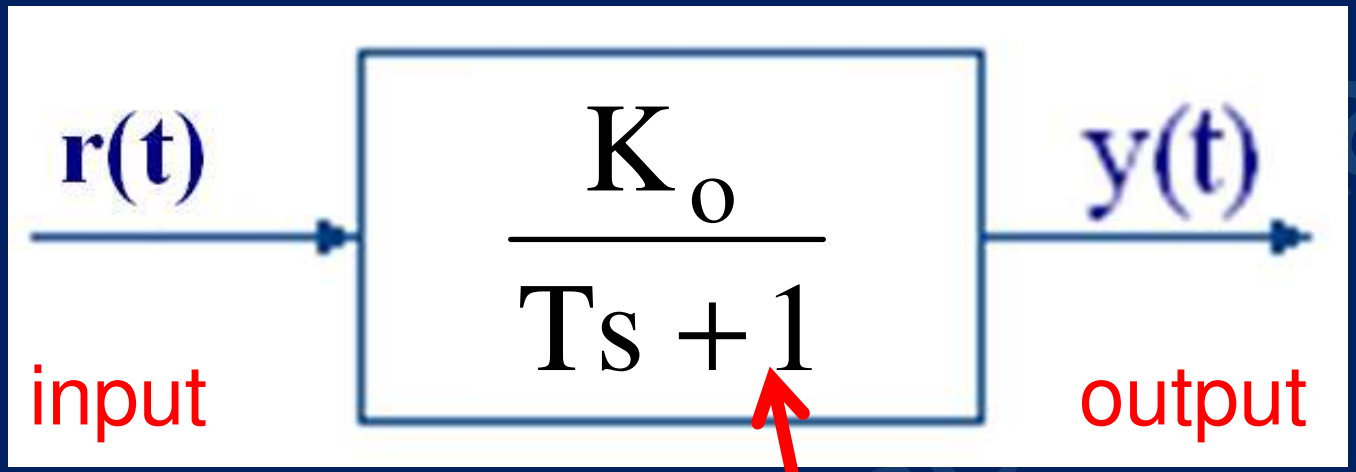
ou seja:

$$\frac{Y(s)}{R(s)} = \frac{a}{bs + c}$$

$$\frac{Y(s)}{R(s)} = \frac{\frac{a}{c}}{\frac{(bs + c)}{c}} = \frac{\frac{a}{c}}{\frac{b}{c}s + 1},$$

K_o

T

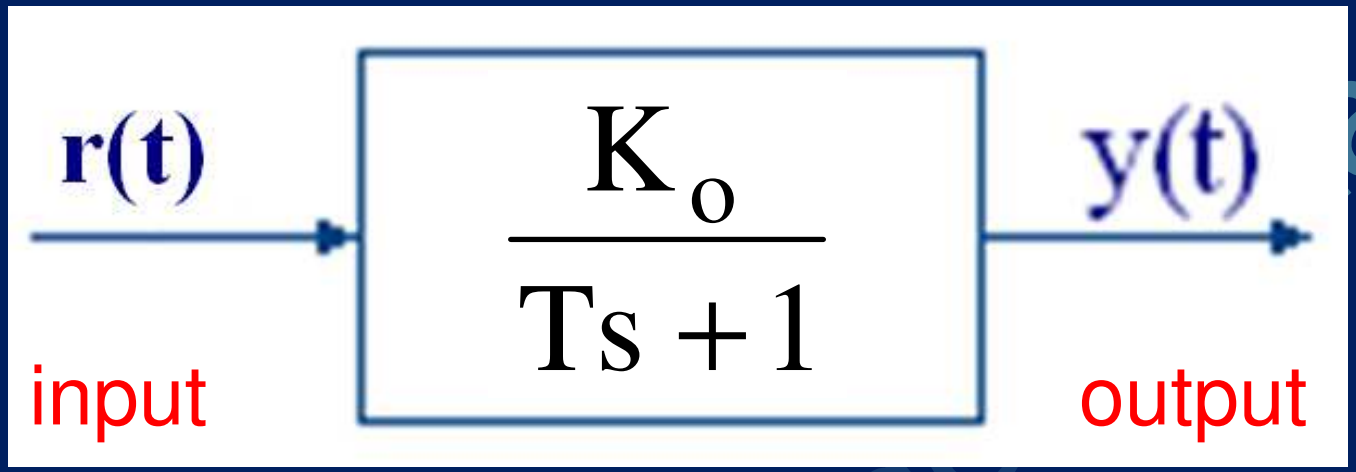


Sistemas de primeira ordem

ou seja:

a *função de transferência* pode ser reescrita como:

$$\frac{Y(s)}{R(s)} = \frac{K_o}{Ts+1}$$



Sistemas de primeira ordem

K_o = ganho do sistema

T = constante de tempo do sistema

a função de transferência:

$$\frac{Y(s)}{R(s)} = \frac{K_o}{Ts+1}$$

Exemplo 1:

$$\frac{Y(s)}{R(s)} = \frac{2}{5s+4}$$

polo:
$s = -0,8$

$K_o = 2/4 = 0,5$ $\qquad$ $T = 5/4 = 1,25$

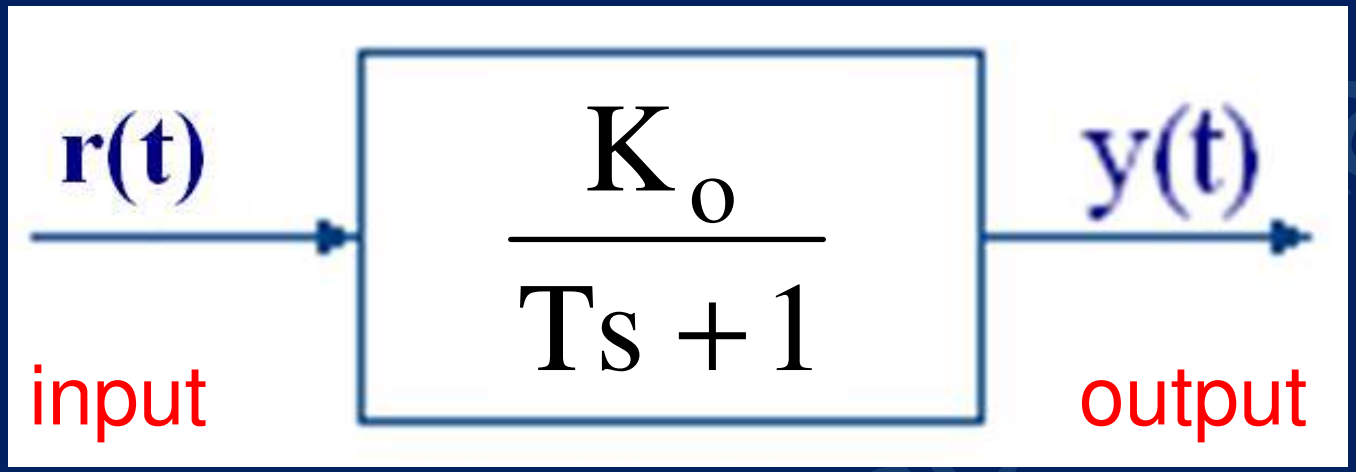
Exemplo 2:

$$\frac{Y(s)}{R(s)} = \frac{12}{s+4}$$

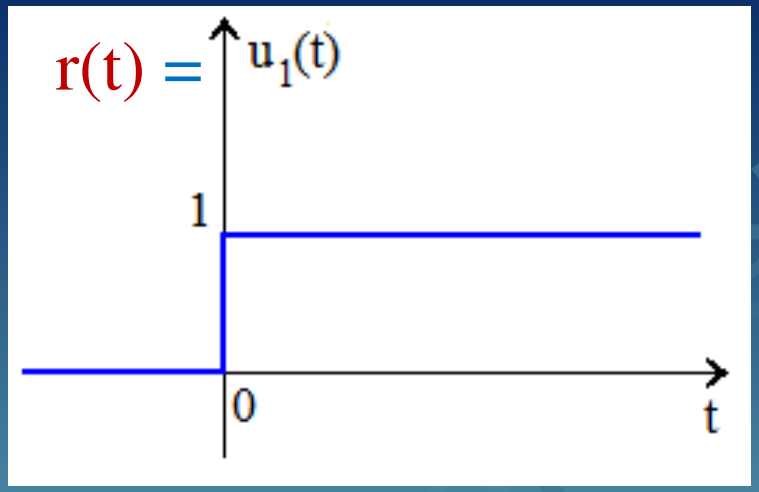
polo:
$s = -4$

$K_o = 3$ $\qquad$ $T = ¼ = 0,25$

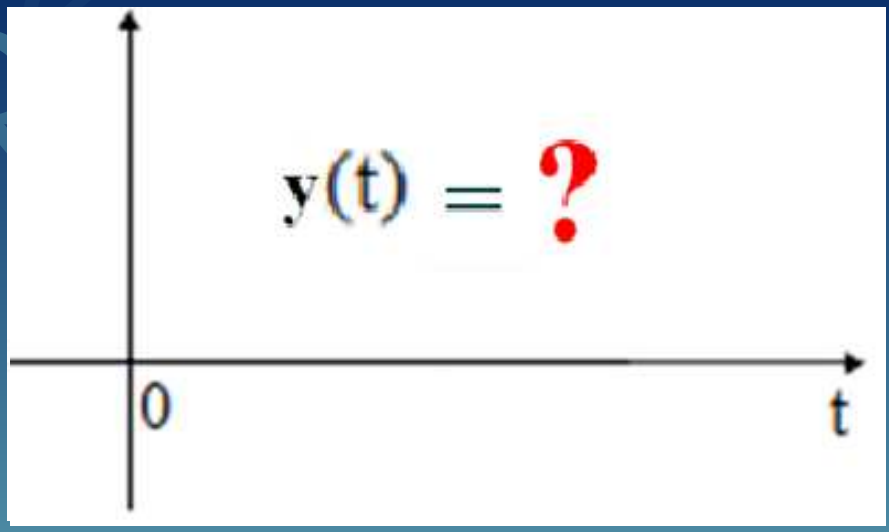
Análise no domínio do tempo - Sistemas de 1ª ordem



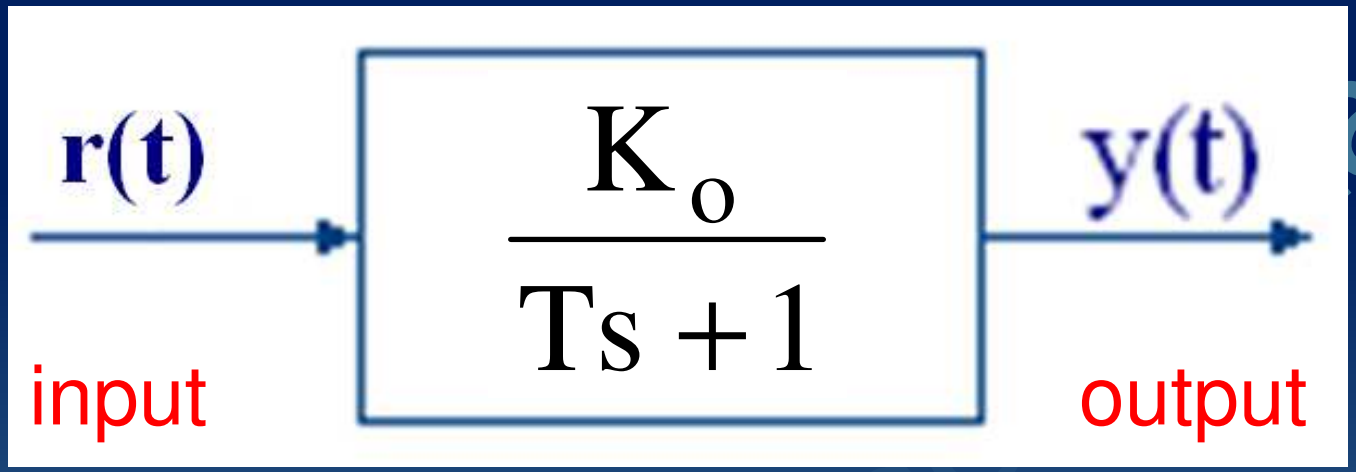
Sistemas de primeira ordem



Entrada degrau unitário



Qual é a resposta ao degrau ?
(*step response*)

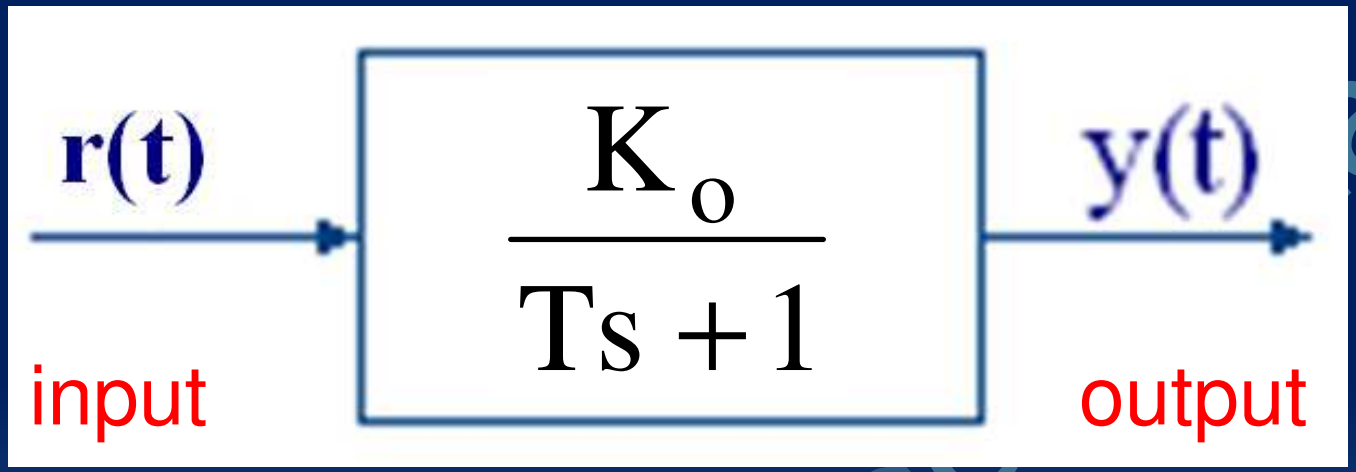


Sistemas de primeira ordem

para calcular fazemos:

$$Y(s) = \frac{K_o}{Ts+1} \cdot R(s)$$

$$Y(s) = \frac{K_o}{(Ts+1)} \cdot \frac{1}{s} = \frac{K_o}{s} - \frac{K_o T}{(Ts+1)}$$



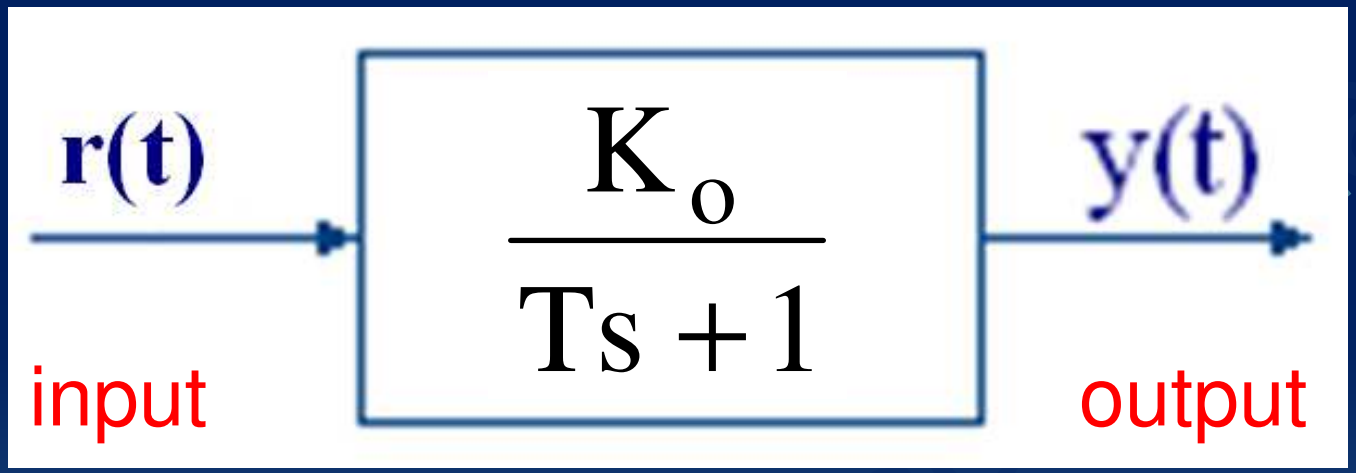
Sistemas de primeira ordem

$$y(t) = \mathcal{L}^{-1}[Y(s)]$$

logo, a resposta ao degrau unitário é:

$$y(t) = K_o(1-e^{-t/T}), \qquad t > 0$$

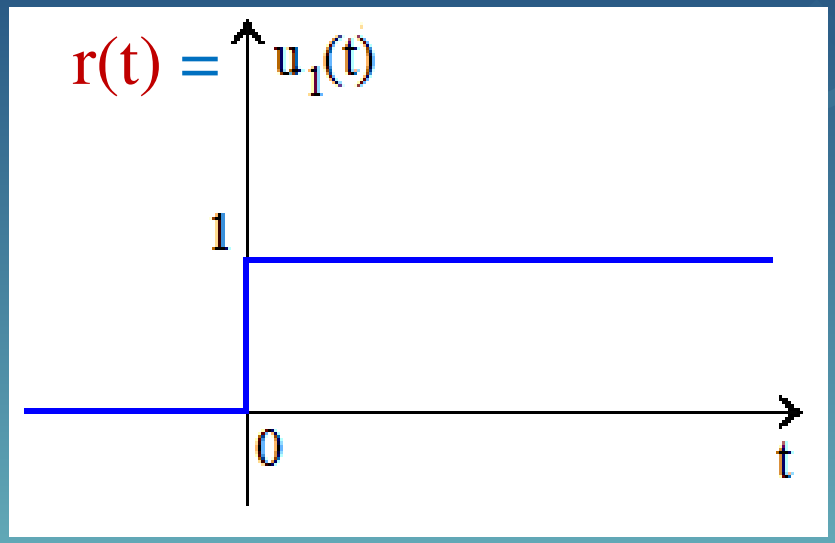
Análise no domínio do tempo - Sistemas de 1ª ordem



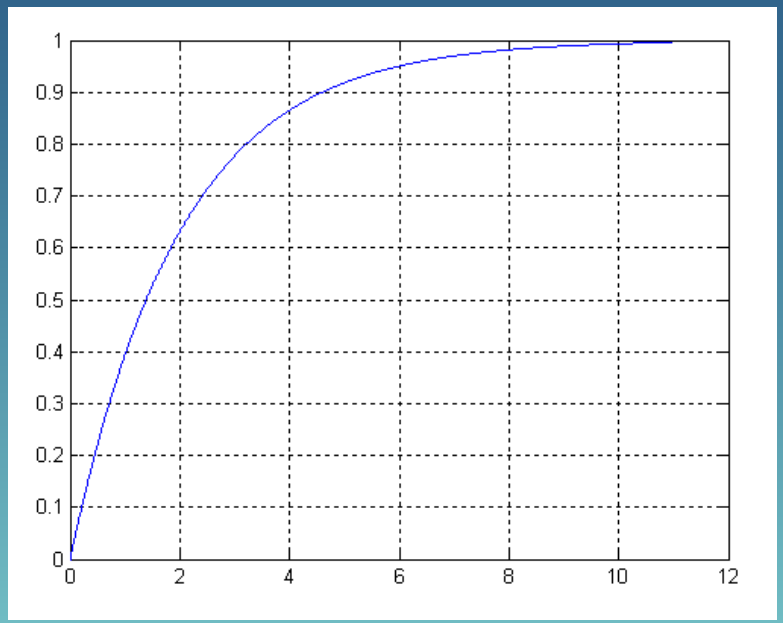
Sistemas de primeira ordem

a resposta ao degrau unitário é:

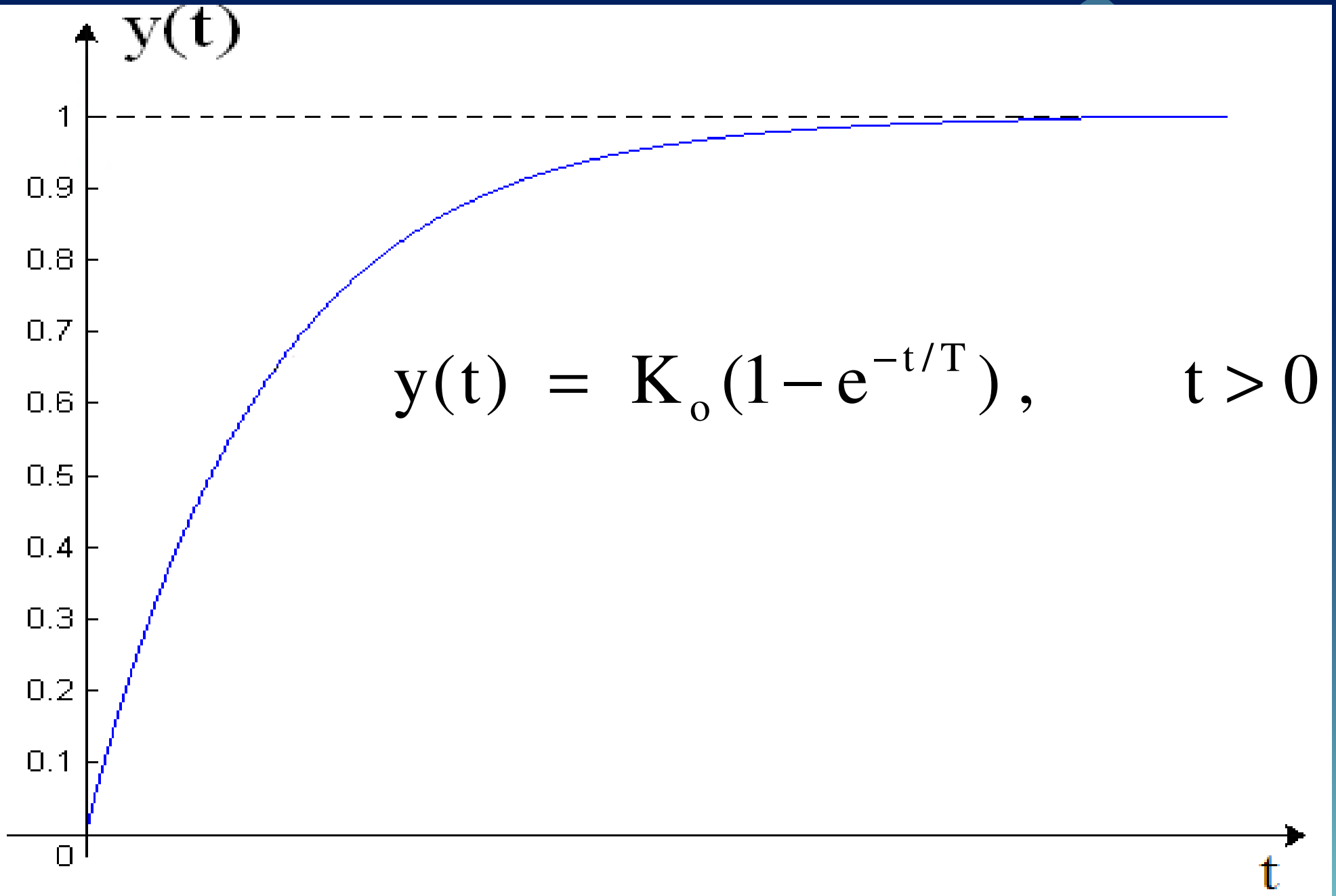
$$y(t) = K_o(1-e^{-t/T}), \quad t > 0$$



entrada degrau unitário



a resposta ao degrau unitário é:



Observe que, para a resposta ao degrau unitário:

$$y(t) = K_o(1-e^{-t/T}), \qquad t>0$$

$$\text{Se } t=T \Rightarrow y(T)=K_o(1-e^{-1})=0{,}632\,K_o$$

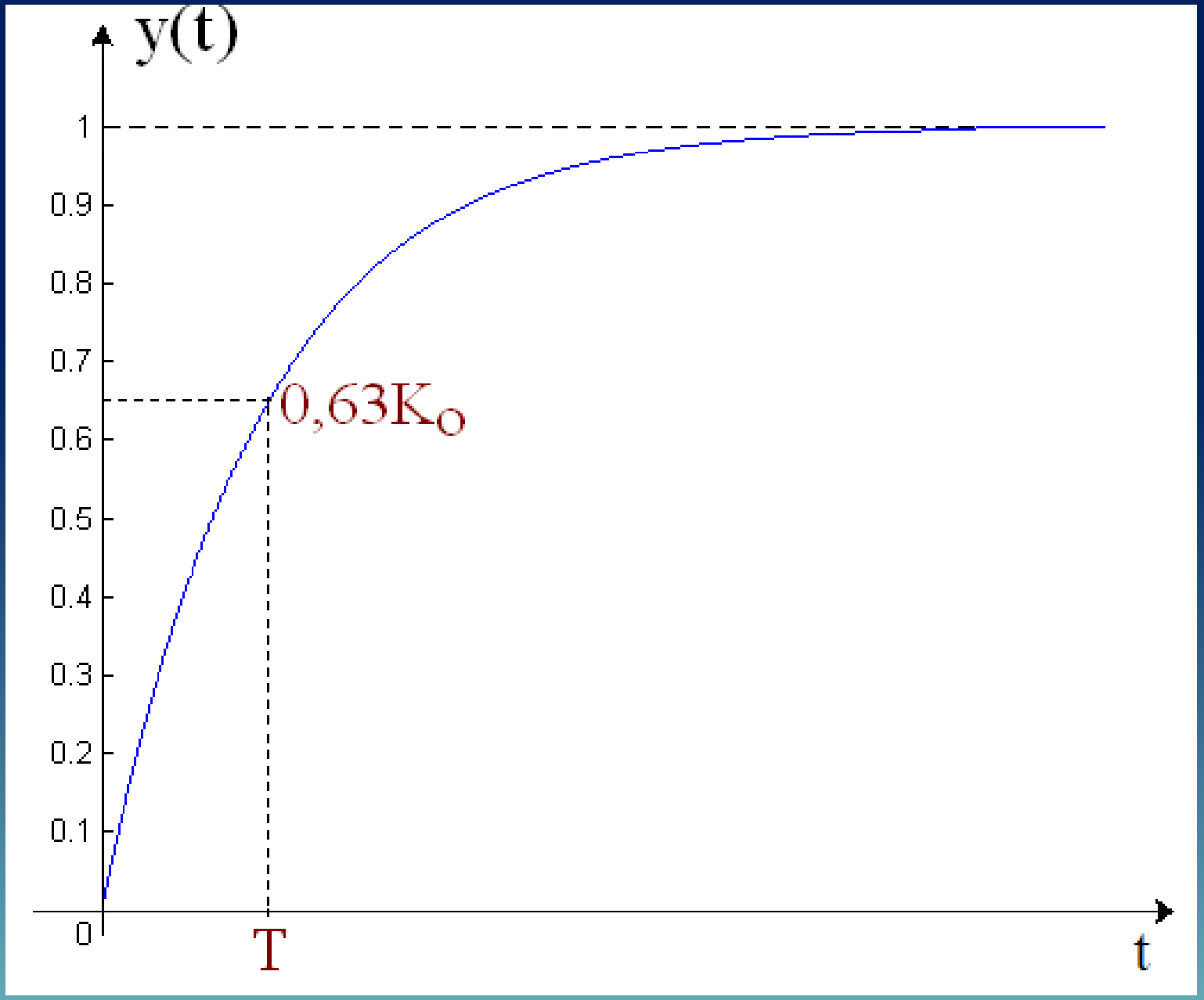
$$\text{Se } t=2T \Rightarrow y(2T)=K_o(1-e^{-2})=0{,}865\,K_o$$

$$\text{Se } t=3T \Rightarrow y(3T)=K_o(1-e^{-3})=0{,}95\,K_o$$

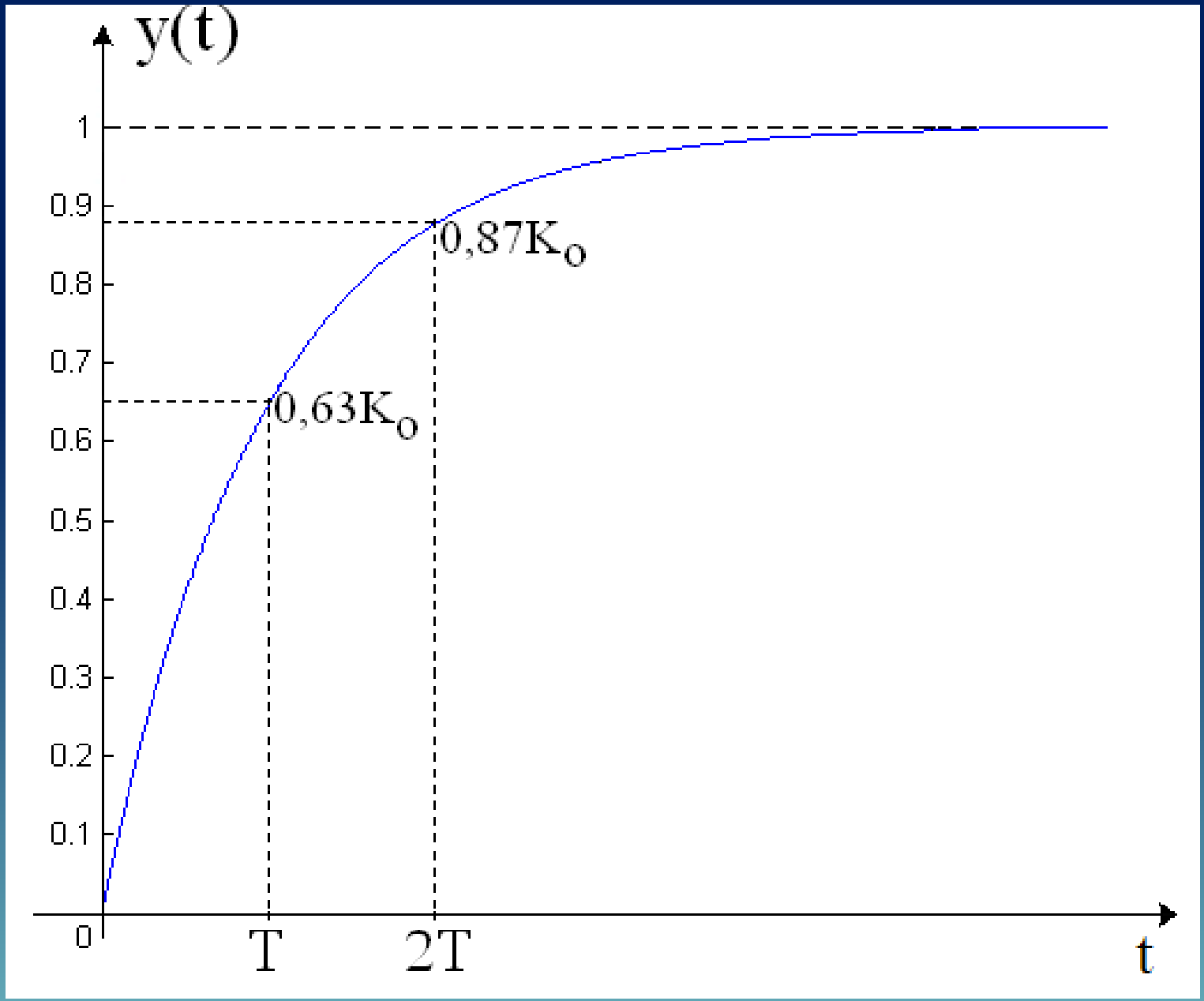
$$\text{Se } t=4T \Rightarrow y(4T)=K_o(1-e^{-4})=0{,}982\,K_o$$

$$\text{Se } t=5T \Rightarrow y(5T)=K_o(1-e^{-5})=0{,}993\,K_o$$

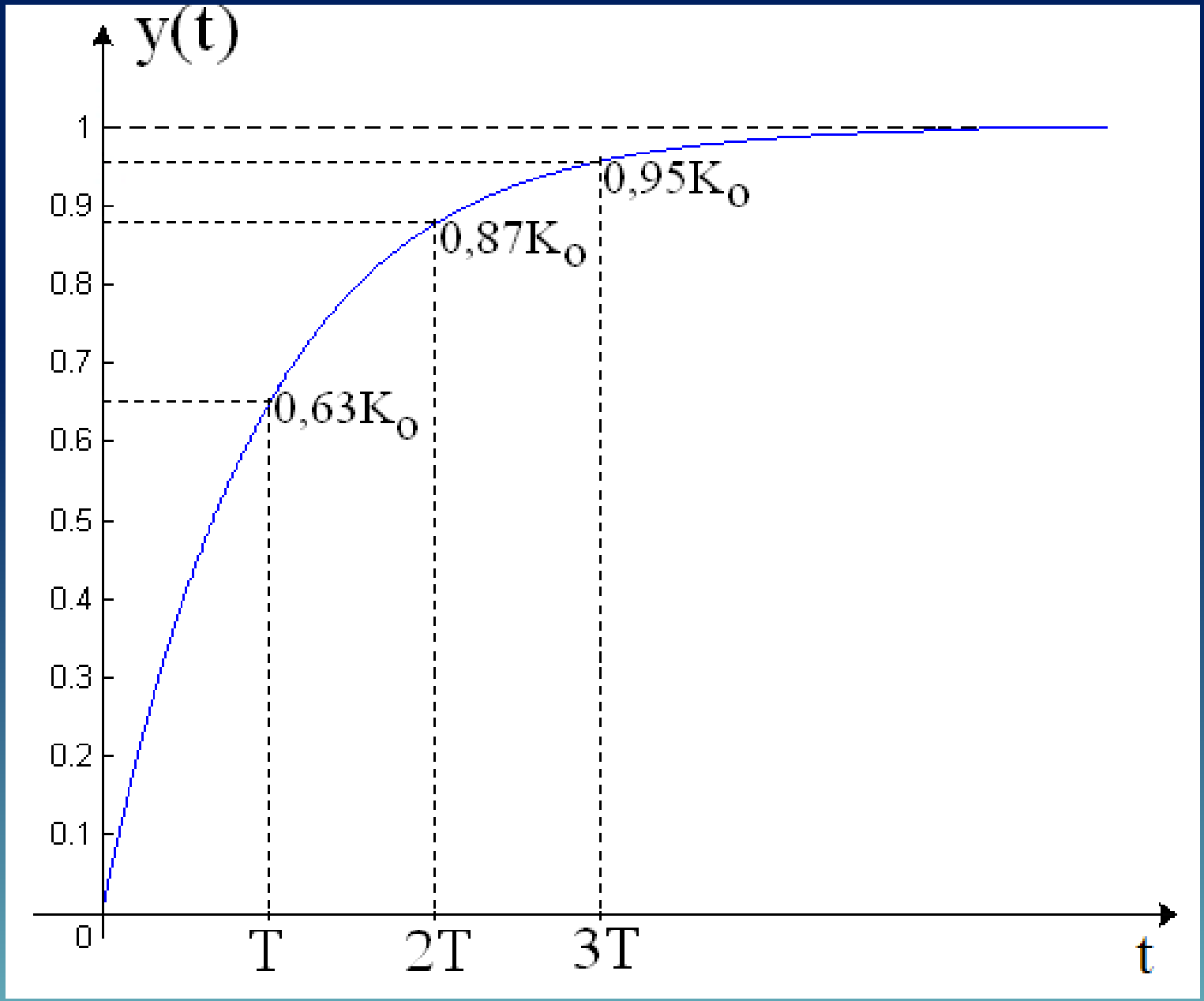
Análise no domínio do tempo - Sistemas de 1ª ordem



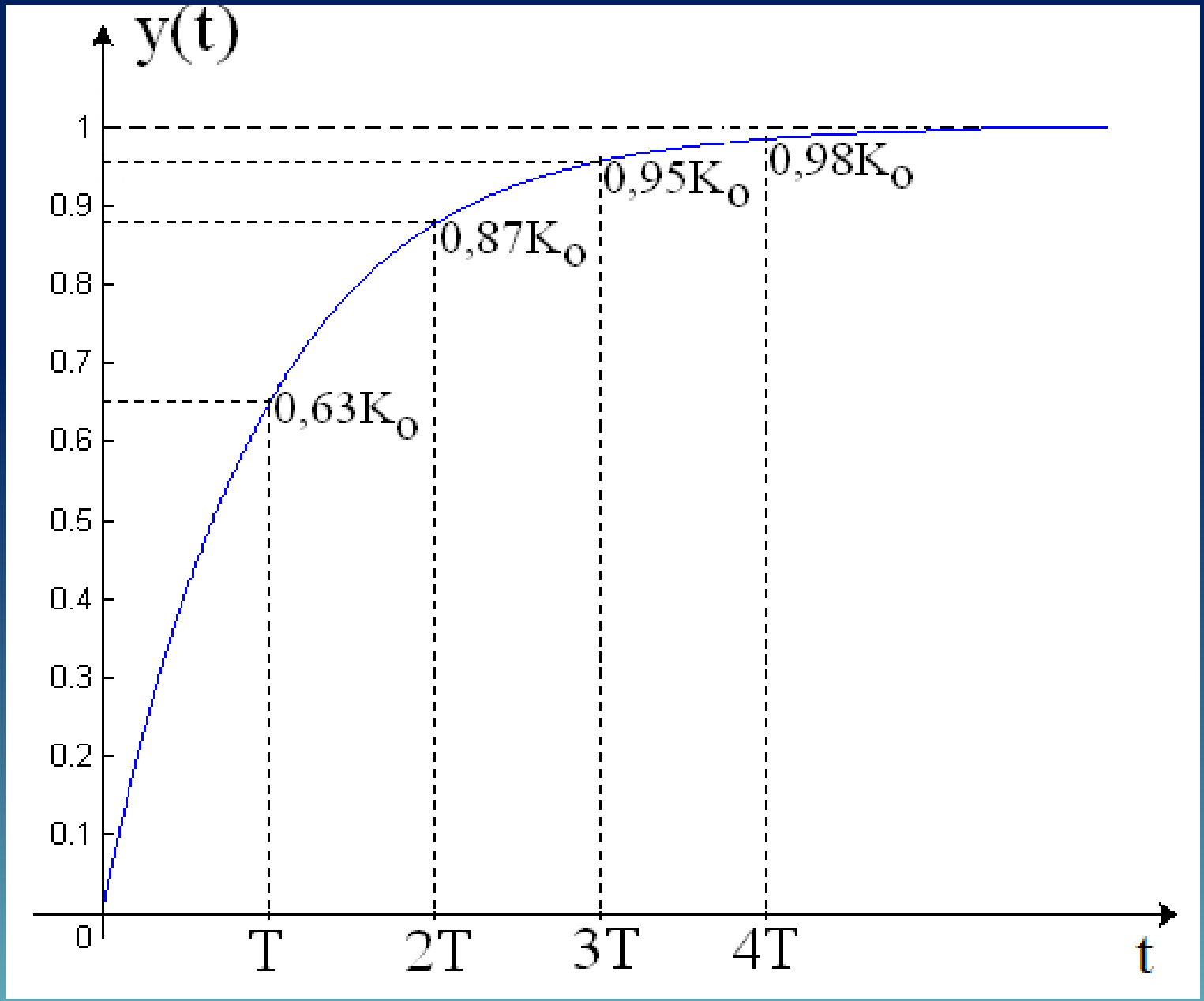
Análise no domínio do tempo - Sistemas de 1ª ordem



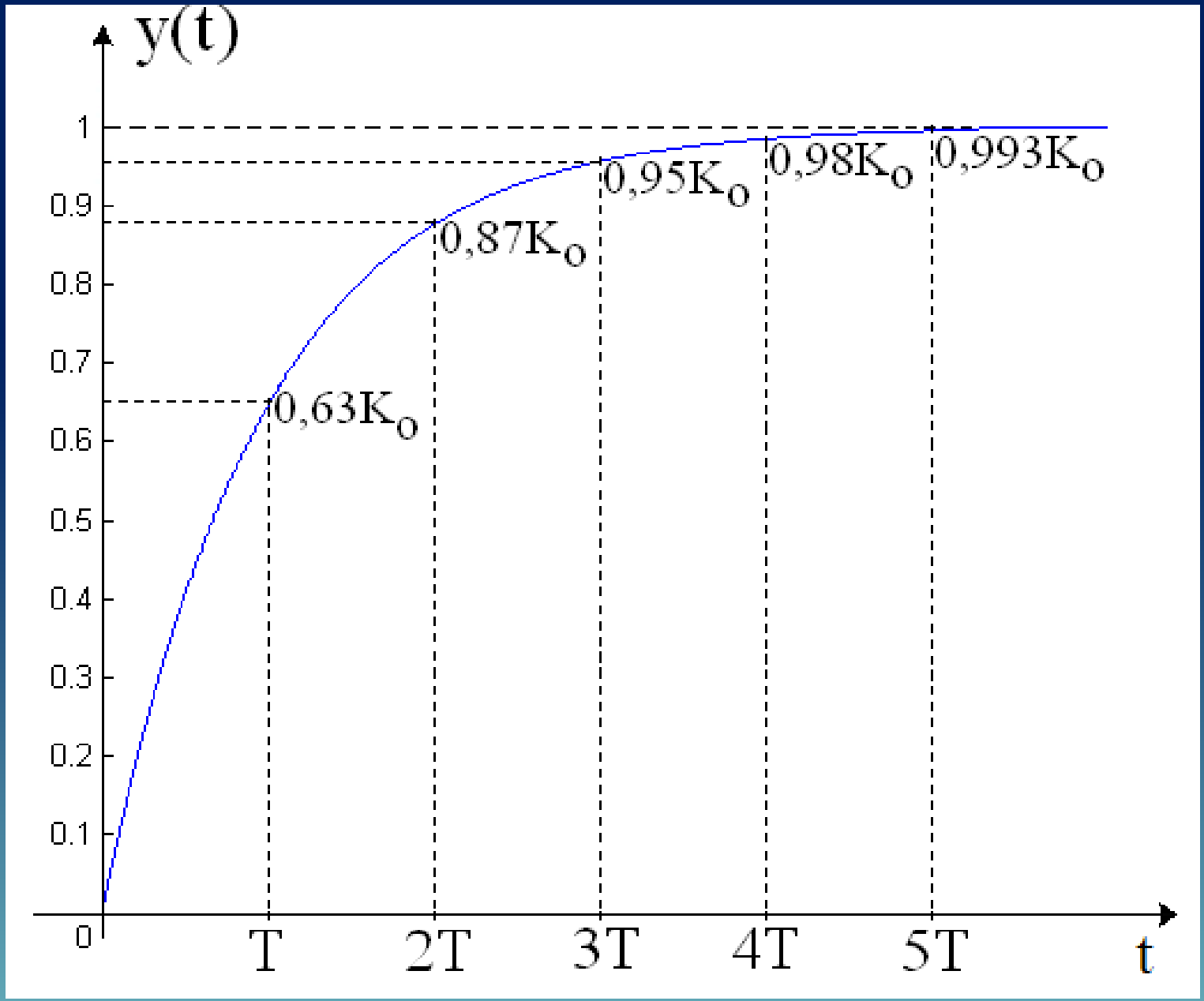
Análise no domínio do tempo - Sistemas de 1ª ordem



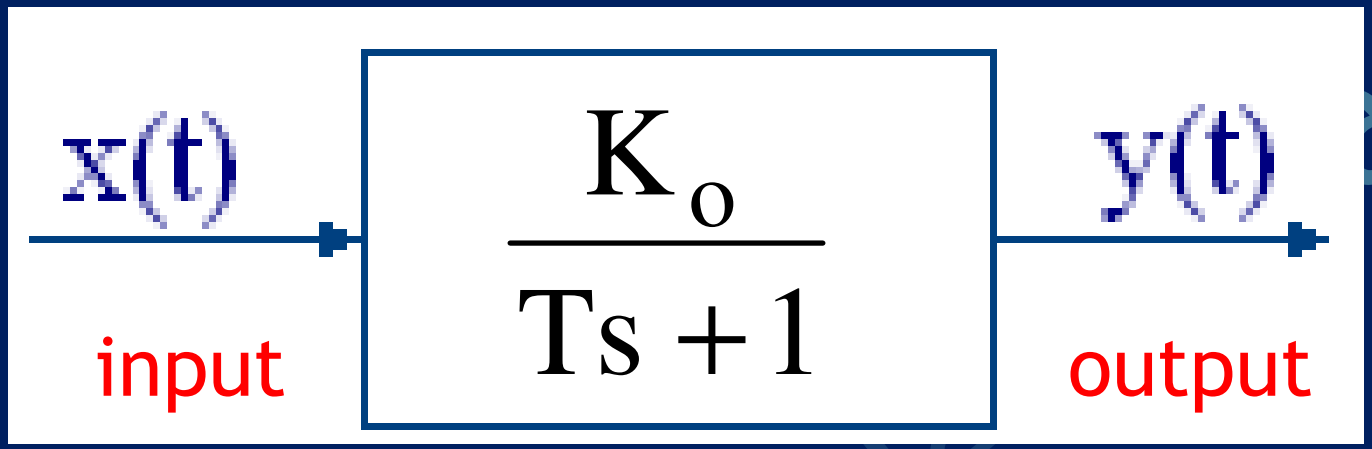
Análise no domínio do tempo - Sistemas de 1ª ordem



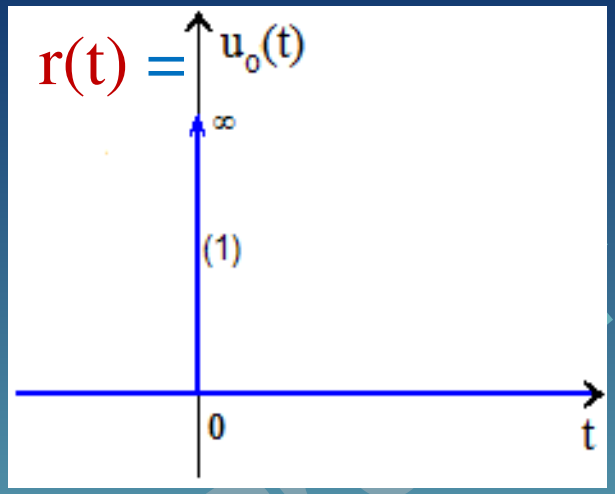
Análise no domínio do tempo - Sistemas de 1ª ordem



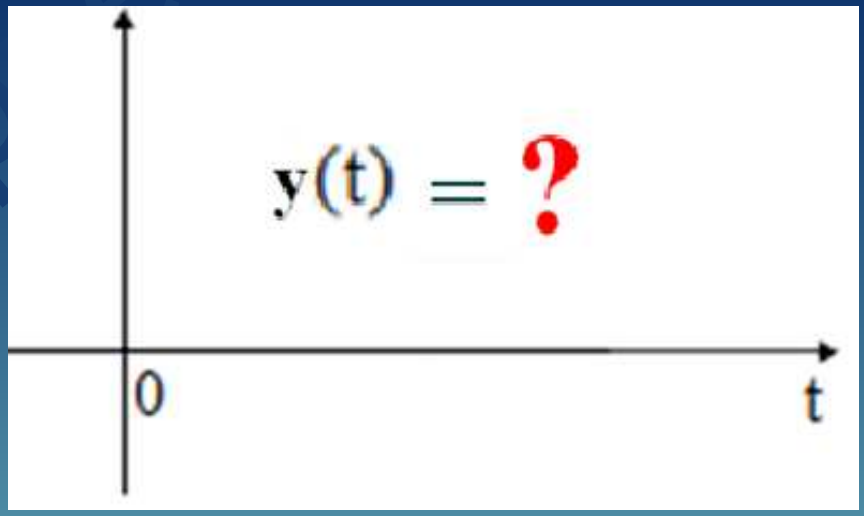
Análise no domínio do tempo - Sistemas de 1ª ordem



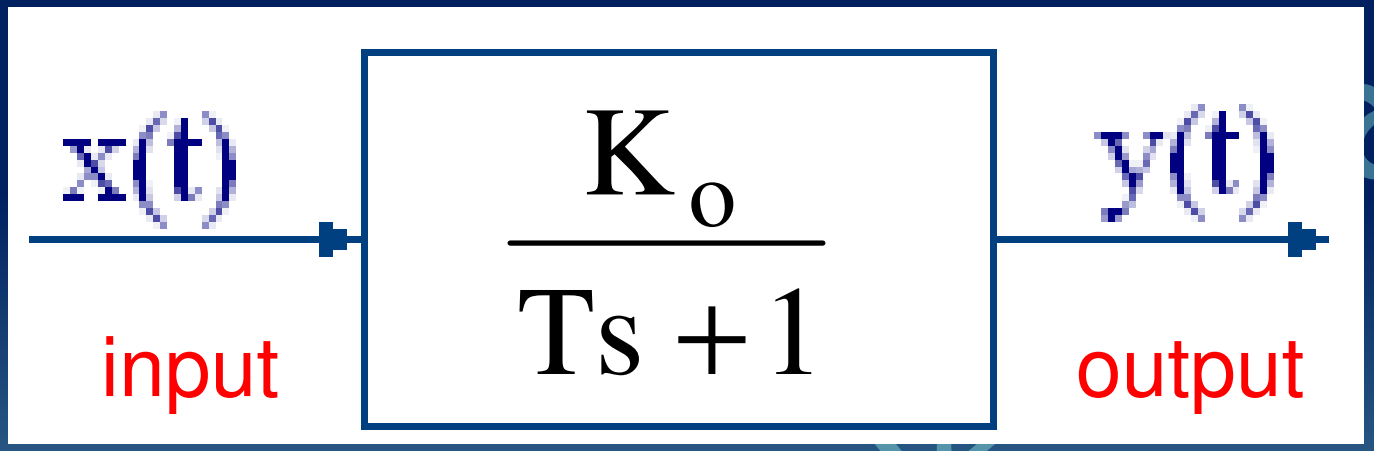
Sistemas de primeira ordem



entrada impulso unitário



Qual é a resposta ao impulso ?
(*impulse response*)

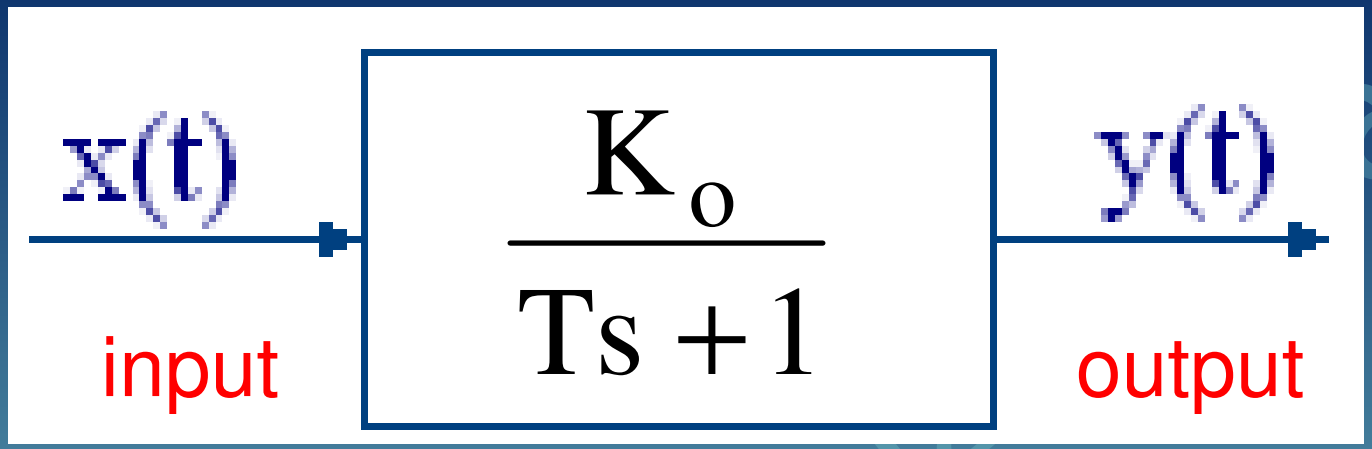


Sistemas de primeira ordem

para calcular fazemos:

$$Y(s) = \frac{K_o}{Ts+1} \cdot R(s)$$

$$Y(s) = \frac{K_o}{(Ts+1)} \cdot 1 = \frac{K_o}{(Ts+1)}$$



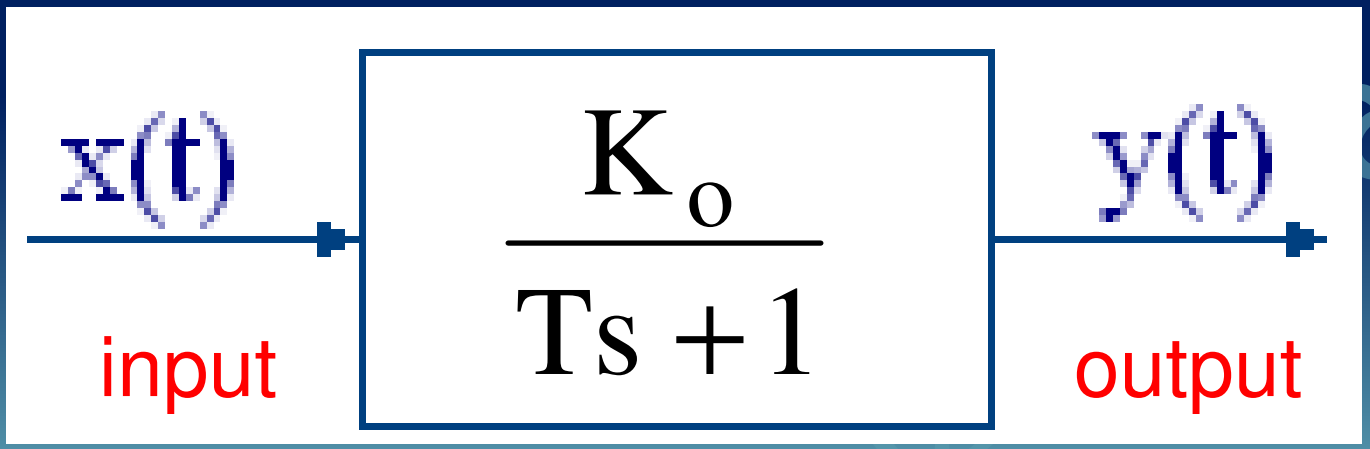
Sistemas de primeira ordem

$$y(t) = \mathcal{L}^{-1}[Y(s)]$$

logo, a resposta ao impulso unitário é:

$$y(t) = \frac{K_o}{T} \cdot e^{-t/T}, \qquad t > 0$$

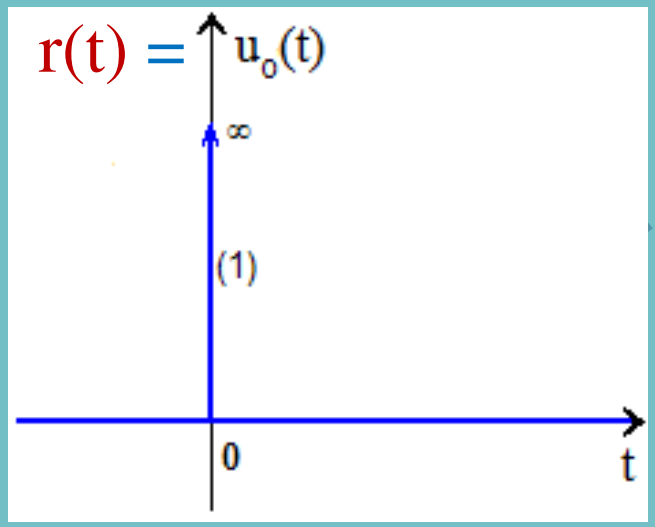
Análise no domínio do tempo - Sistemas de 1ª ordem



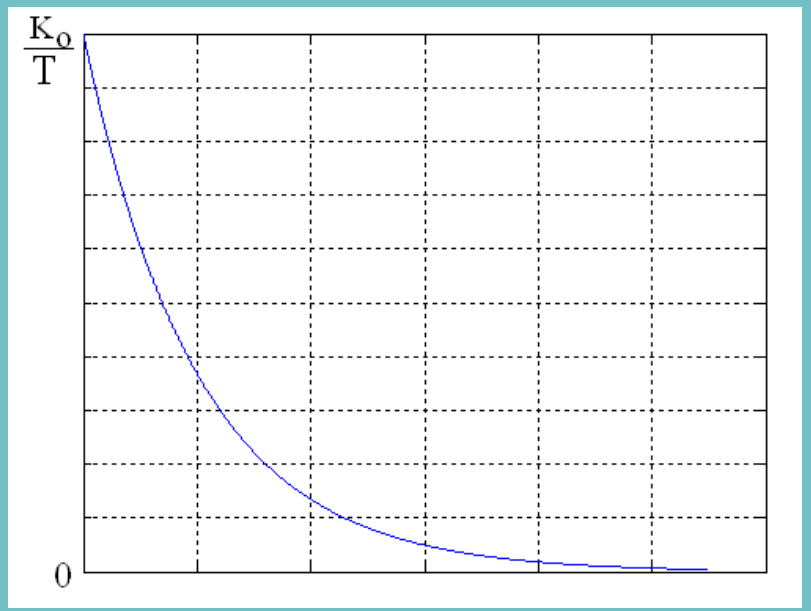
Sistemas de primeira ordem

a resposta ao impulso unitário é:

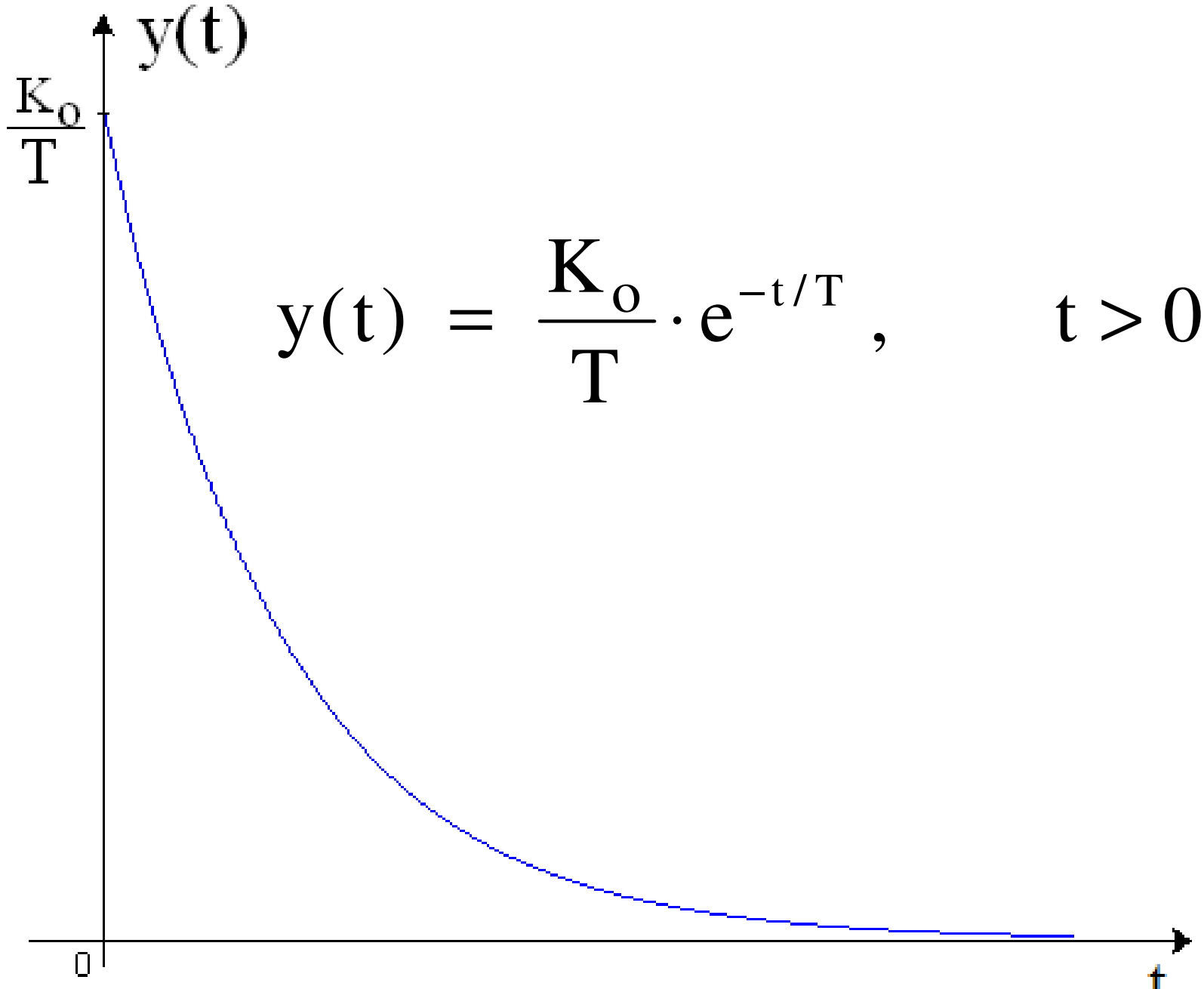
$$y(t) = \frac{K_o}{T} \cdot e^{-t/T}, \qquad t > 0$$



entrada impulso unitário



a resposta ao impulso unitário é:



$$y(t) = \frac{K_o}{T} \cdot e^{-t/T}, \qquad t > 0$$

Observe que, para a resposta ao impulso:

$$y(t) = \frac{K_o}{T} \cdot e^{-t/T}, \qquad t > 0$$

$$\text{Se } t = T \Rightarrow y(T) = (K_o / T) \cdot e^{-1} = 0,368 \cdot (K_o / T)$$

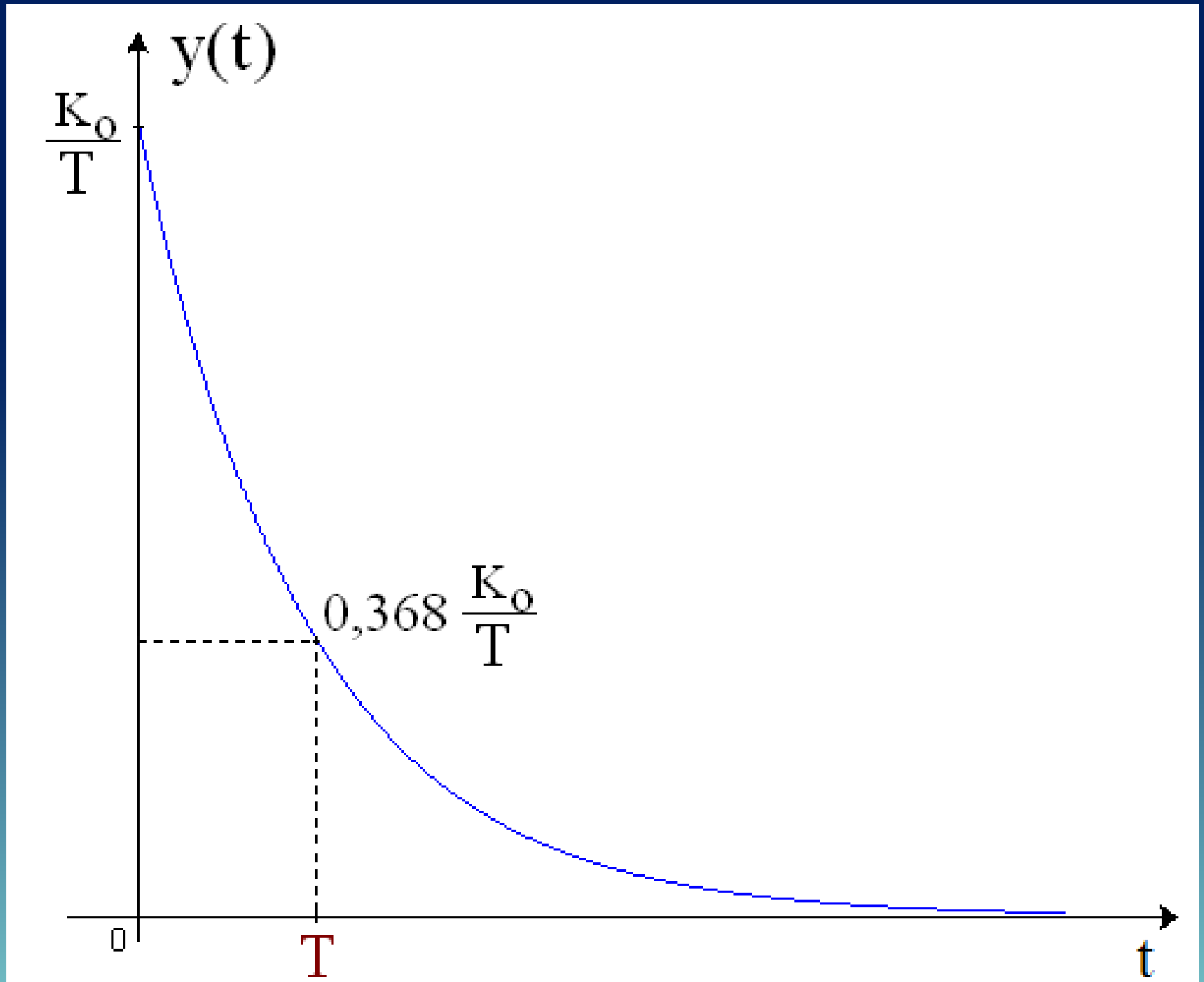
$$\text{Se } t = 2T \Rightarrow y(2T) = (K_o / T) \cdot e^{-2} = 0,135 \cdot (K_o / T)$$

$$\text{Se } t = 3T \Rightarrow y(3T) = (K_o / T) \cdot e^{-3} = 0,05 \cdot (K_o / T)$$

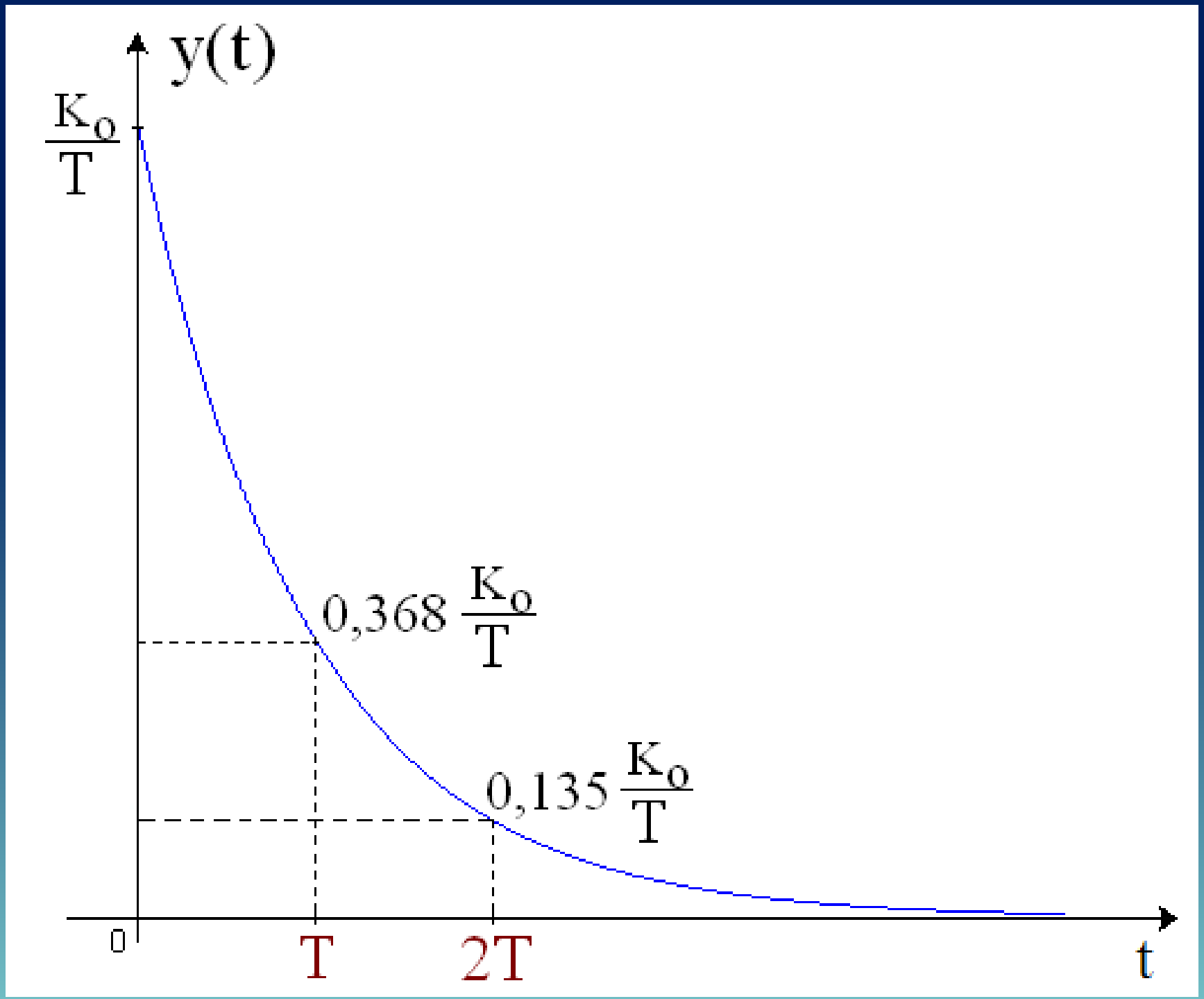
$$\text{Se } t = 4T \Rightarrow y(4T) = (K_o / T) \cdot e^{-4} = 0,02 \cdot (K_o / T)$$

$$\text{Se } t = 5T \Rightarrow y(5T) = (K_o / T) \cdot e^{-5} = 0,007 \cdot (K_o / T)$$

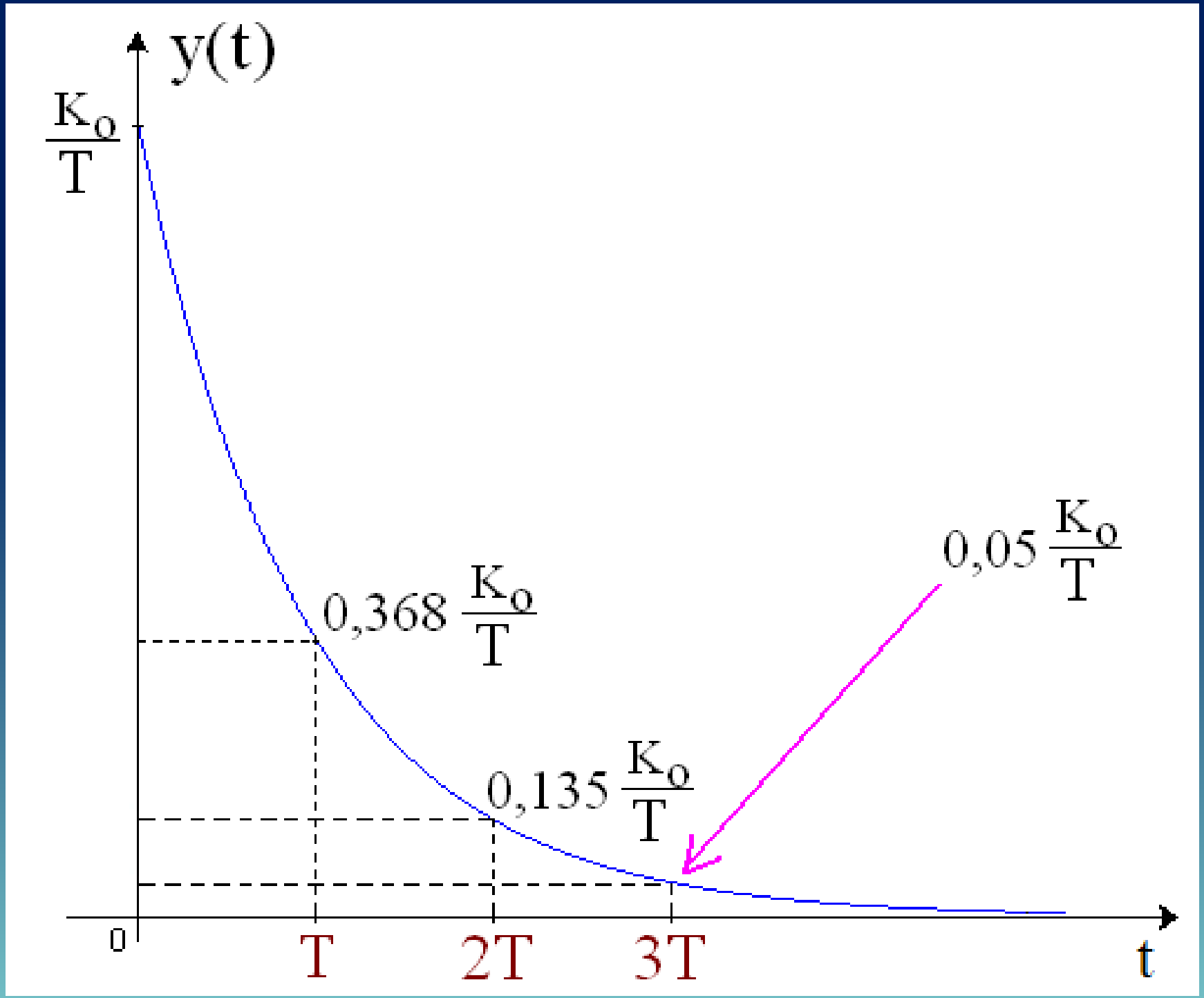
Análise no domínio do tempo - Sistemas de 1ª ordem



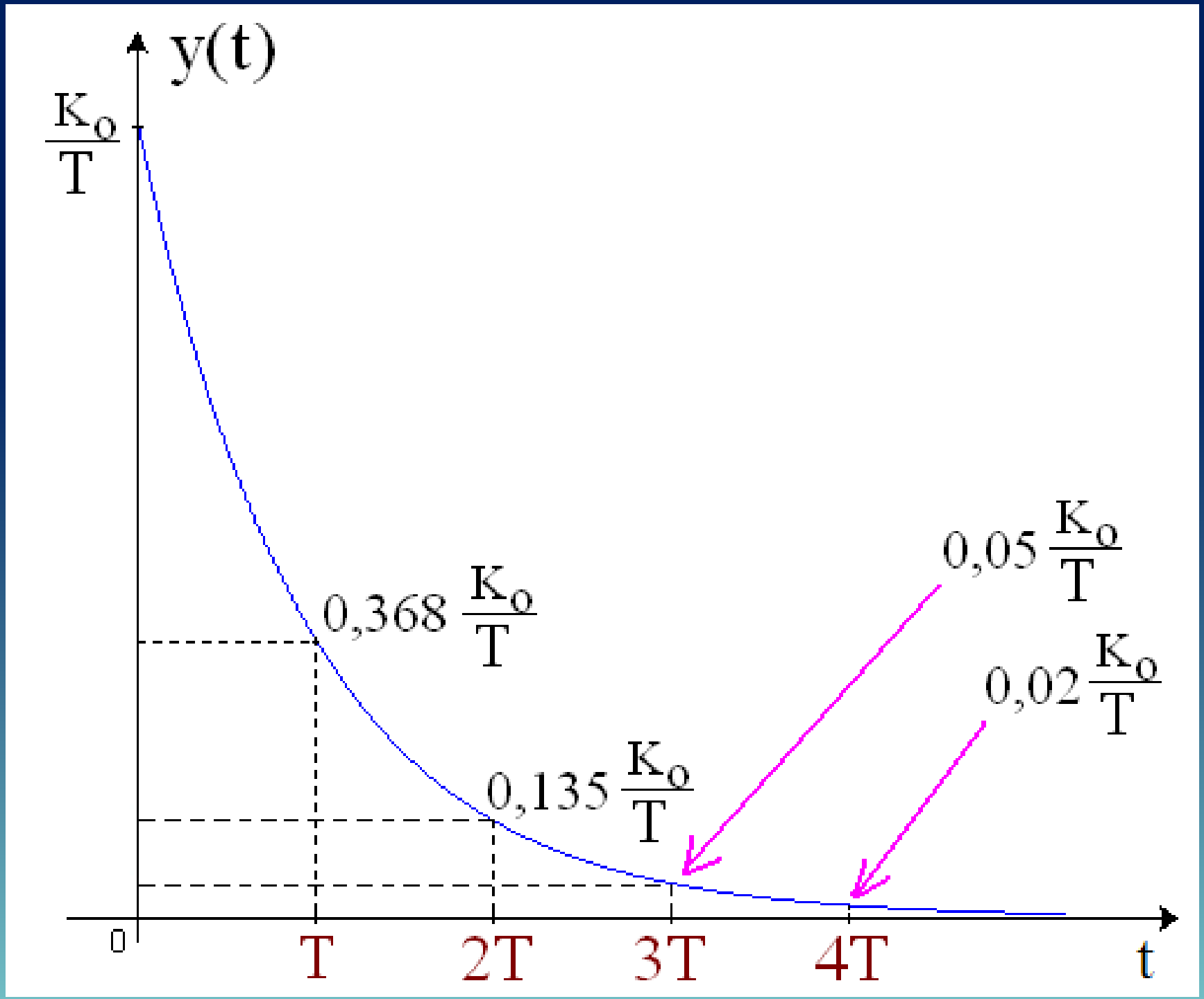
Análise no domínio do tempo - Sistemas de 1ª ordem



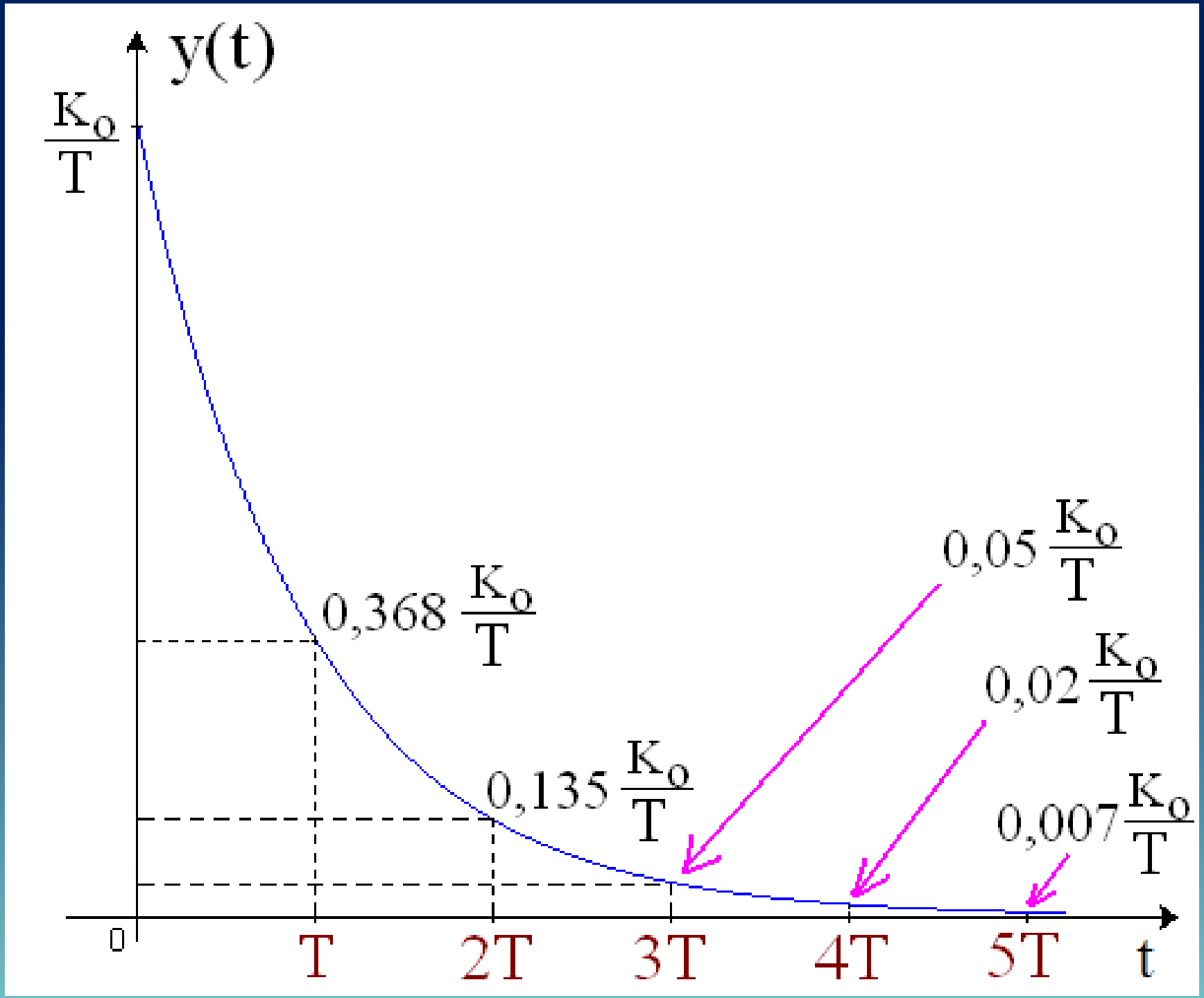
Análise no domínio do tempo - Sistemas de 1ª ordem



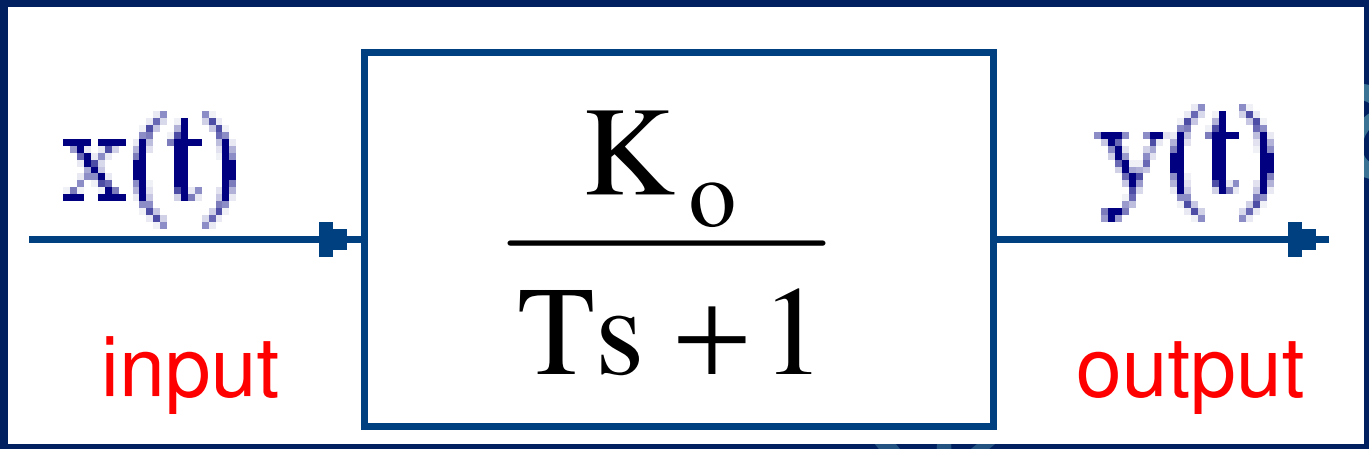
Análise no domínio do tempo - Sistemas de 1ª ordem



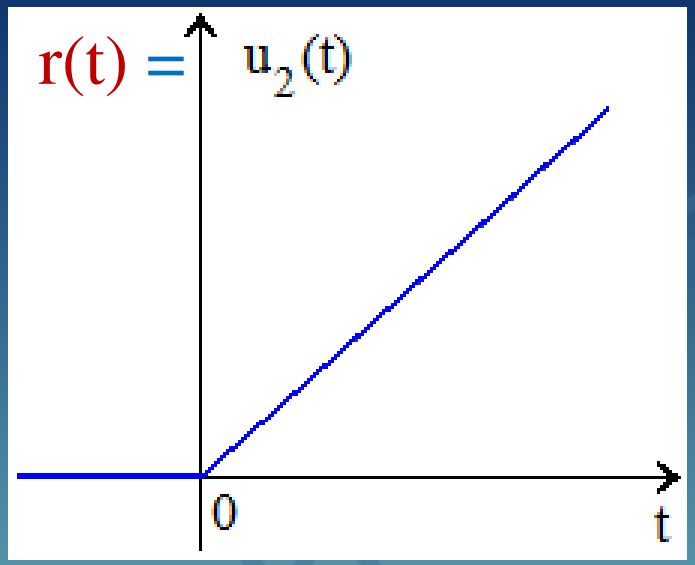
Análise no domínio do tempo - Sistemas de 1ª ordem



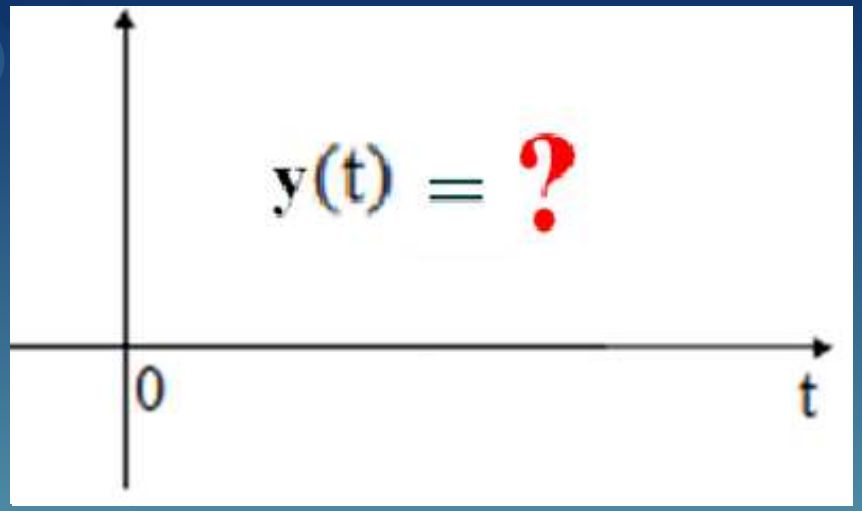
Análise no domínio do tempo - Sistemas de 1ª ordem



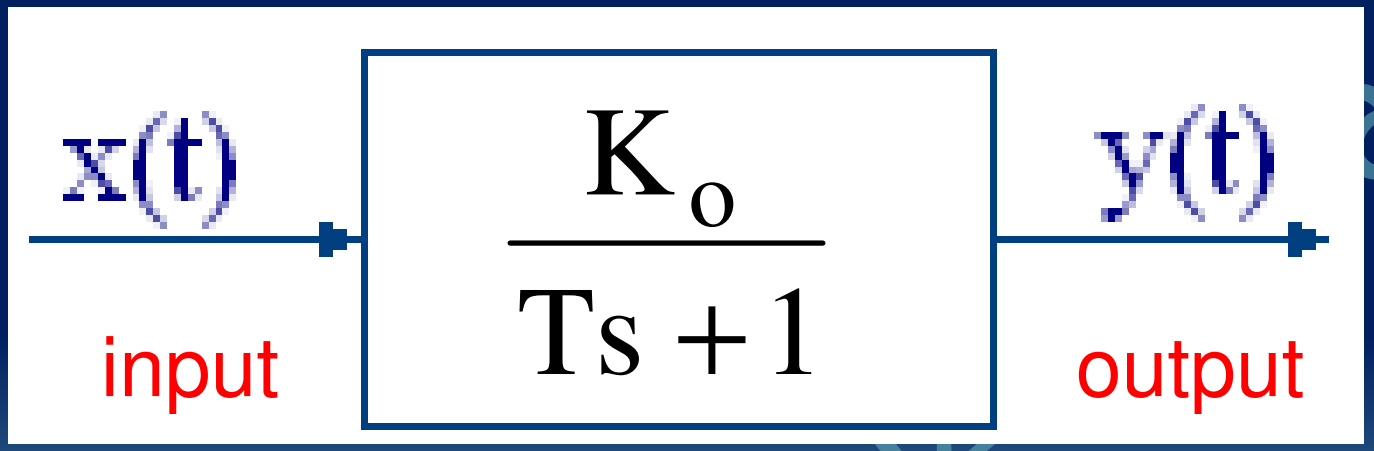
Sistemas de primeira ordem



entrada rampa unitária



Qual é a resposta à rampa ?
(*ramp response*)

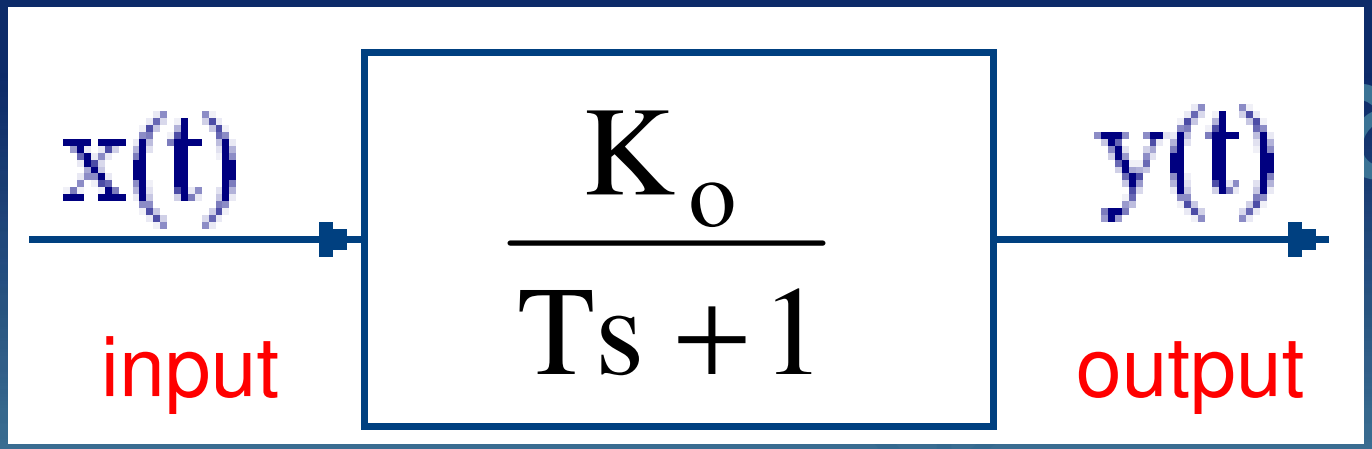


Sistemas de primeira ordem

para calcular fazemos:

$$Y(s) = \frac{K_o}{Ts+1} \cdot R(s)$$

$$Y(s) = \frac{K_o}{(Ts+1)} \cdot \frac{1}{s^2} = \frac{K_o}{s^2} - \frac{K_o T}{s} + \frac{K_o T^2}{(Ts+1)}$$

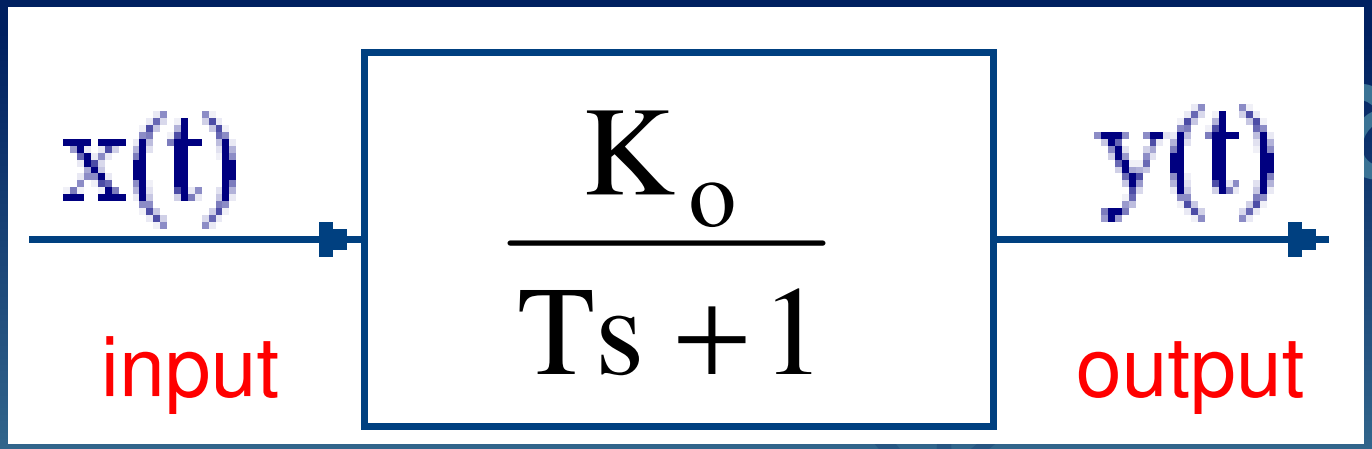


Sistemas de primeira ordem

$$y(t) = \mathcal{L}^{-1}[Y(s)]$$

logo, a resposta à rampa unitária é:

$$y(t) = K_o(t - T + T \cdot e^{-t/T}), \qquad t > 0$$

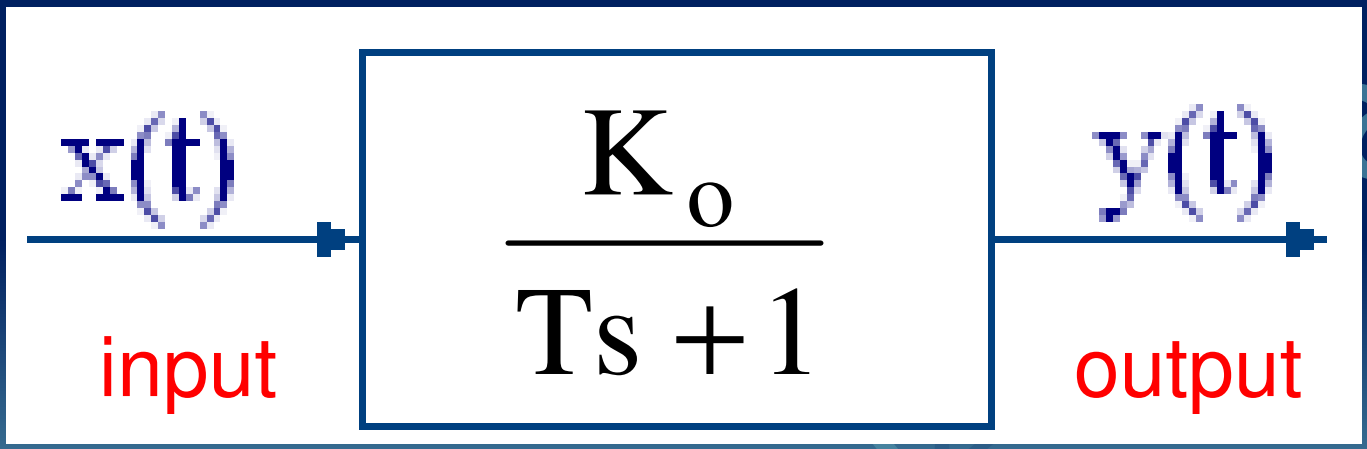


Sistemas de primeira ordem

Se $K_o = 1$, a resposta à rampa unitária é:

$$y(t) = t - T + T \cdot e^{-t/T}, \qquad t > 0$$

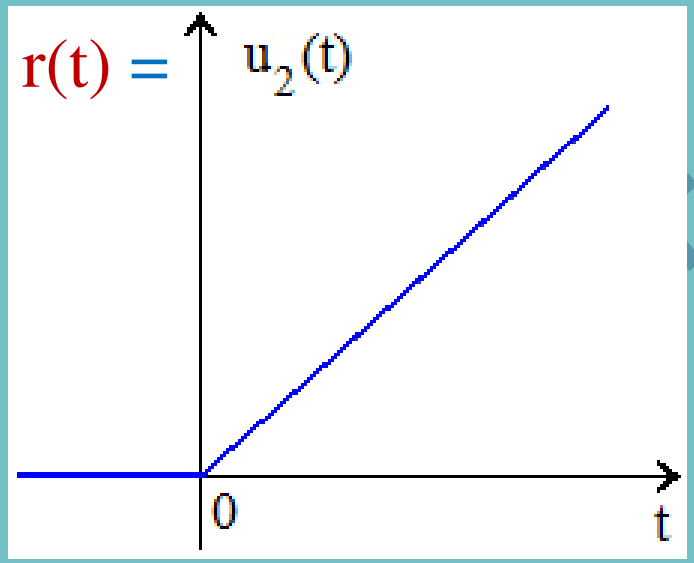
Análise no domínio do tempo - Sistemas de 1ª ordem



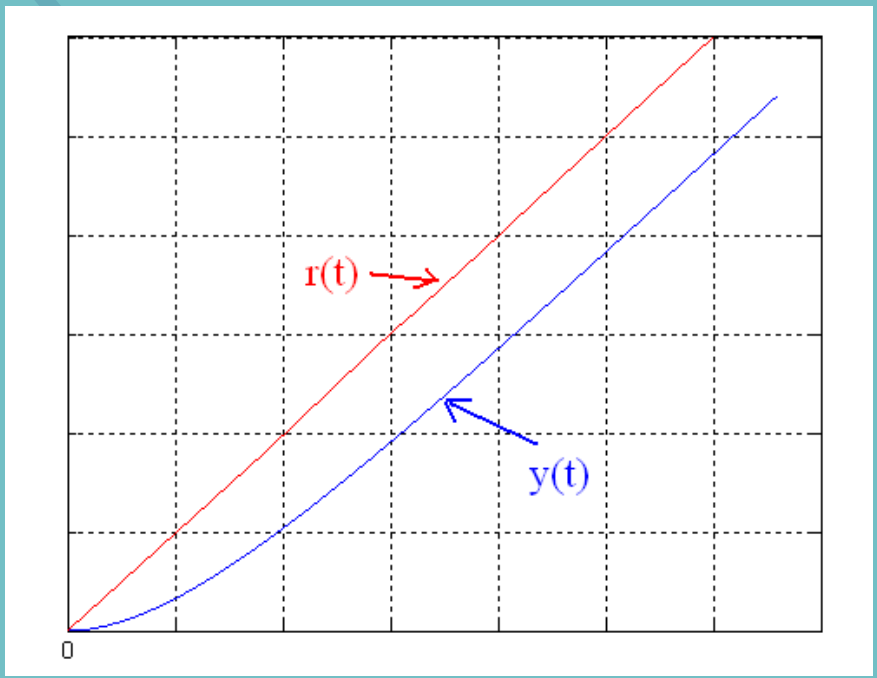
Sistemas de primeira ordem

resposta à rampa unitária para $K_o = 1$:

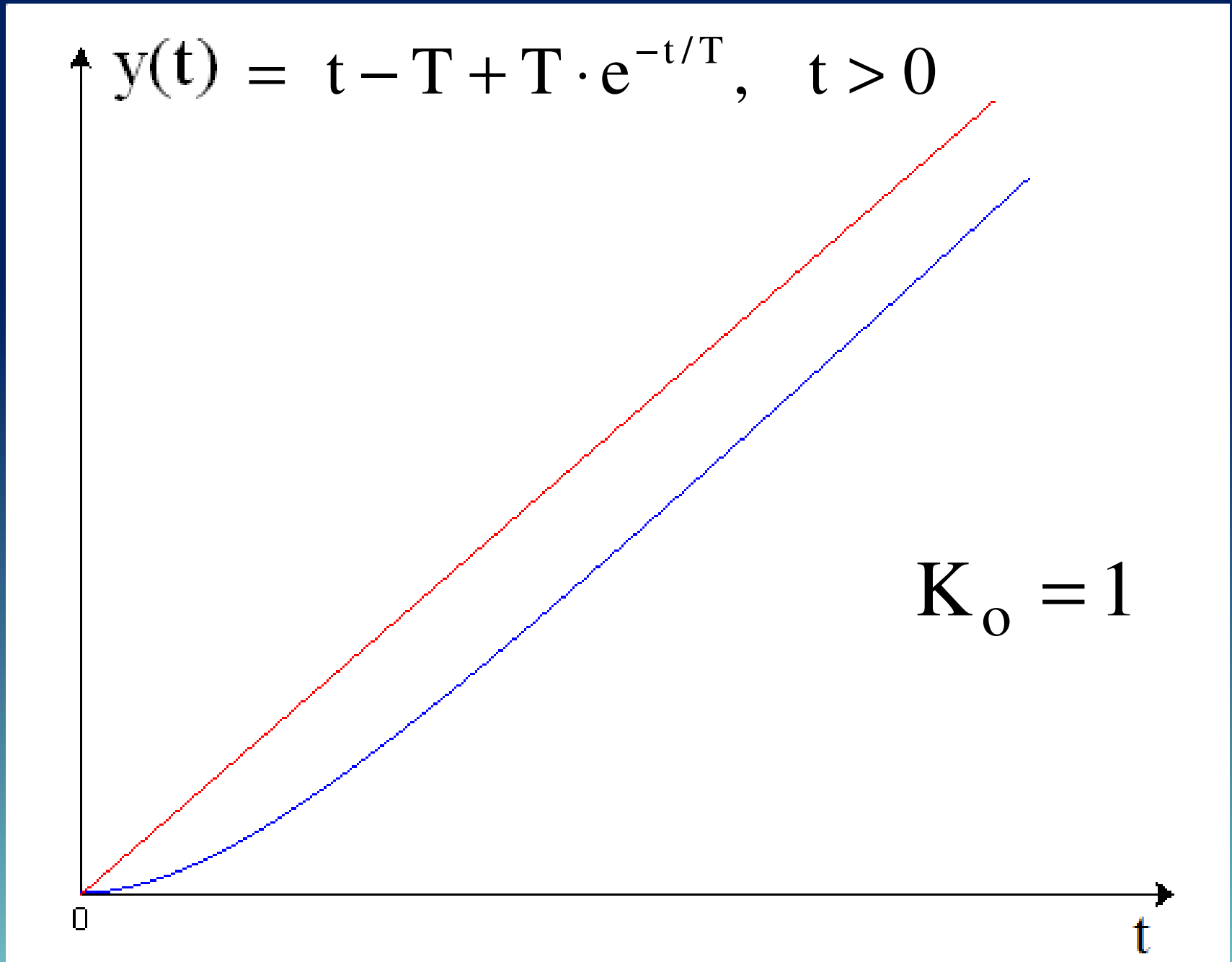
$$y(t) = t - T + T \cdot e^{-t/T}, \quad t > 0$$



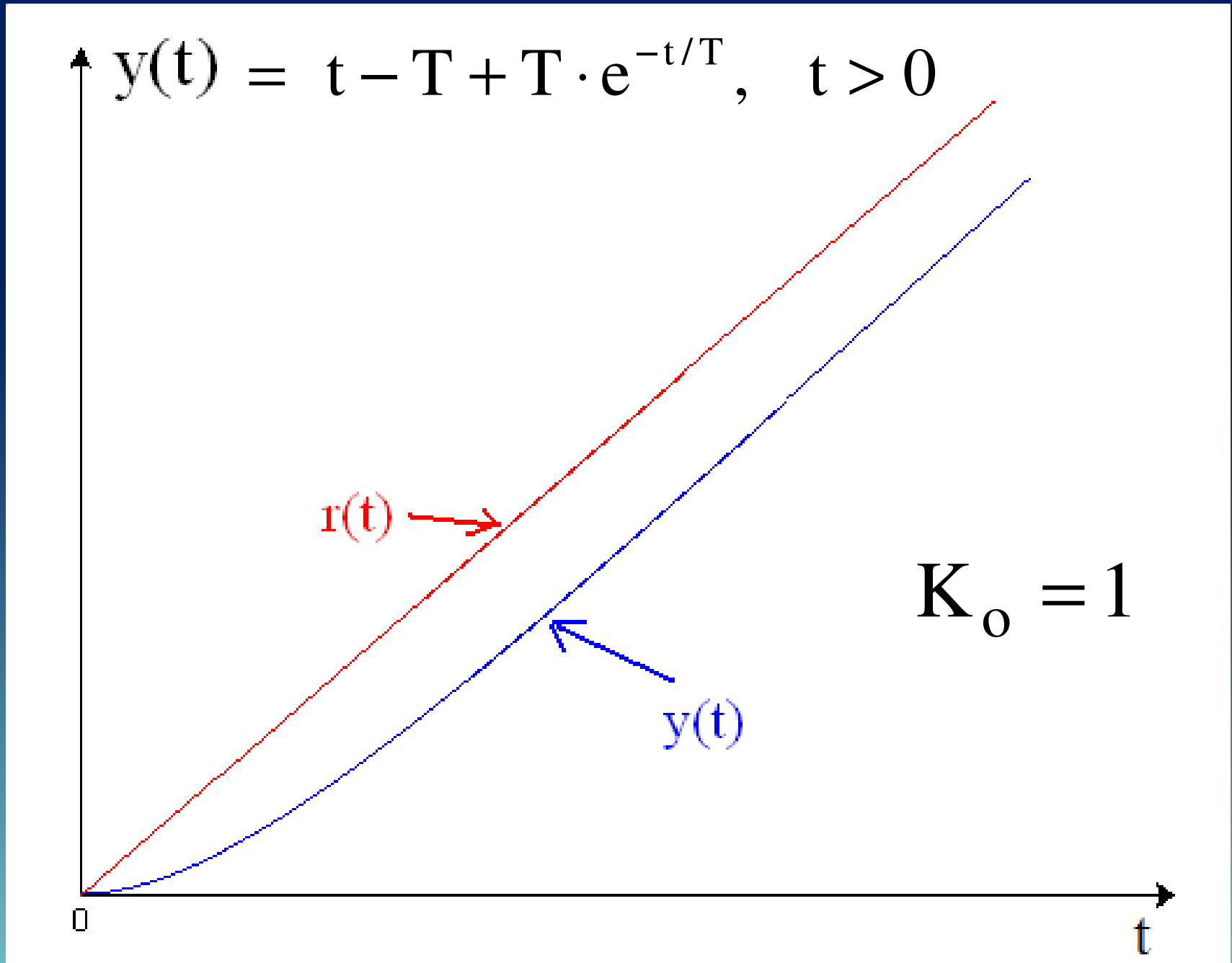
entrada rampa unitária



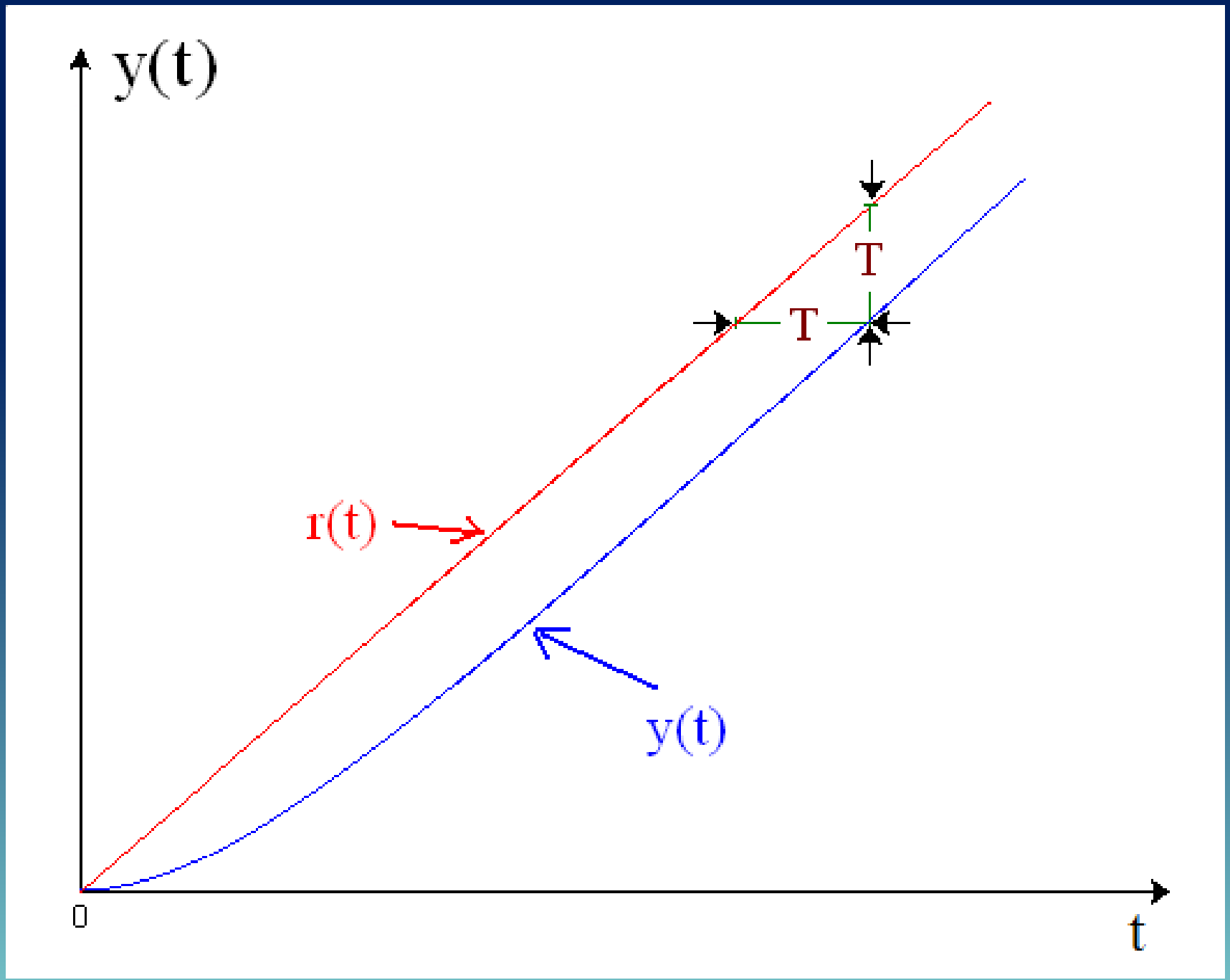
for the unit ramp input, the output is:

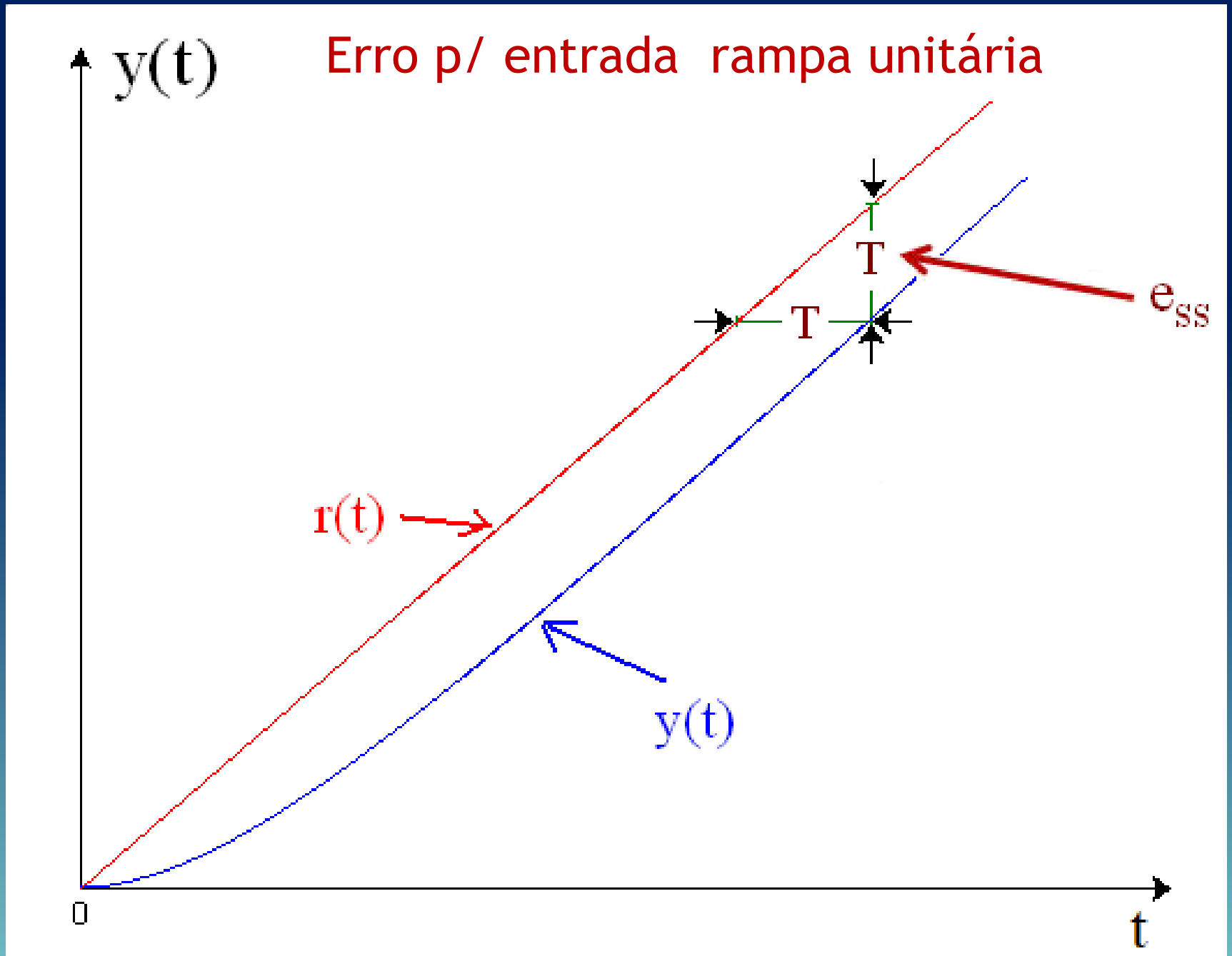


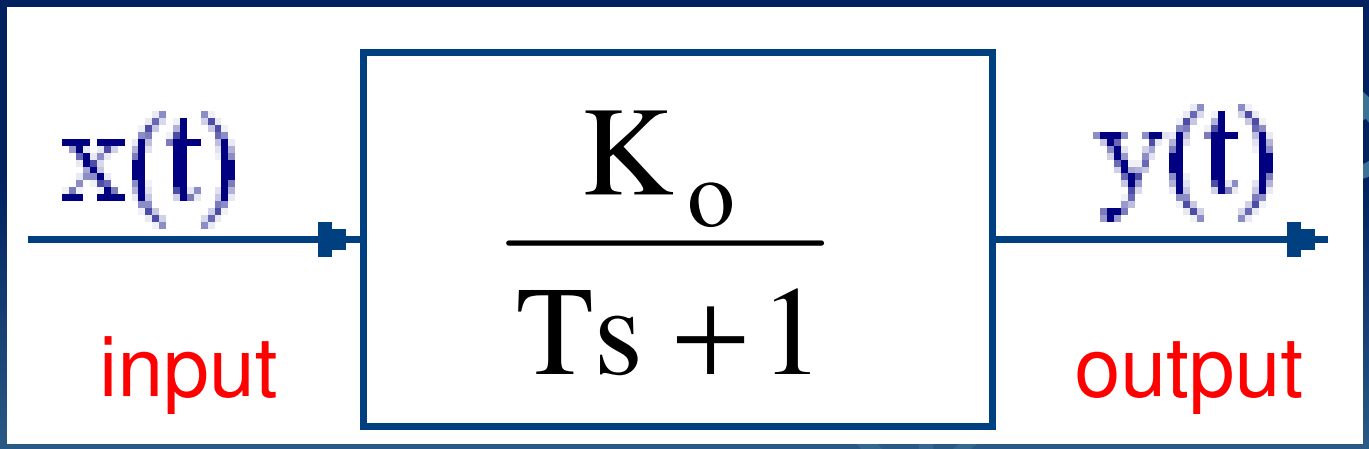
for the unit ramp input, the output is:



Análise no domínio do tempo - Sistemas de 1ª ordem




Erro p/ entrada rampa unitária
y(t)
T
T
e_{ss}
r(t)
y(t)
0
t

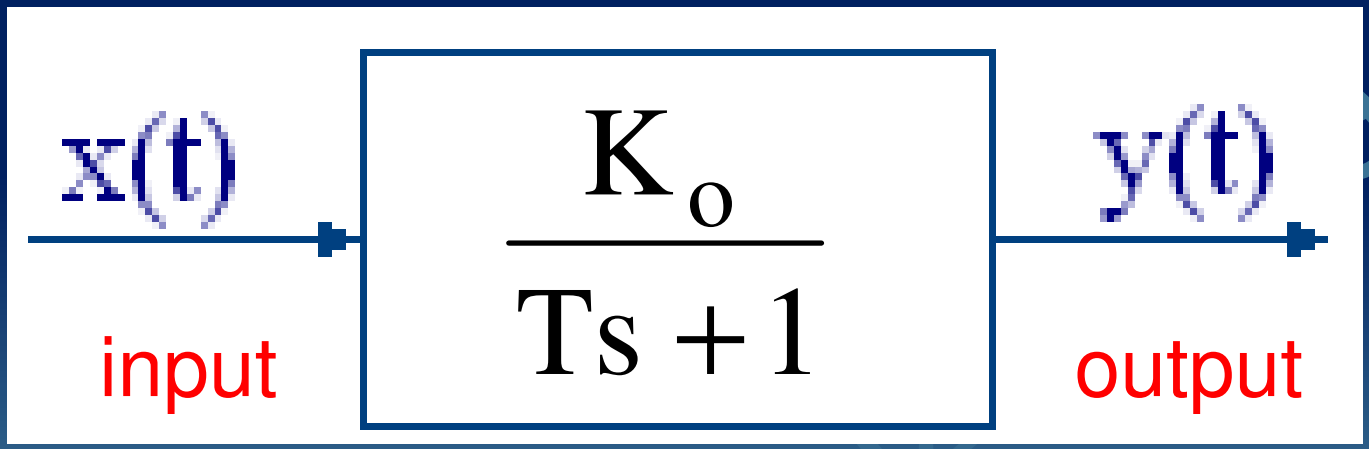


Sistemas de primeira ordem

Erro para entrada rampa unitária, $K_o = 1$:

$$E(s) = \frac{1}{s^2} - \frac{1}{Ts+1} \cdot \frac{1}{s^2} = \frac{1}{s^2} \cdot \left(1 - \frac{1}{Ts+1}\right) =$$

$$= \frac{1}{s^2} \cdot \frac{(Ts+1-1)}{Ts+1} = \frac{1}{s^2} \cdot \frac{Ts}{Ts+1}$$



Sistemas de primeira ordem

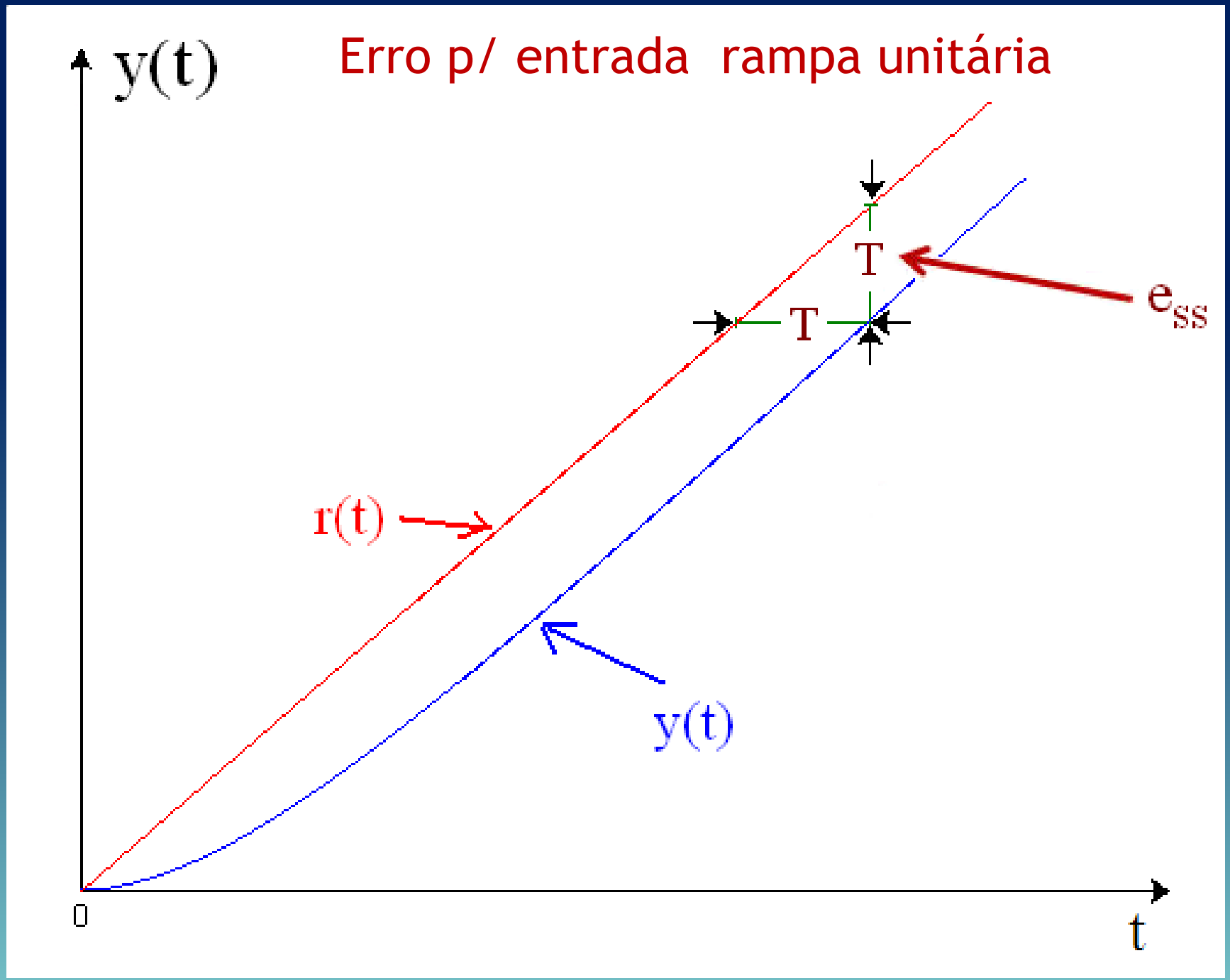
Erro em estado estacionário:

$$e_{ss} = \lim_{s\to 0} s \cdot E(s) = \lim_{s\to 0} s \cdot \frac{1}{s^2} \cdot \frac{Ts}{Ts+1} =$$

$$= \lim_{s\to 0} \frac{T}{Ts+1} = T$$

$$\boxed{e_{ss} = T}$$

Análise no domínio do tempo - Sistemas de 1ª ordem

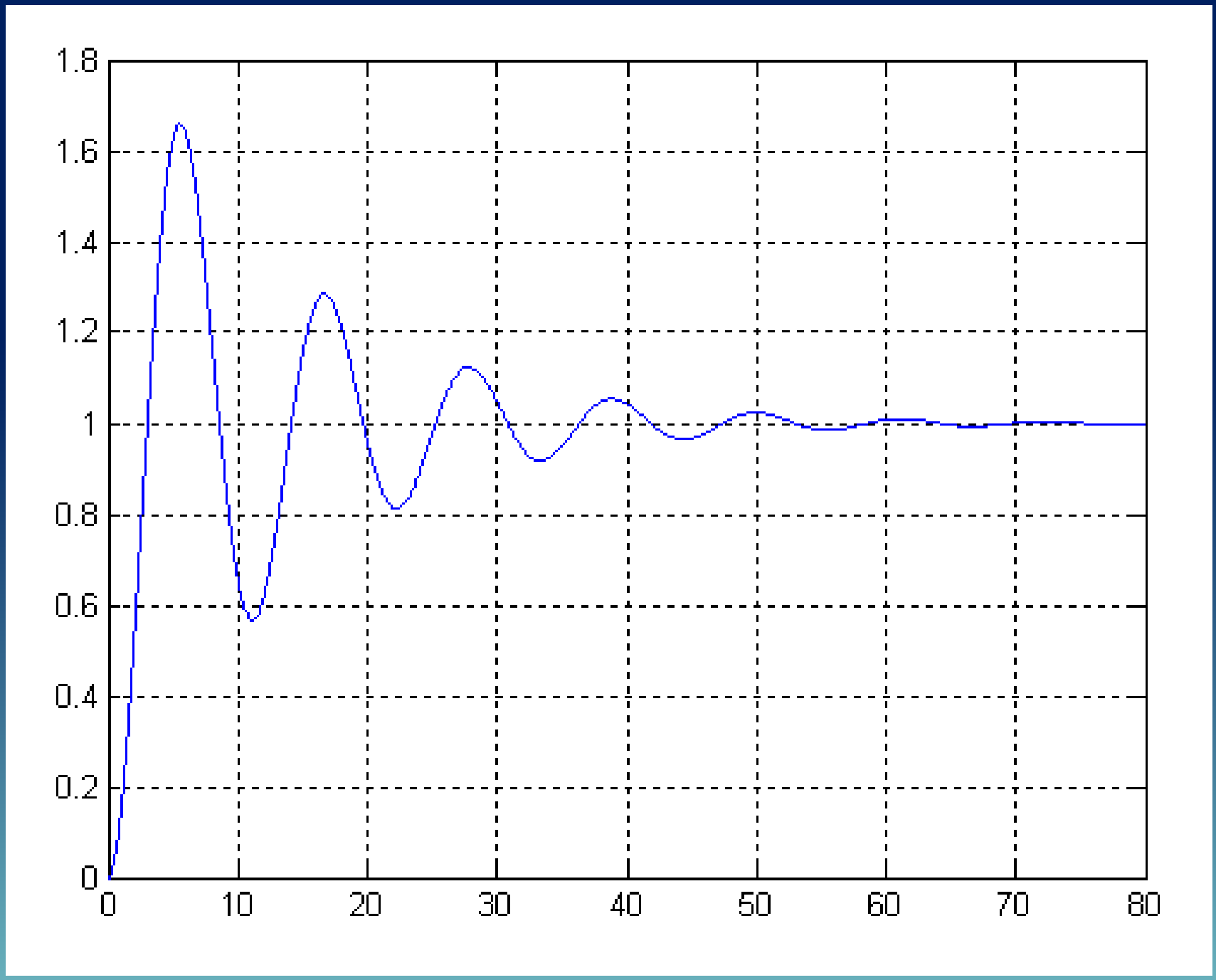


Departamento de
Engenharia Eletromecânica

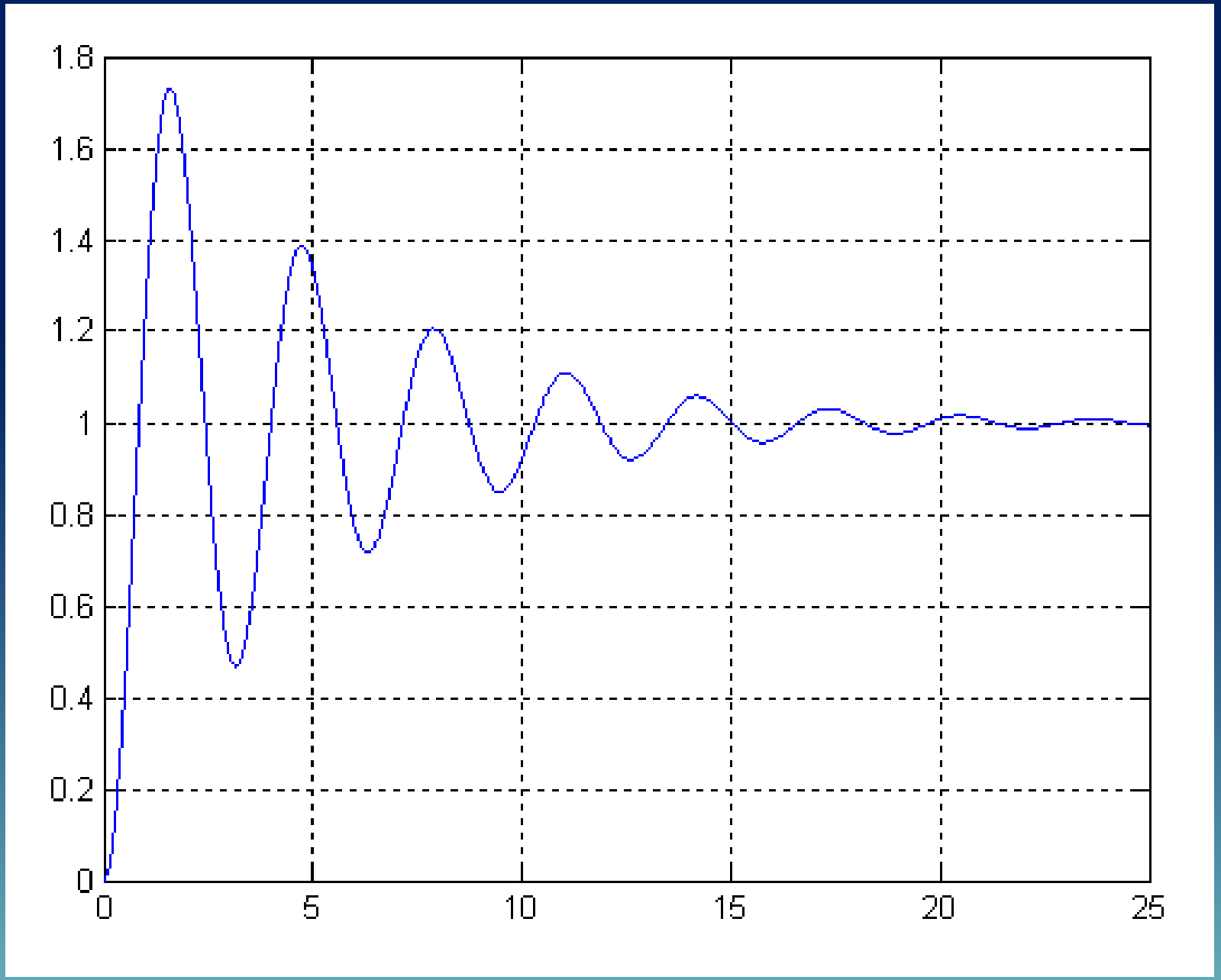
Obrigado!

Felippe de Souza
felippe@ubi.pt

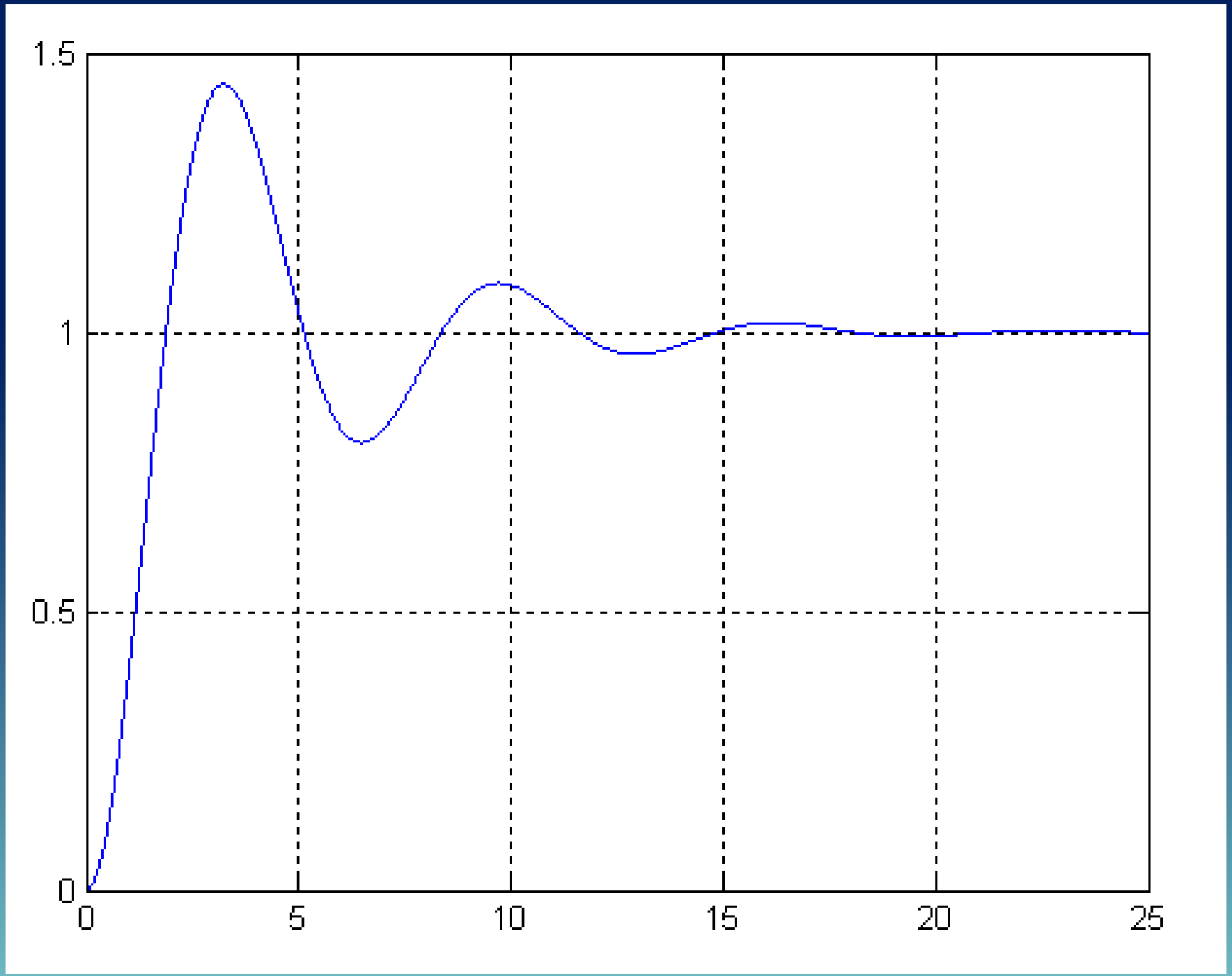
Análise no domínio do tempo



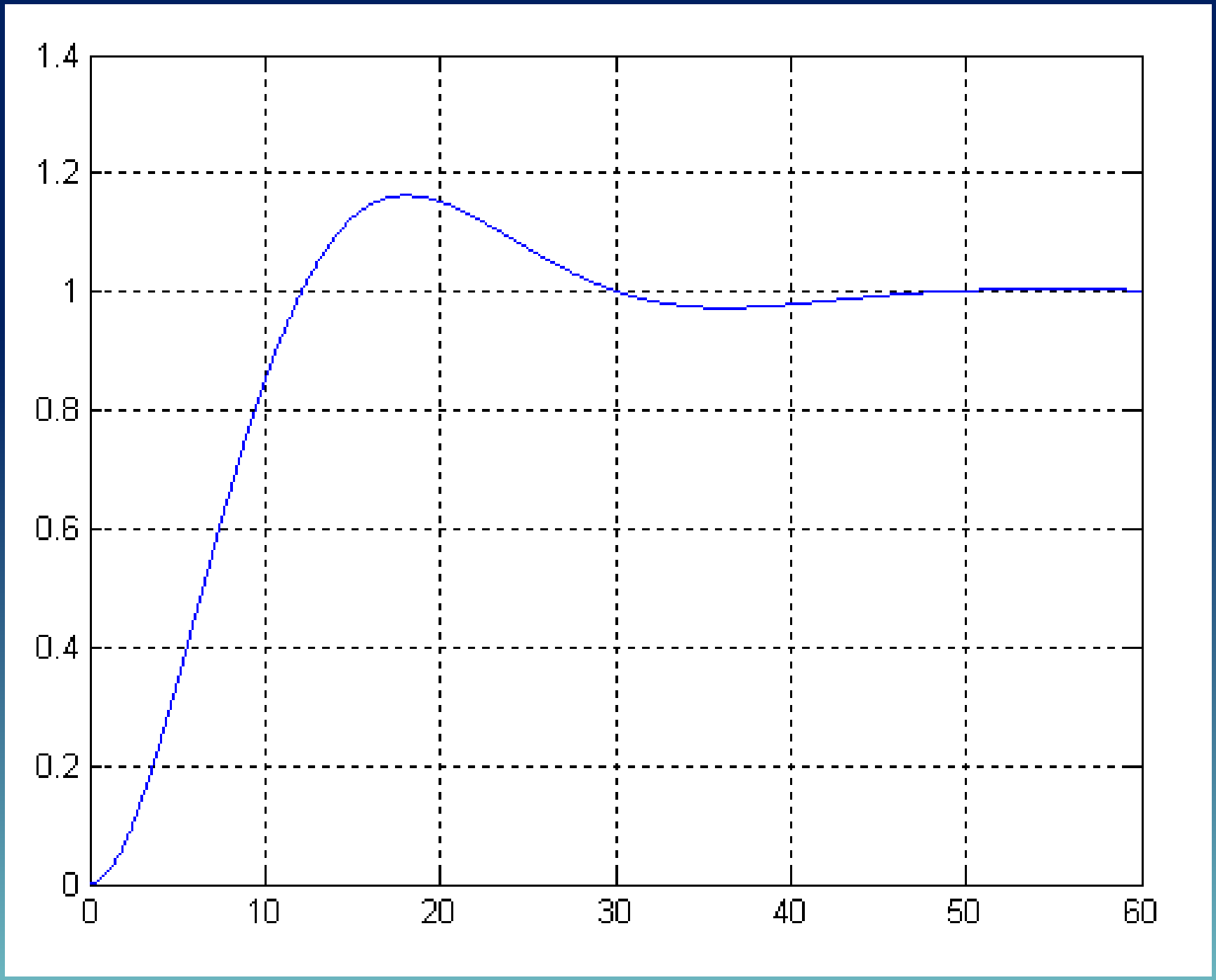
Análise no domínio do tempo



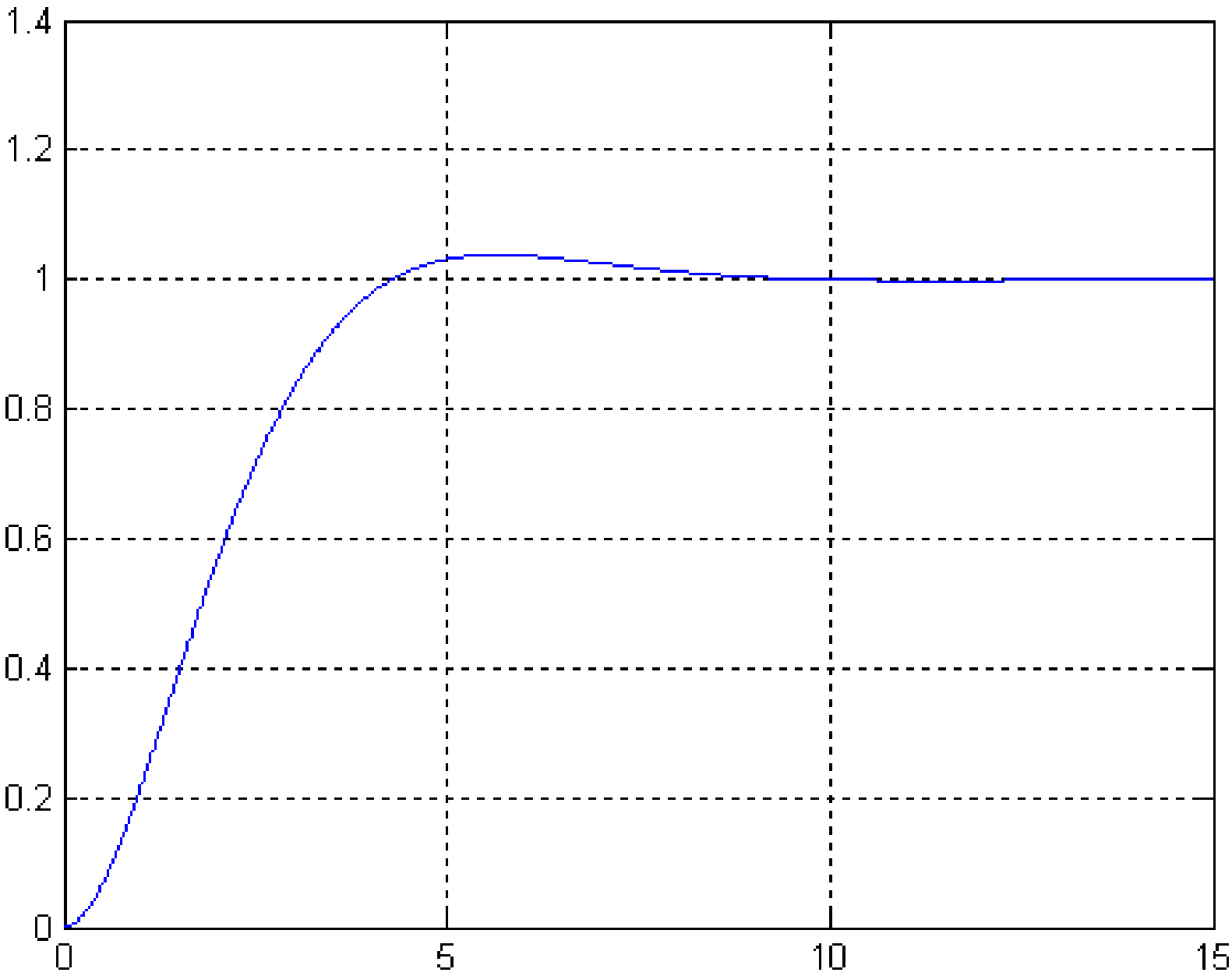
Análise no domínio do tempo



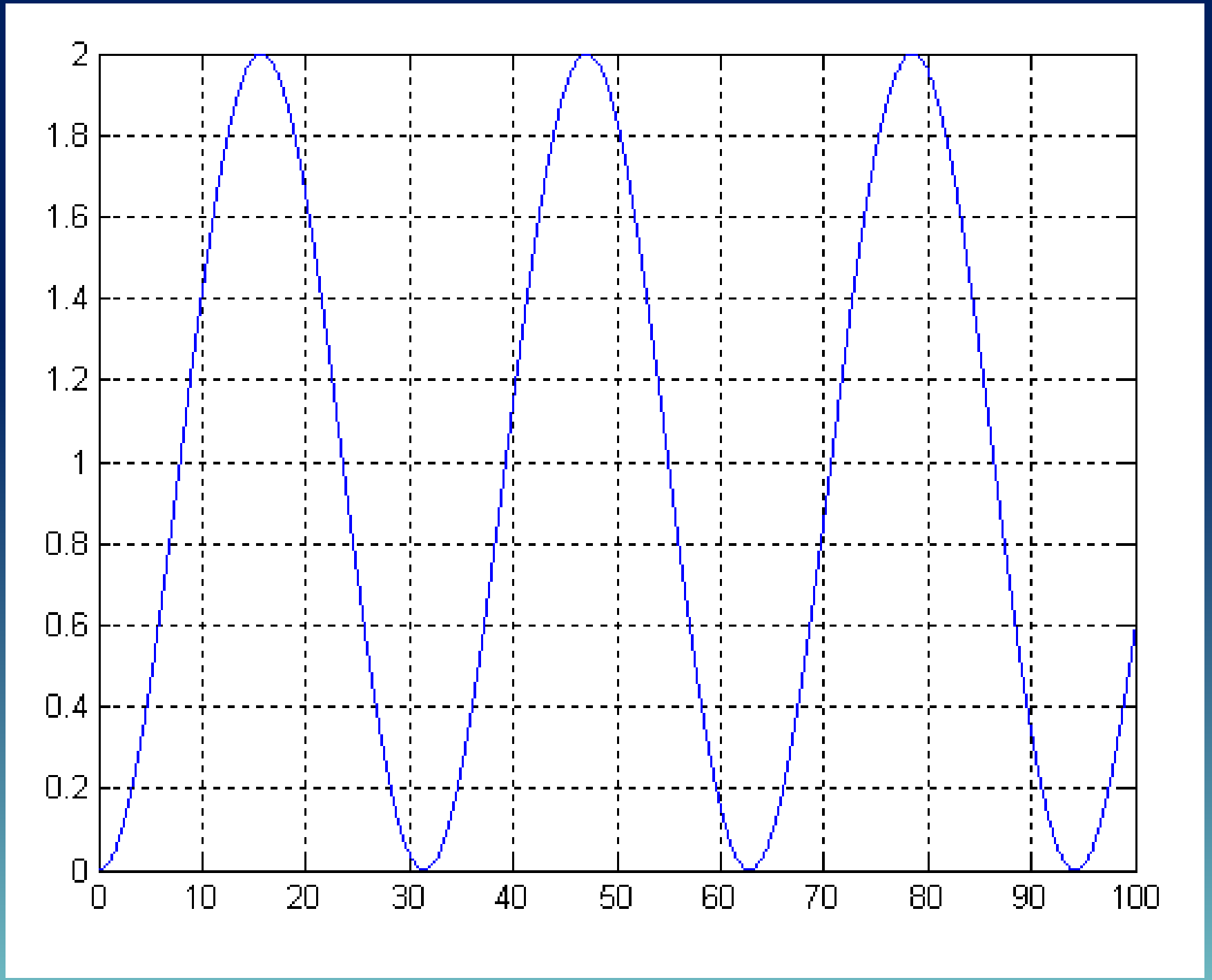
Análise no domínio do tempo



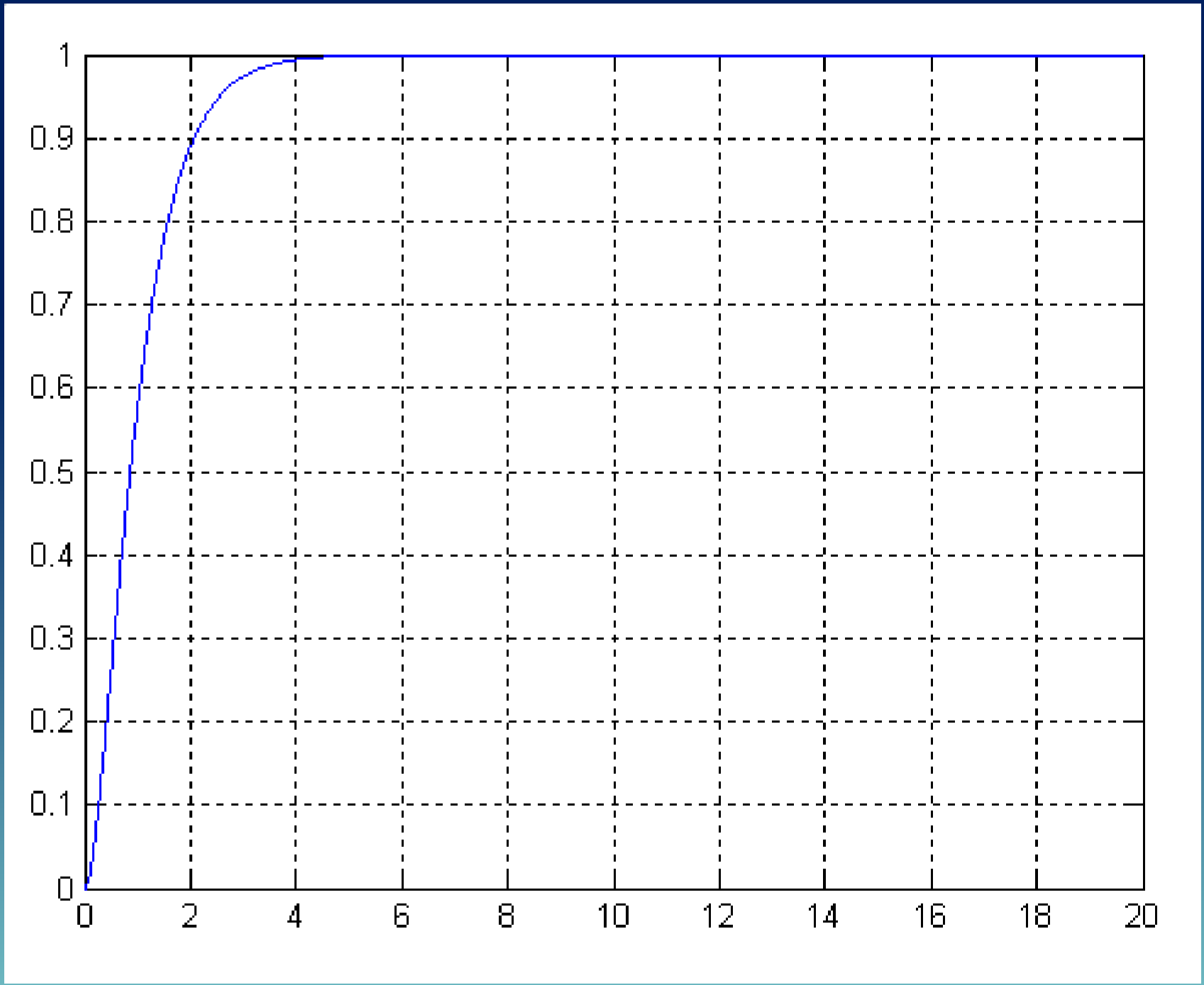
Análise no domínio do tempo



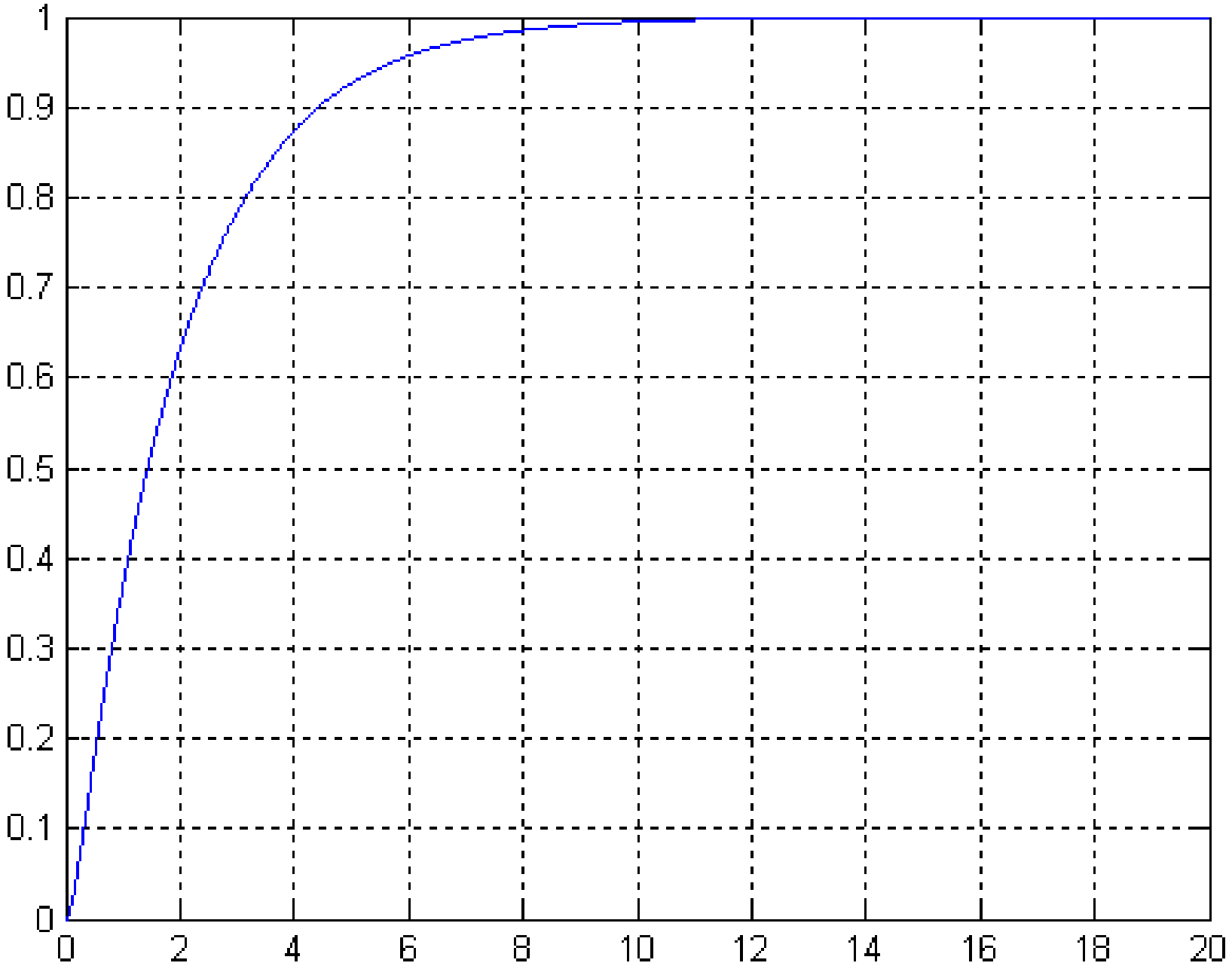
Análise no domínio do tempo



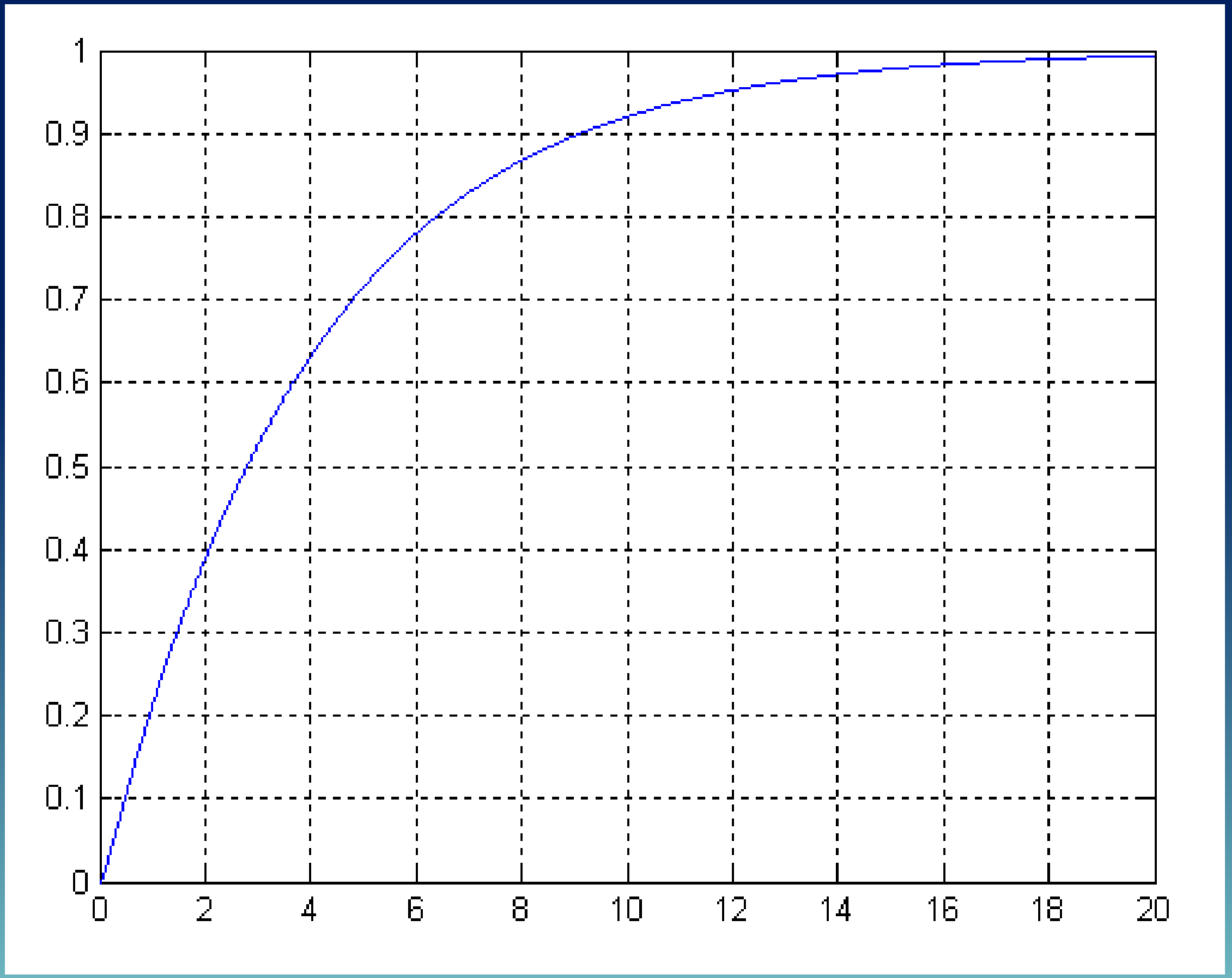
Análise no domínio do tempo



Análise no domínio do tempo

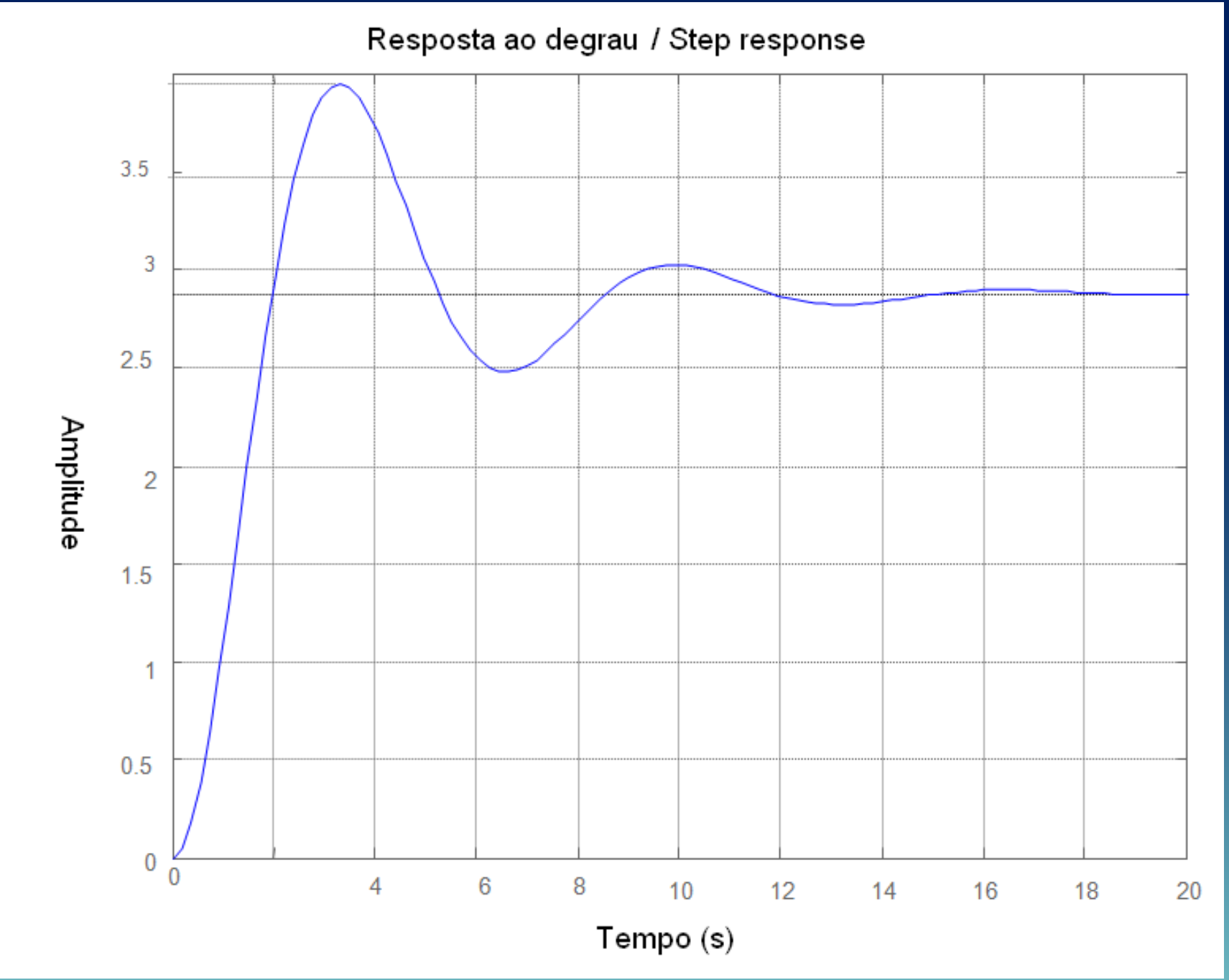


Análise no domínio do tempo



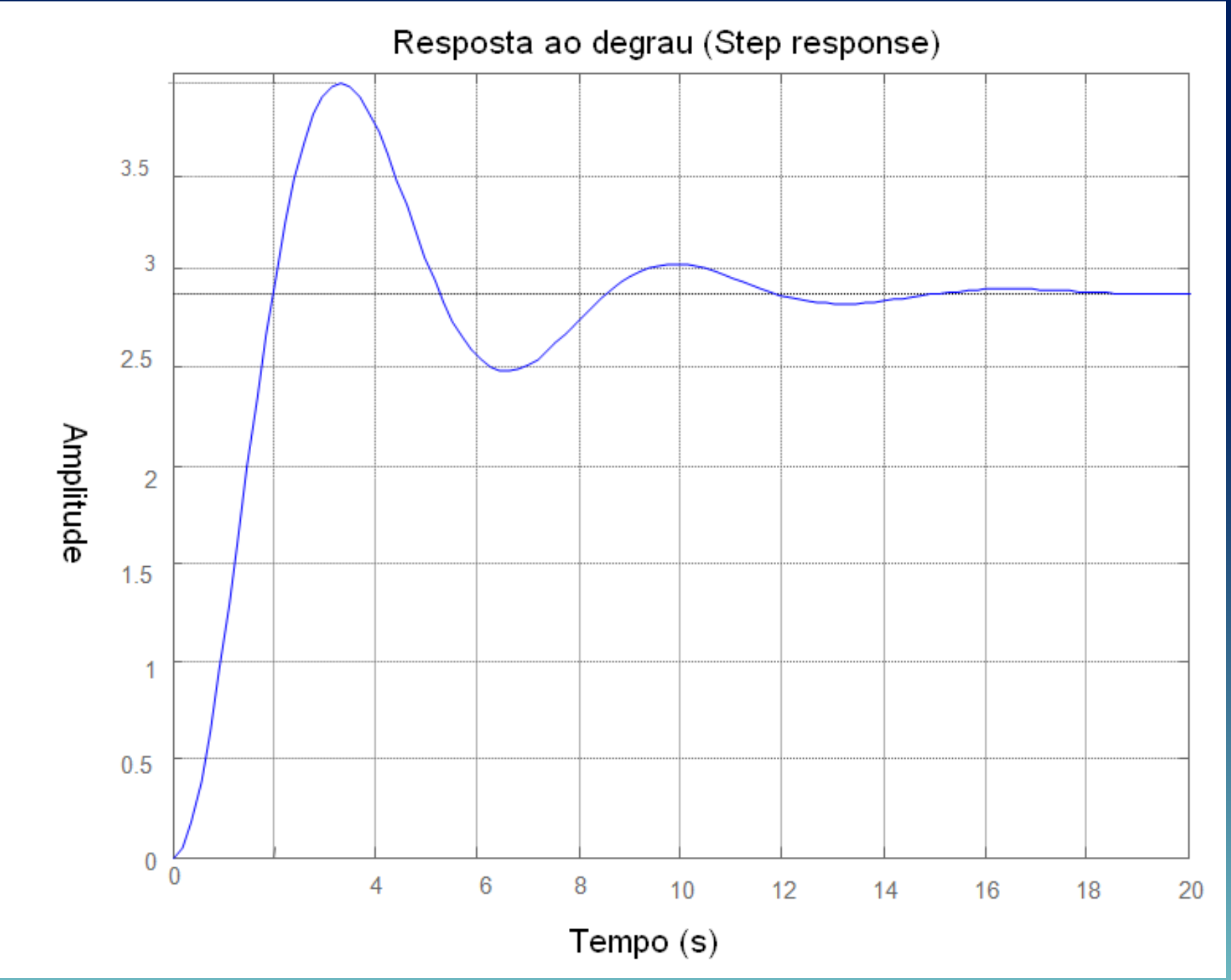
Análise no domínio do tempo

Análise no domínio do tempo

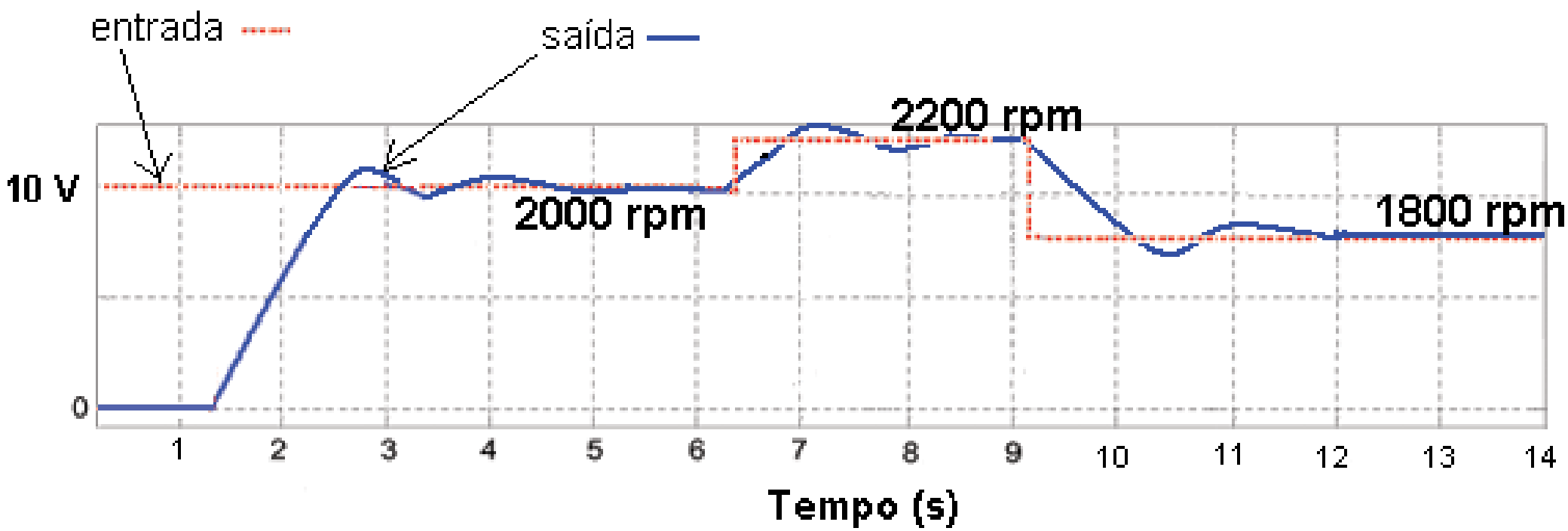


Análise no domínio do tempo

Análise no domínio do tempo

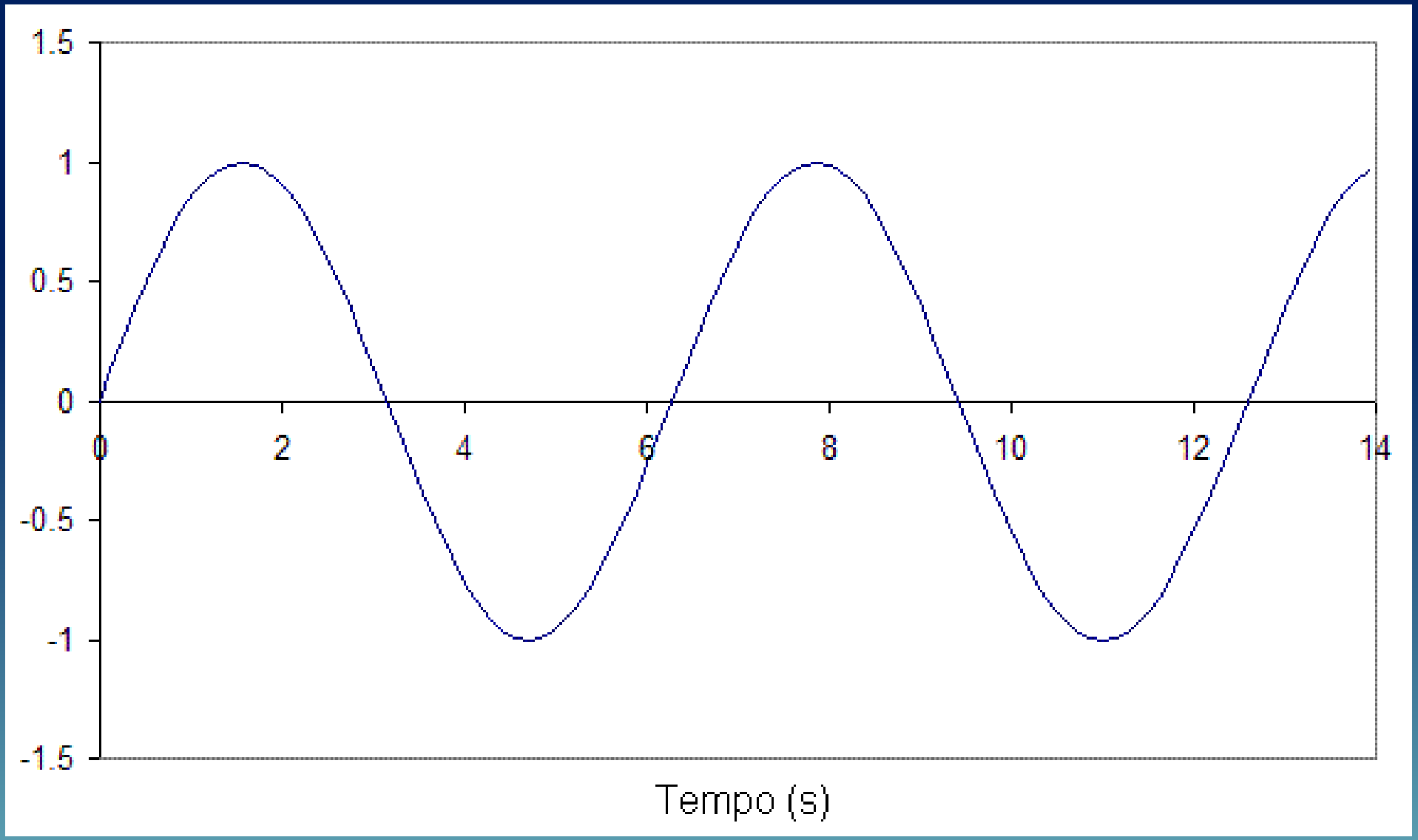


Análise no domínio do tempo

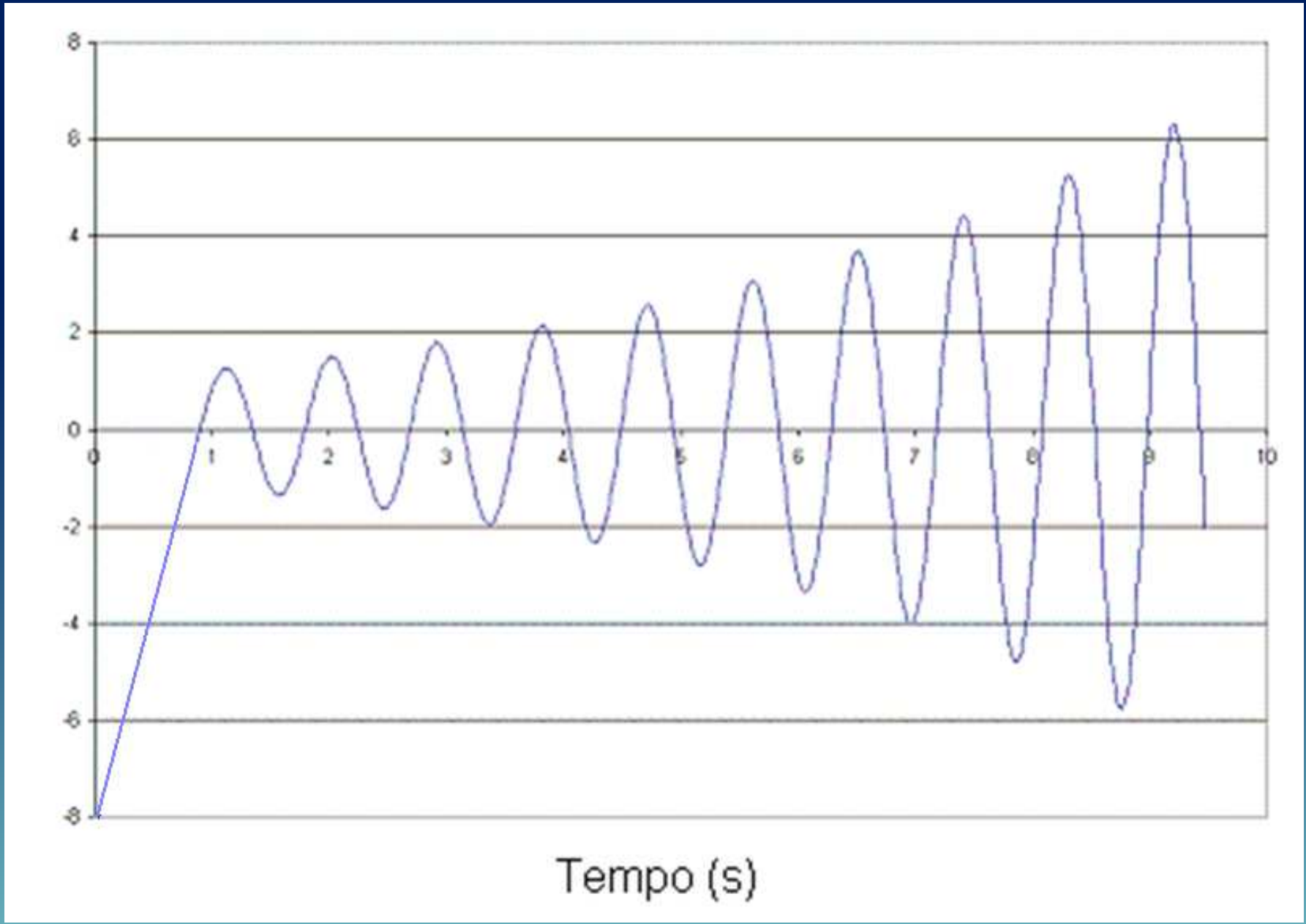


Análise no domínio do tempo

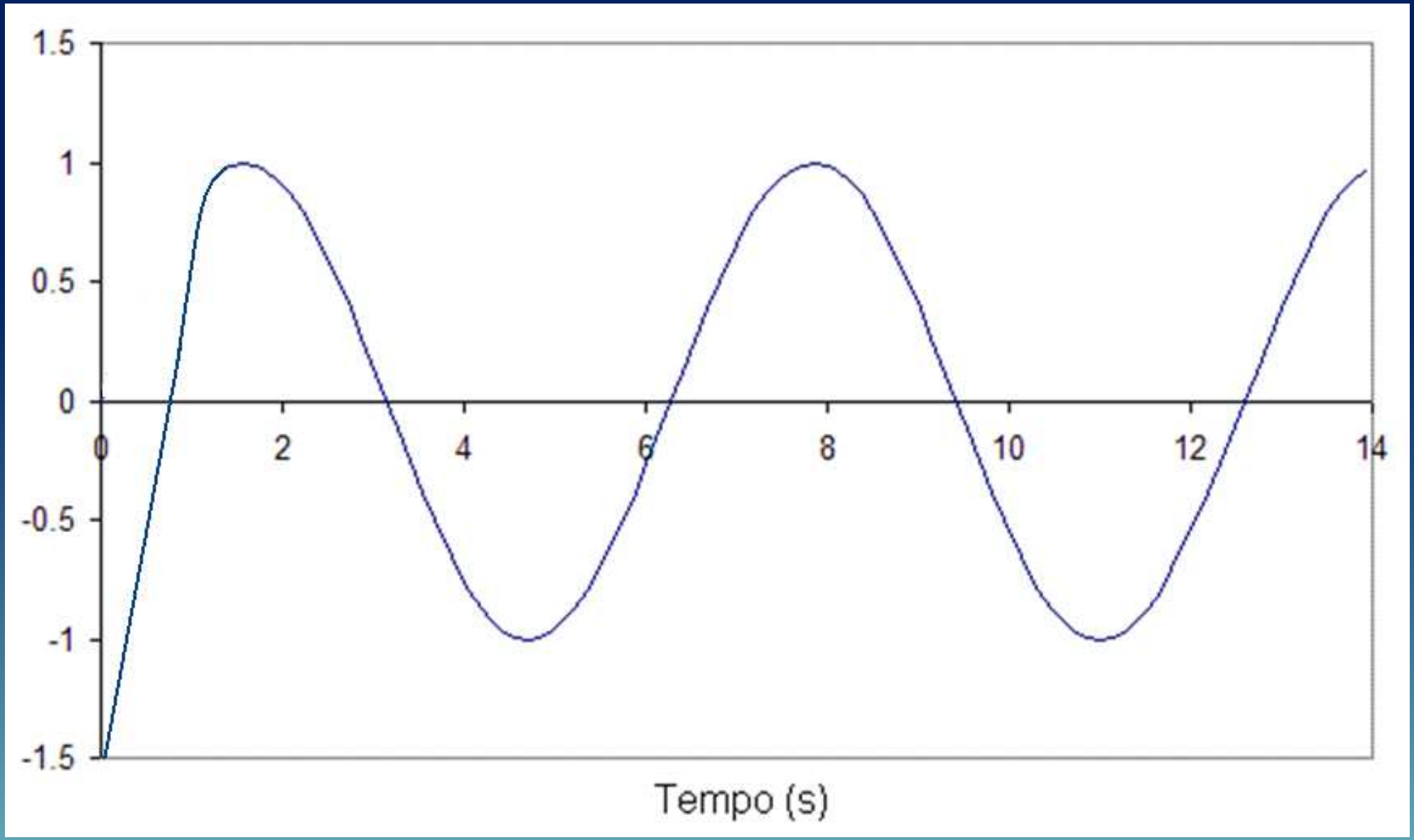
Análise no domínio do tempo



Análise no domínio do tempo



Análise no domínio do tempo



Análise no domínio do tempo

Departamento de
Engenharia Eletromecânica

Obrigado!

Felippe de Souza
felippe@ubi.pt