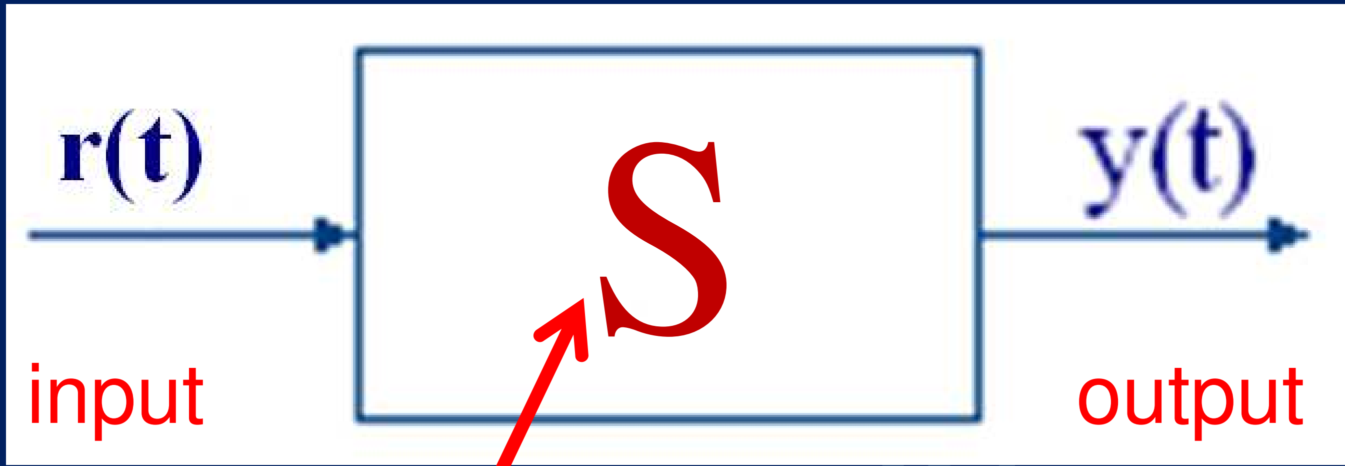
Control Systems

6

"Time domain analysis"

part I – 1^{st} order systems

J. A. M. Felippe de Souza

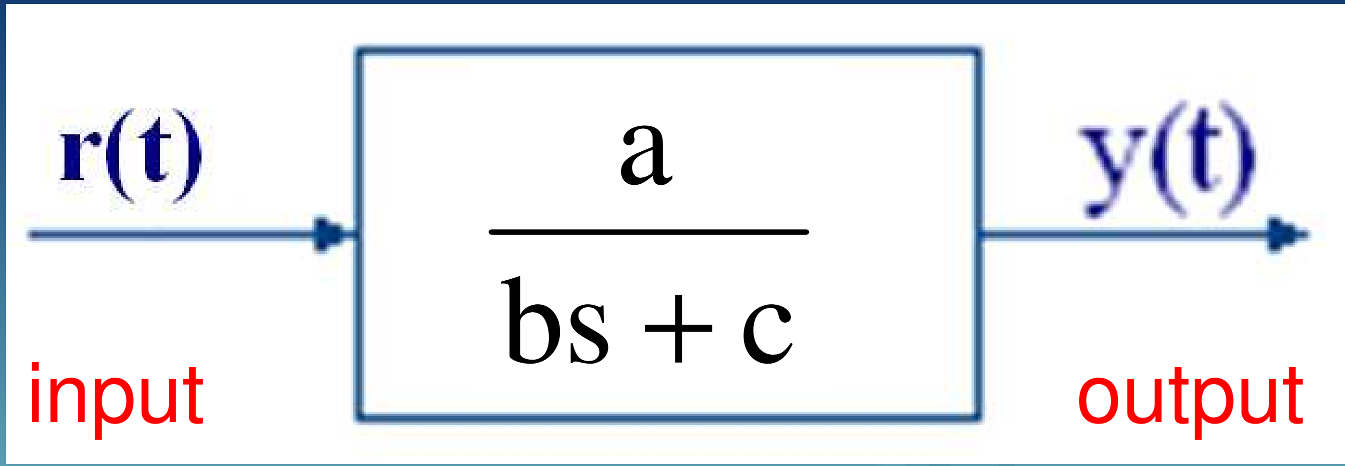


1st order systems

First order systems

of type

$$G(s) = \frac{a}{bs + c}$$



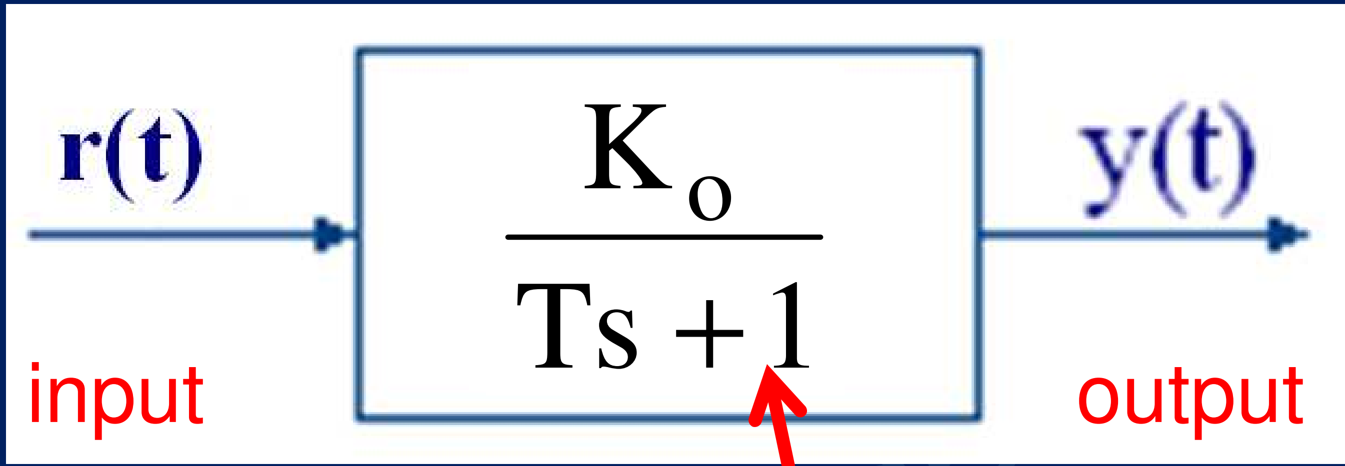
1st order systems

that is:

$$\frac{Y(s)}{R(s)} = \frac{a}{bs + c}$$

$$\Rightarrow \quad \frac{Y(s)}{R(s)} = \frac{\frac{a}{c}}{\frac{(bs+c)}{c}} = \frac{\frac{a}{c}}{\frac{b}{c}s + 1},$$

$K_o = \frac{a}{c}$, $T = \frac{b}{c}$

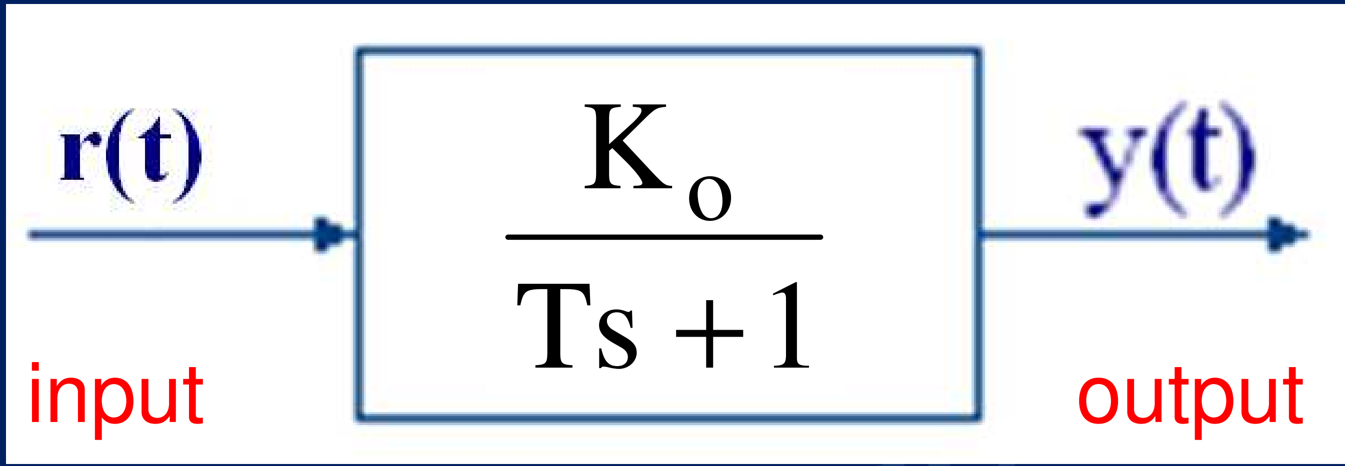


1st order systems

that is:

the *transfer function* can be rewritten as:

$$\frac{Y(s)}{R(s)} = \frac{K_o}{Ts+1}$$



1st order systems

K_o = gain of the system

T = time constant of the system

the transfer function:

$$\frac{Y(s)}{R(s)} = \frac{K_o}{Ts+1}$$

Example 1:

$$\frac{Y(s)}{R(s)} = \frac{2}{5s+4}$$

pole:
$s = -0,8$

$K_o = 2/4 = 0,5$

$T = 5/4 = 1,25$

Example 2:

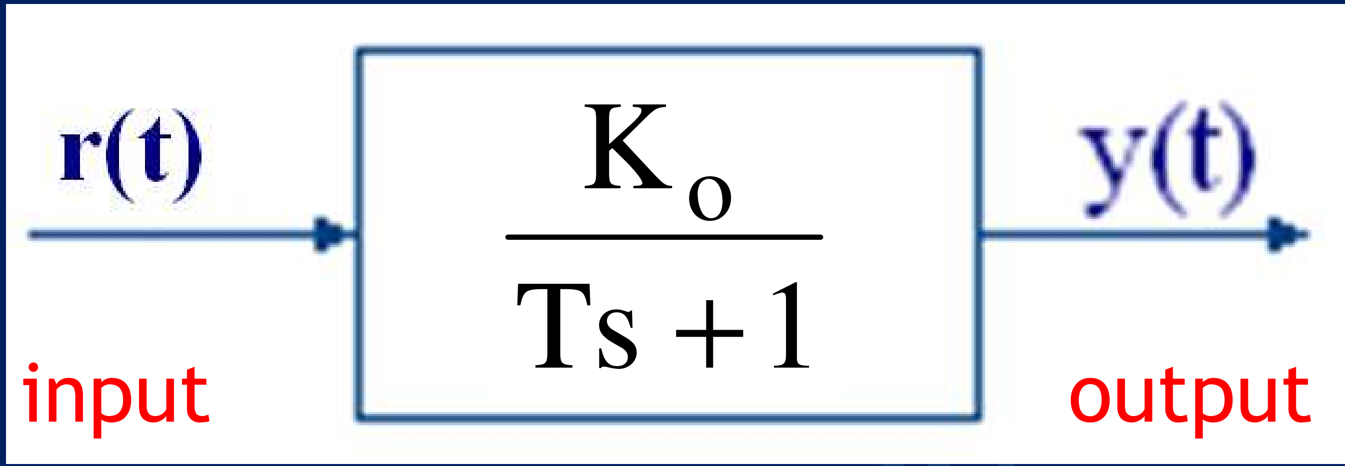
$$\frac{Y(s)}{R(s)} = \frac{12}{s+4}$$

pole:
$s = -4$

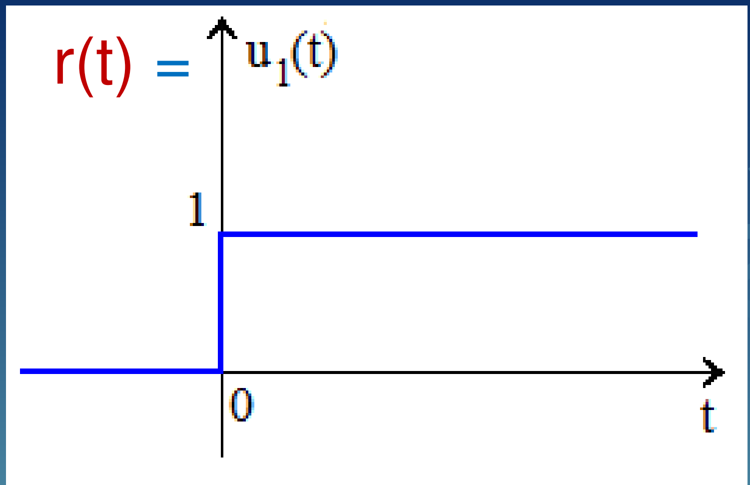
$K_o = 3$

$T = 1/4 = 0,25$

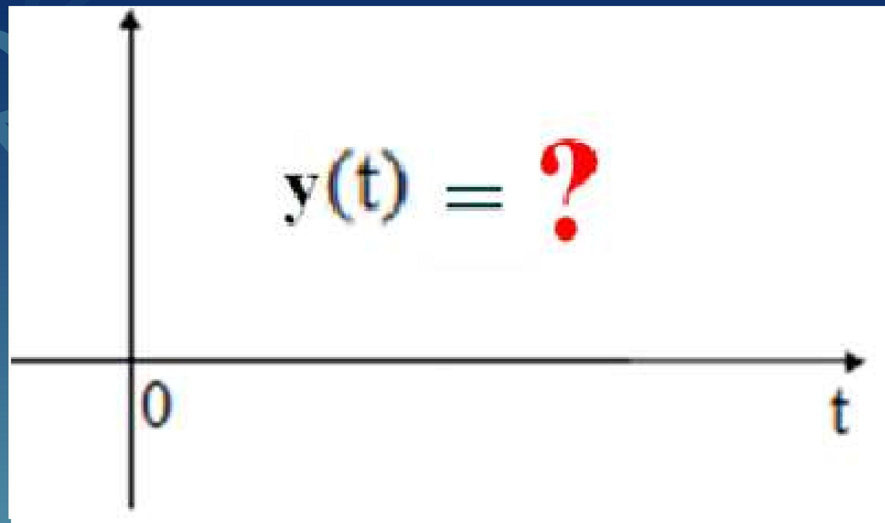
Time domain analysis - 1st order systems



1st order systems

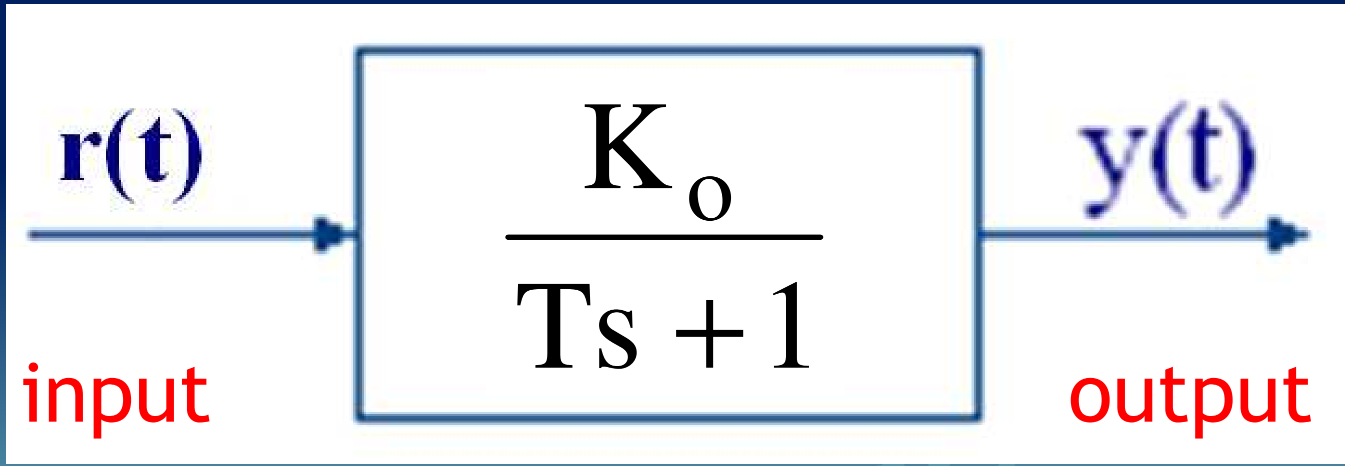


unit step input



What is the output?
(*step response*)

Time domain analysis - 1st order systems

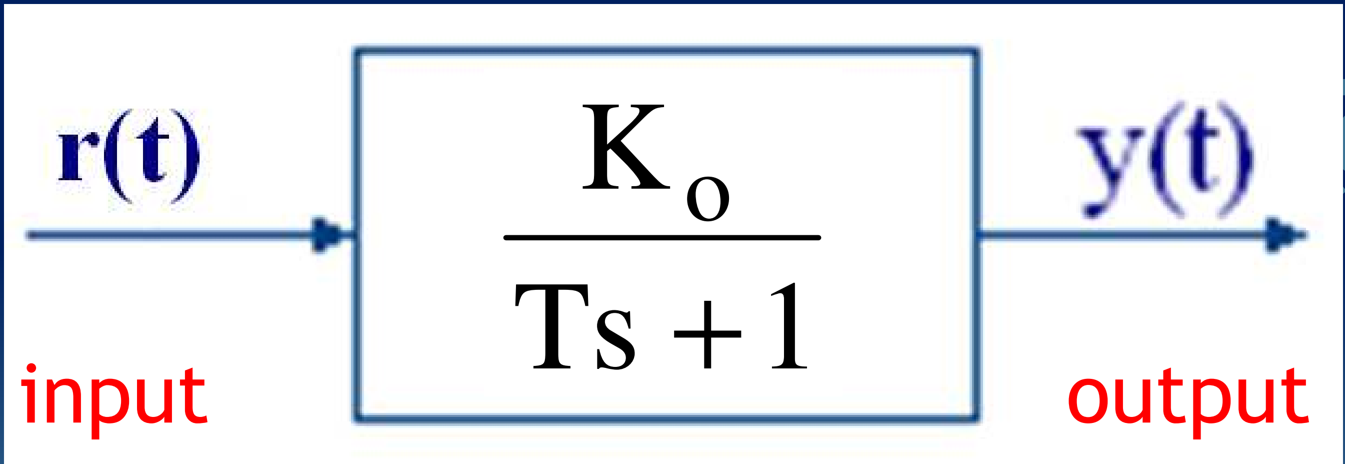


1st order systems

in order to calculate:

$$Y(s) = \frac{K_o}{Ts+1} \cdot R(s)$$

$$Y(s) = \frac{K_o}{(Ts+1)} \cdot \frac{1}{s} = \frac{K_o}{s} - \frac{K_o T}{(Ts+1)}$$



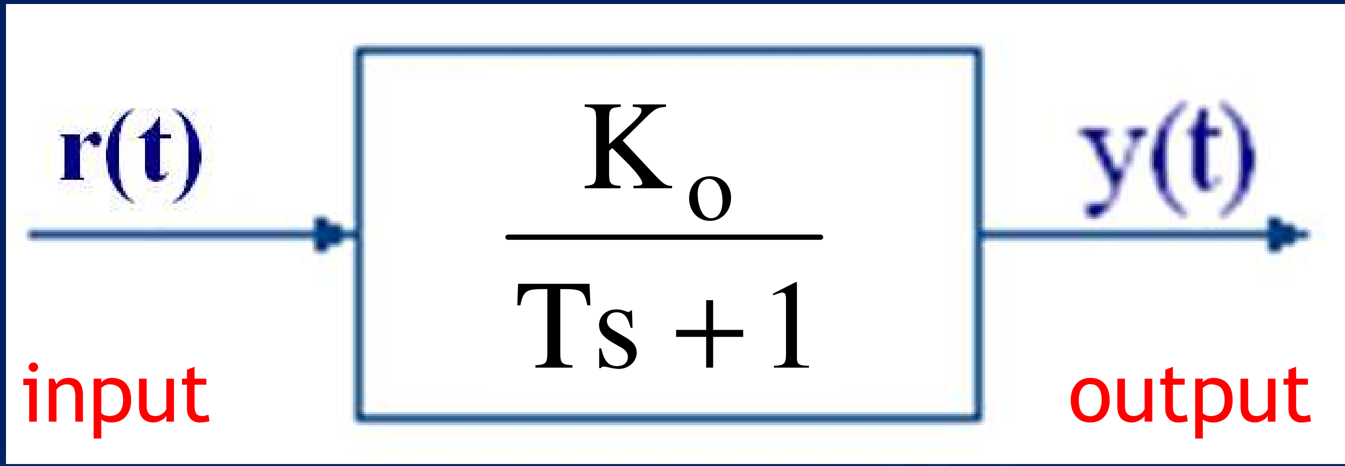
1st order systems

$$y(t) = \mathscr{L}^{-1}[Y(s)]$$

hence, the unit step response is:

$$y(t) = K_o(1 - e^{-t/T}), \qquad t > 0$$

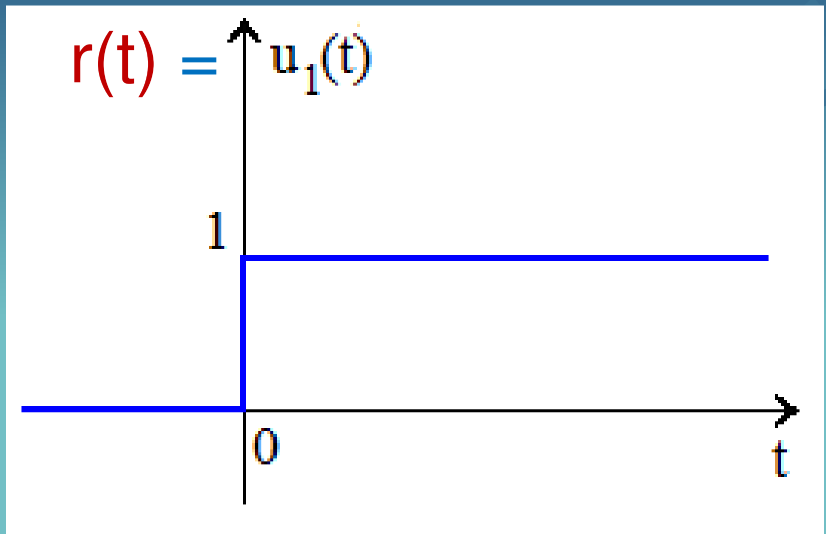
Time domain analysis - 1st order systems



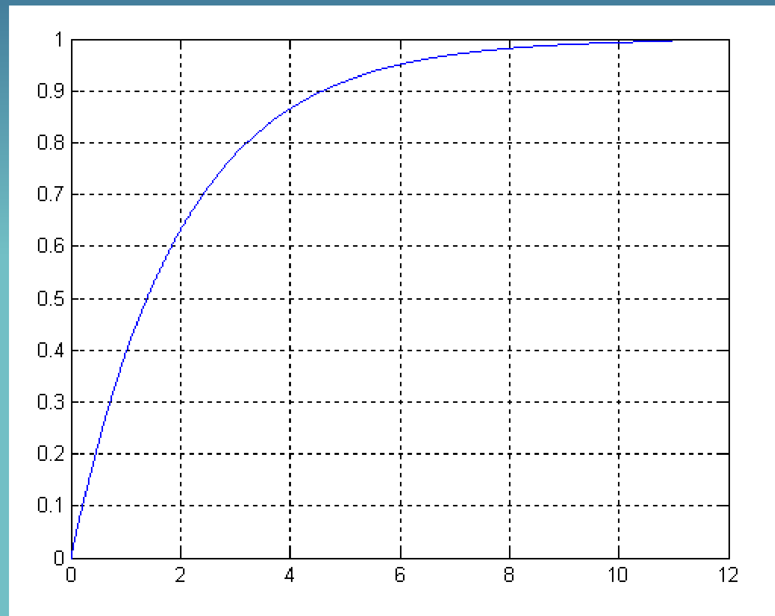
1st order systems

the unit step response is:

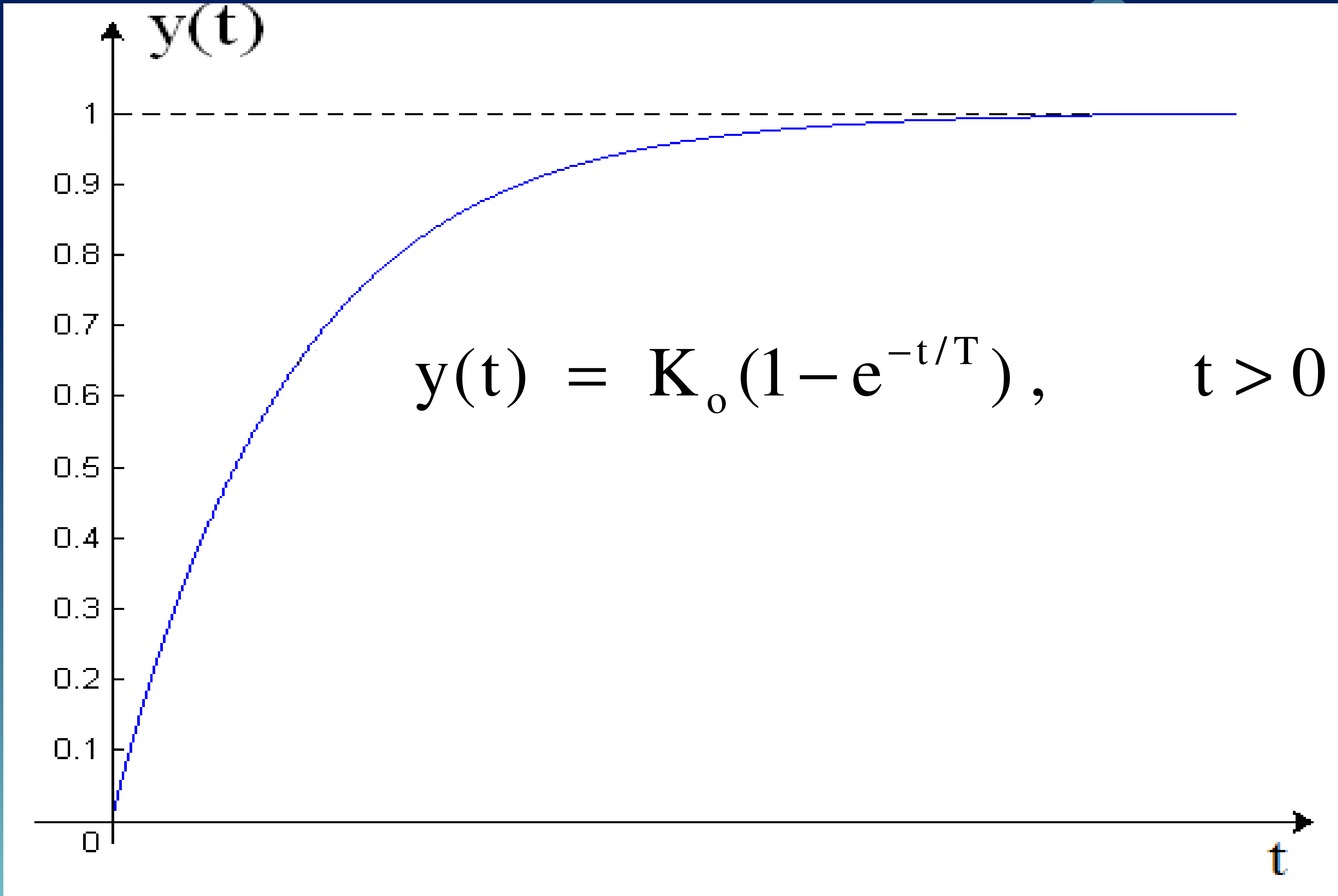
$$y(t) = K_o(1 - e^{-t/T}), \quad t > 0$$



unit step input



the unit step response is:



Observe that, for the unit step response:

$$y(t) = K_o(1 - e^{-t/T}), \qquad t > 0$$

$$\text{If} \quad t = T \quad \Rightarrow \quad y(T) = K_o(1 - e^{-1}) = 0{,}632\,K_o$$

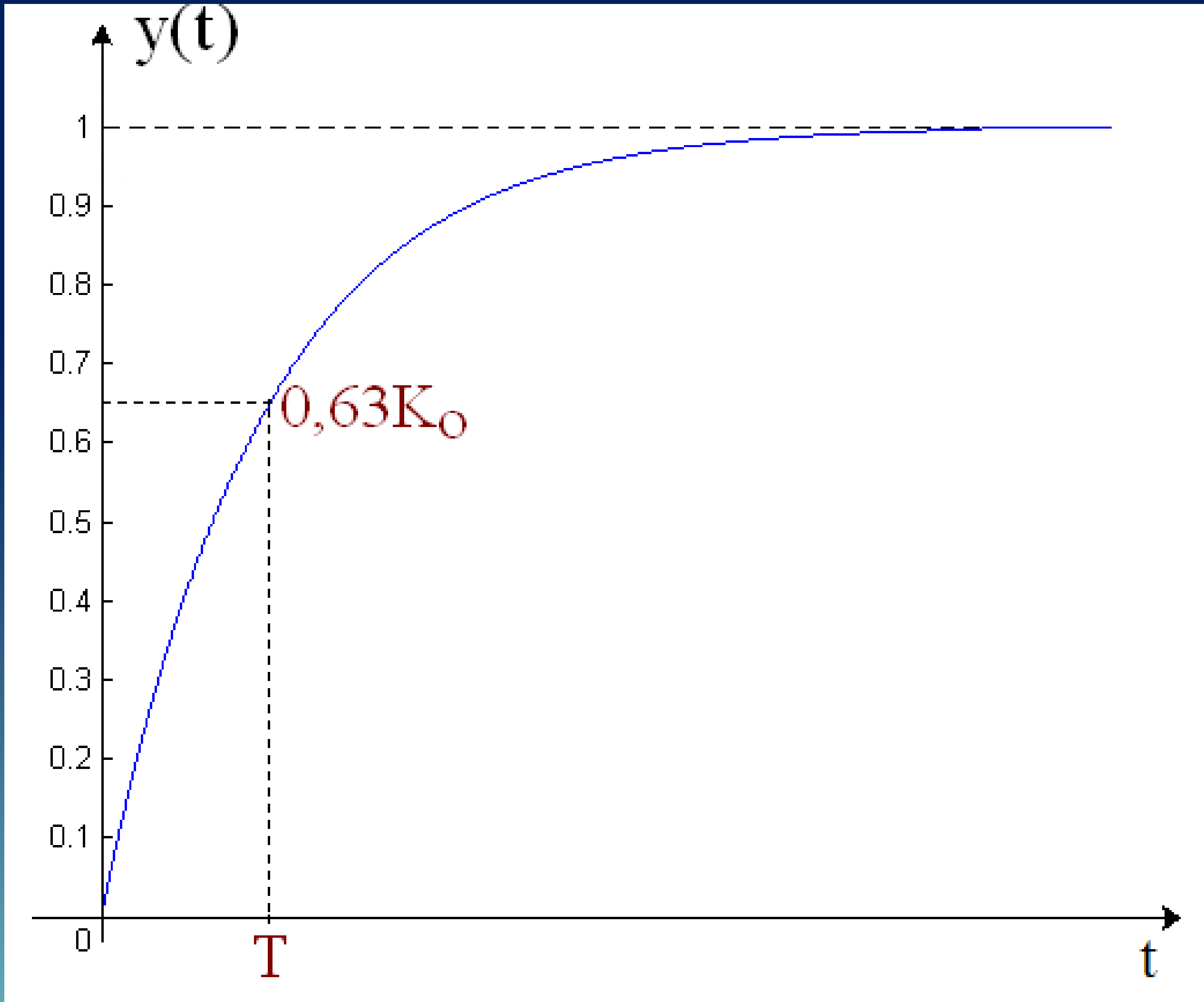
$$\text{If} \quad t = 2T \quad \Rightarrow \quad y(2T) = K_o(1 - e^{-2}) = 0{,}865\,K_o$$

$$\text{If} \quad t = 3T \quad \Rightarrow \quad y(3T) = K_o(1 - e^{-3}) = 0{,}95\,K_o$$

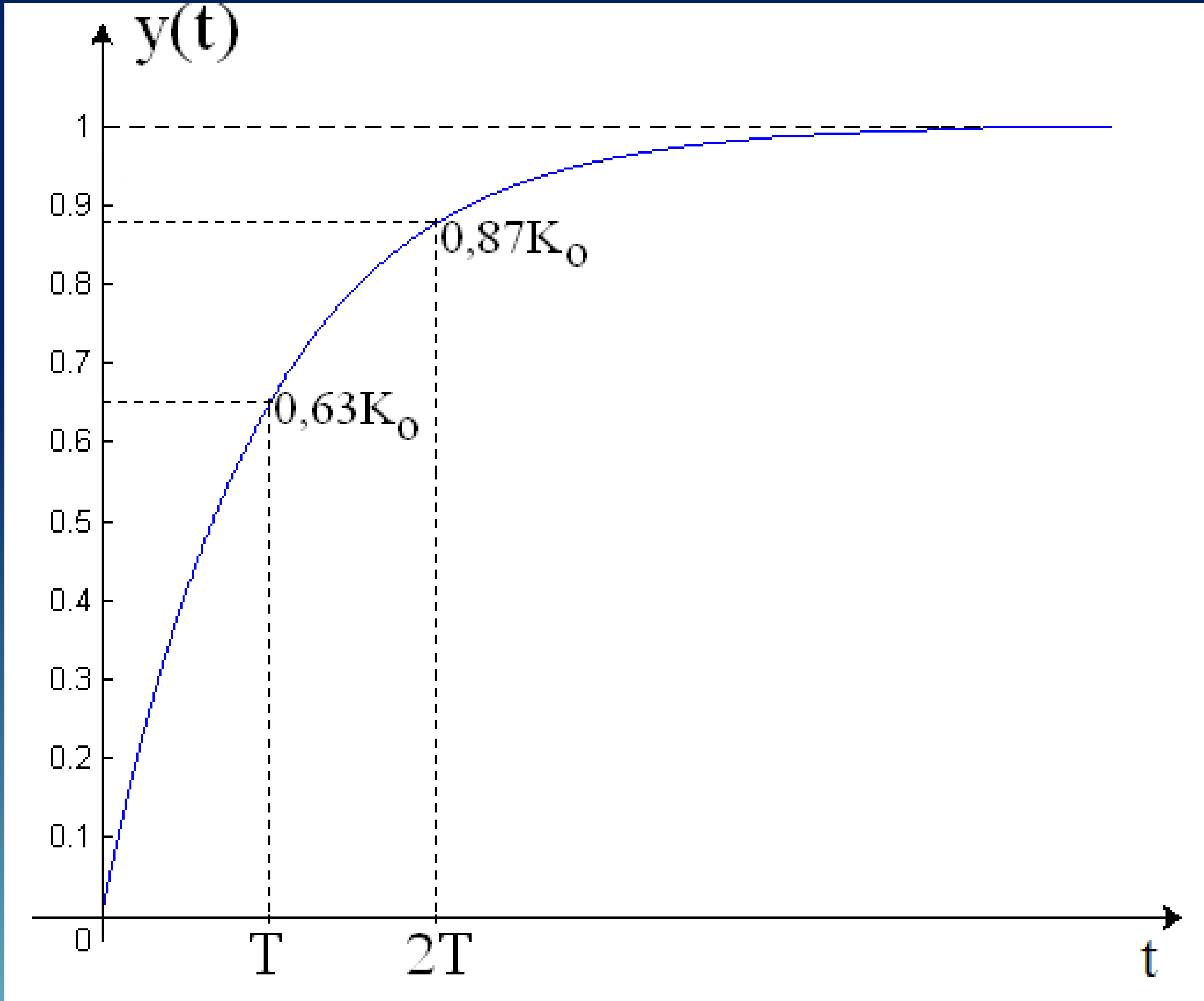
$$\text{If} \quad t = 4T \quad \Rightarrow \quad y(4T) = K_o(1 - e^{-4}) = 0{,}982\,K_o$$

$$\text{If} \quad t = 5T \quad \Rightarrow \quad y(5T) = K_o(1 - e^{-5}) = 0{,}993\,K_o$$

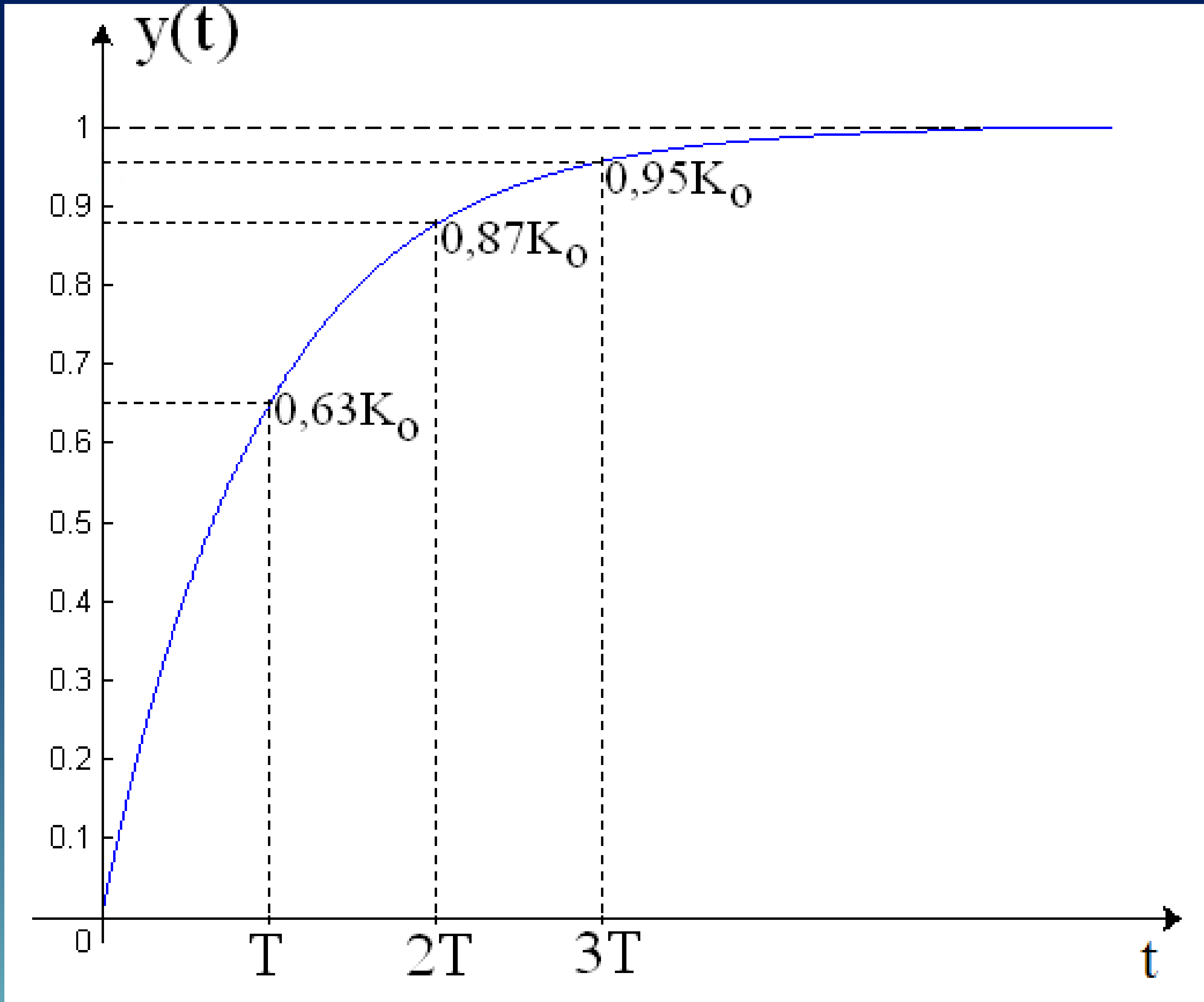
Time domain analysis - 1st order systems



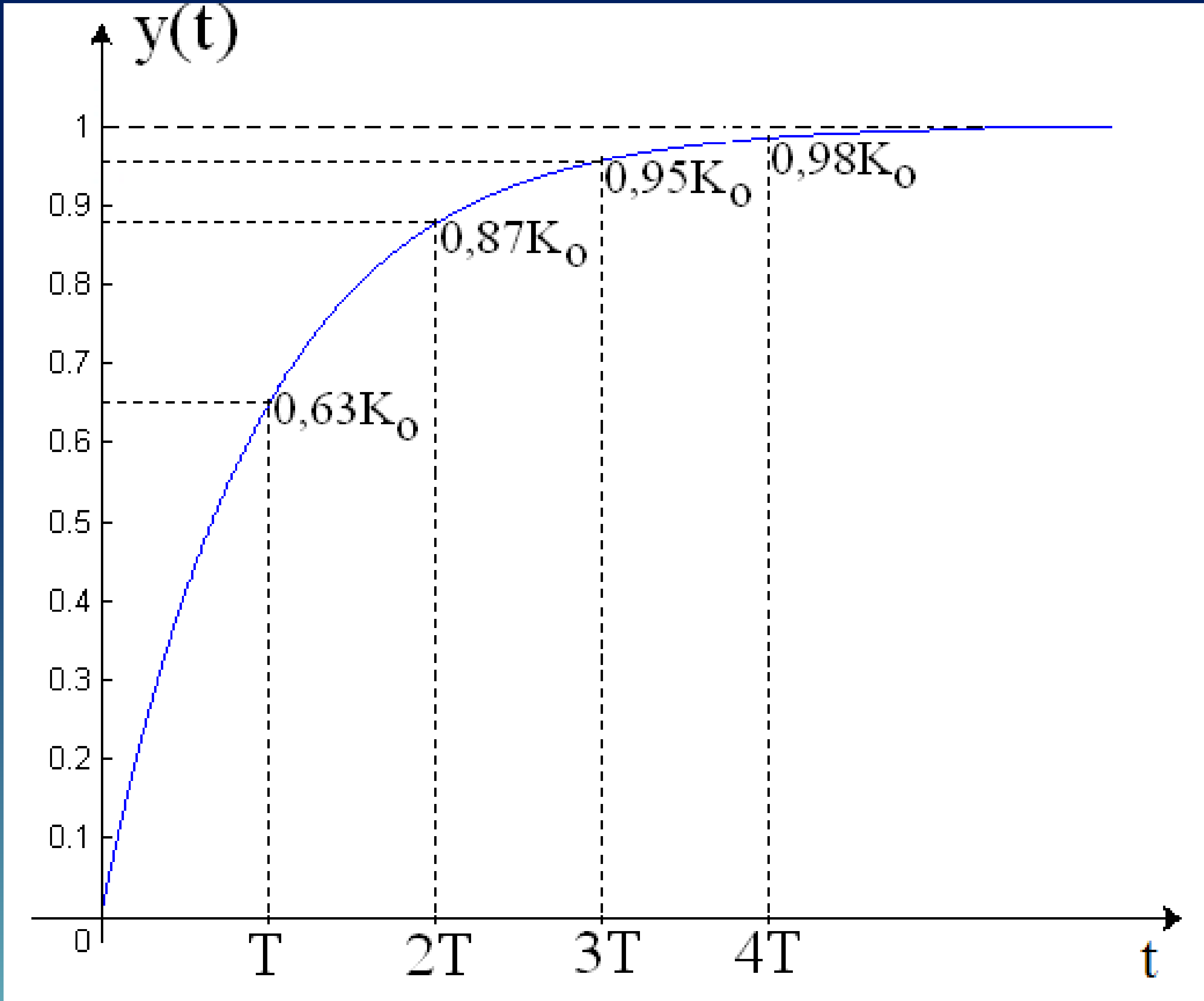
Time domain analysis - 1st order systems



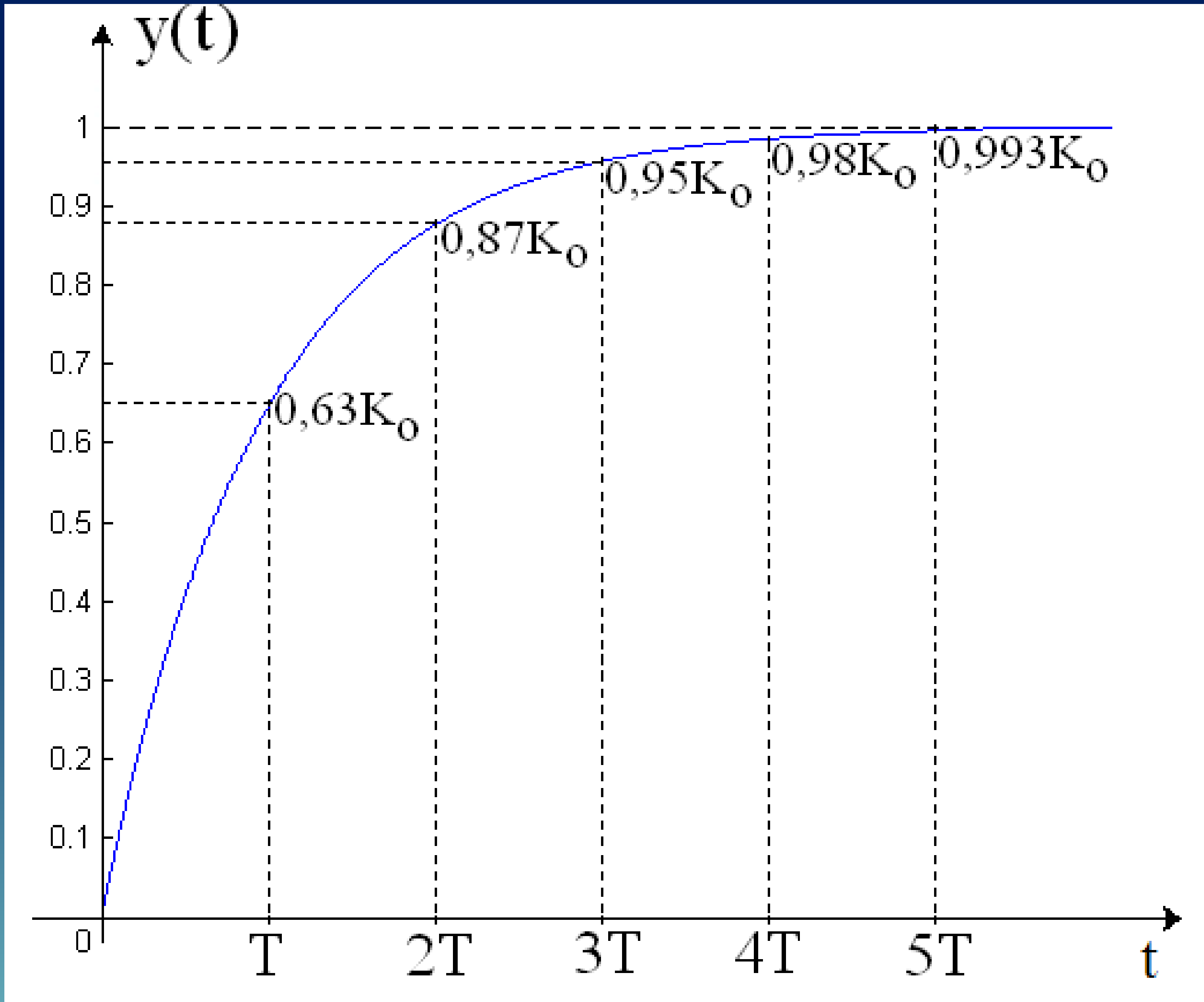
Time domain analysis - 1st order systems



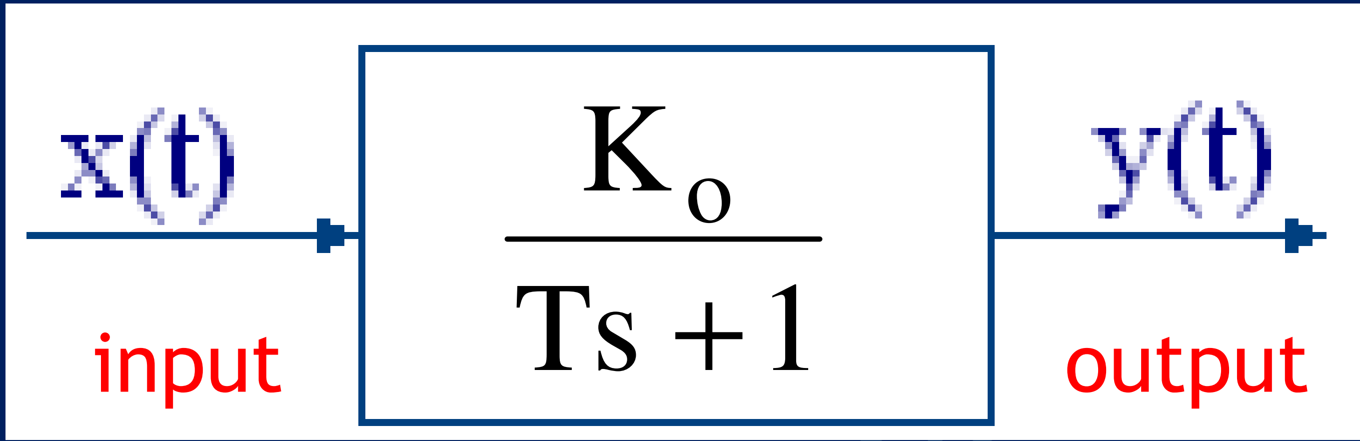
Time domain analysis - 1st order systems



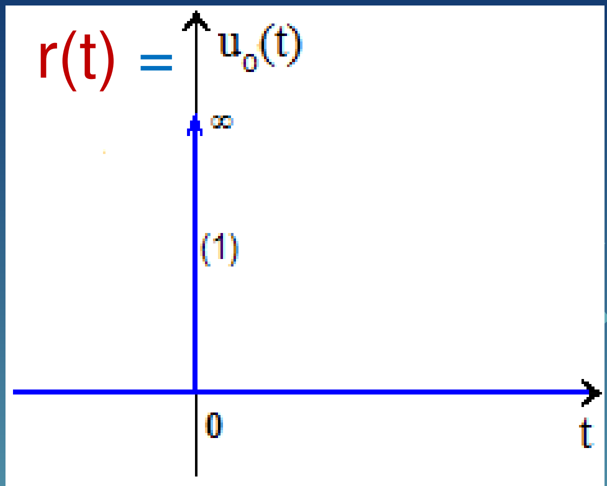
Time domain analysis - 1st order systems



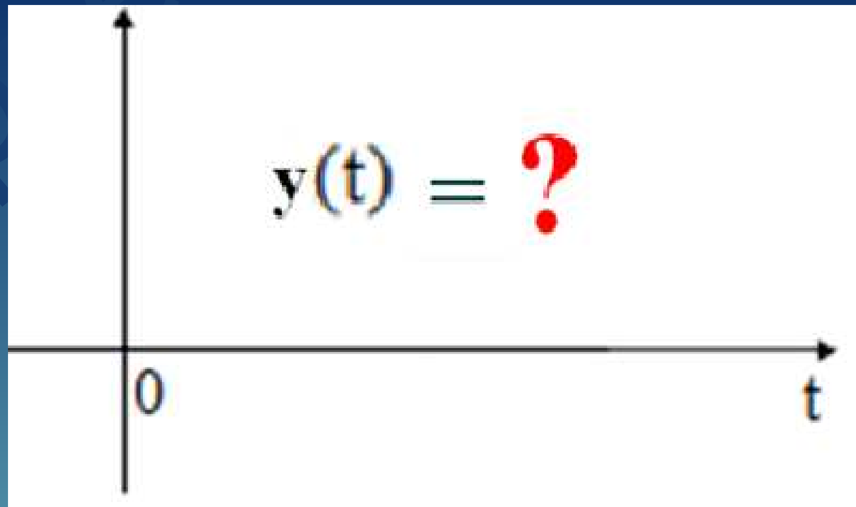
Time domain analysis - 1st order systems



1st order systems

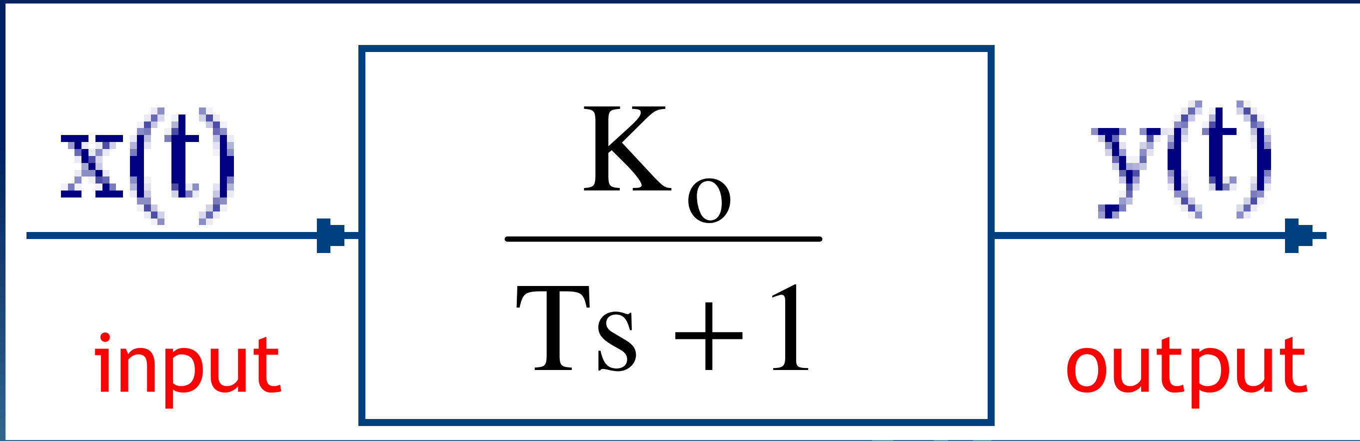


unit impulse input



What is the output?

(*impulse response*)



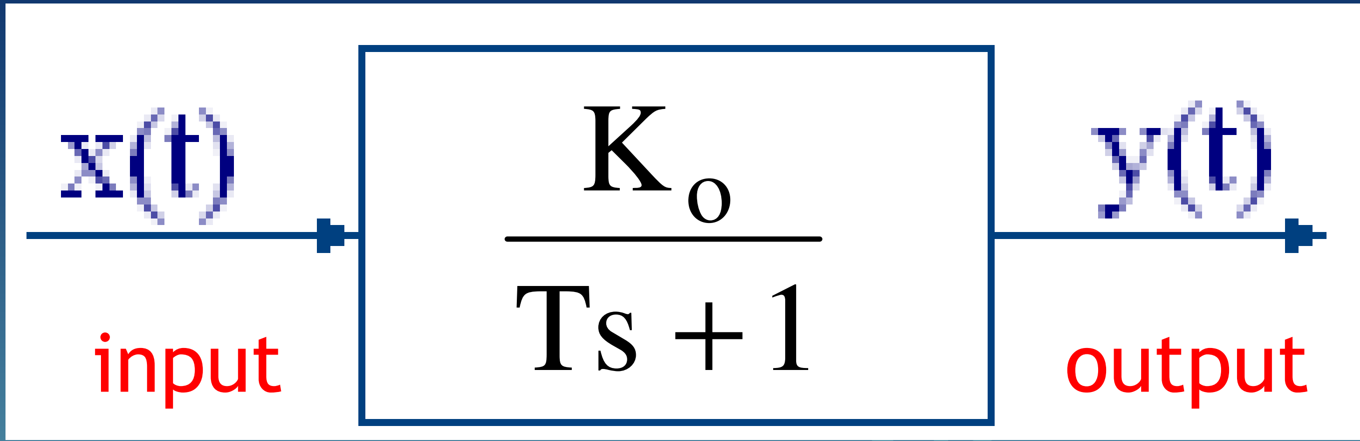
1st order systems

in order to calculate:

$$Y(s) = \frac{K_o}{Ts+1} \cdot R(s)$$

$$Y(s) = \frac{K_o}{(Ts+1)} \cdot 1 = \frac{K_o}{(Ts+1)}$$

Time domain analysis - 1st order systems



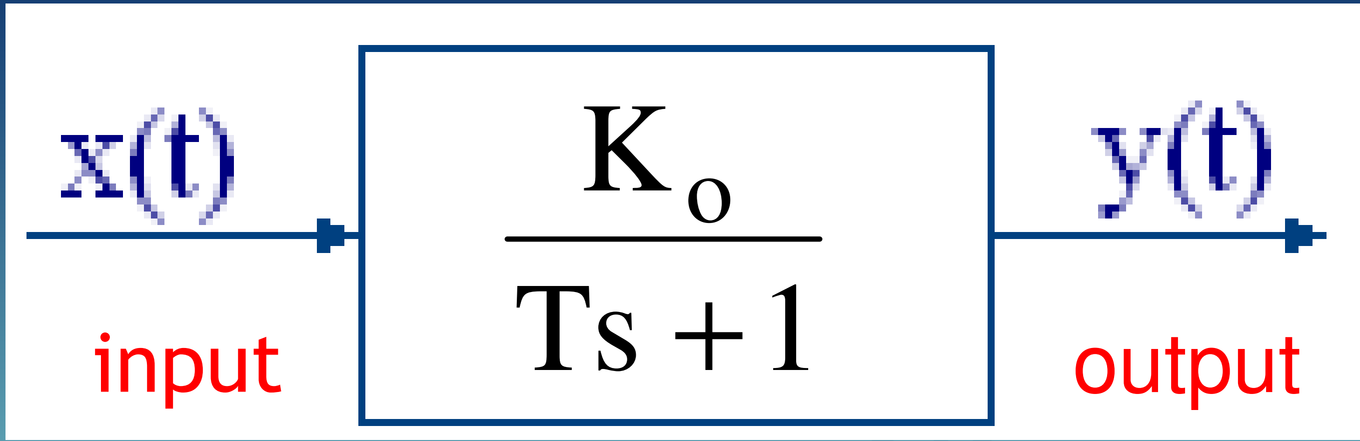
1st order systems

$$y(t) = \mathscr{L}^{-1}\left[Y(s)\right]$$

hence, the unit impulse response is:

$$y(t) = \frac{K_o}{T} \cdot e^{-t/T}, \qquad t > 0$$

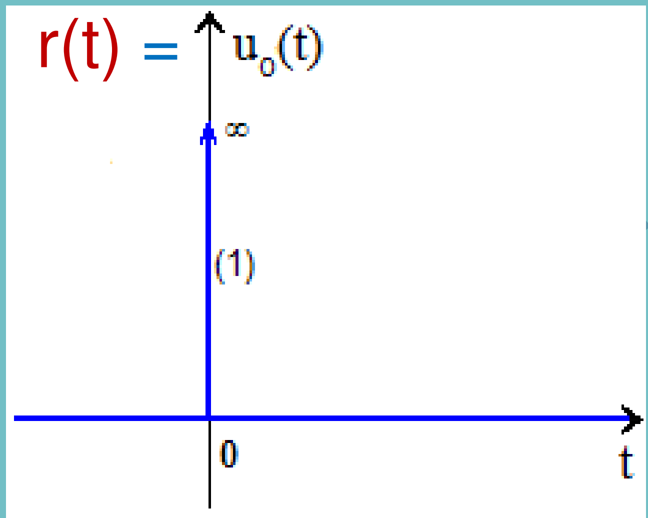
Time domain analysis - 1st order systems



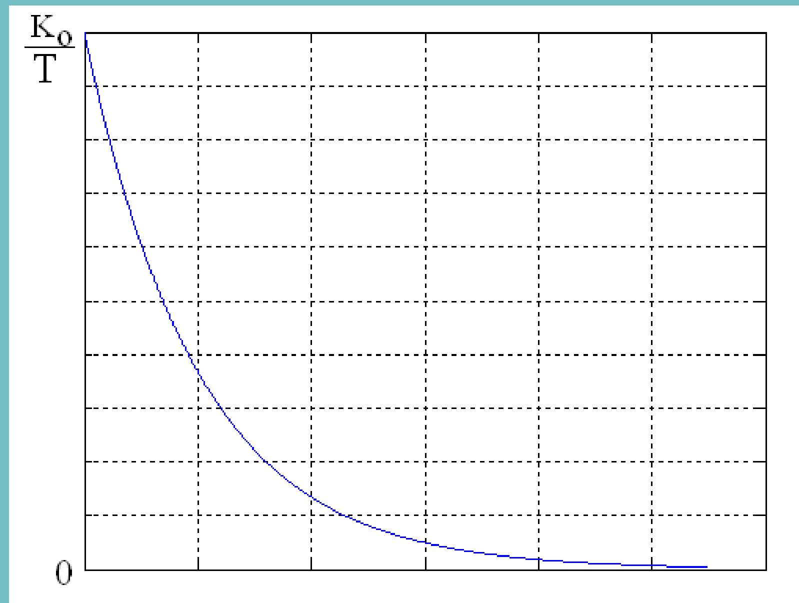
1st order systems

the unit impulse response is:

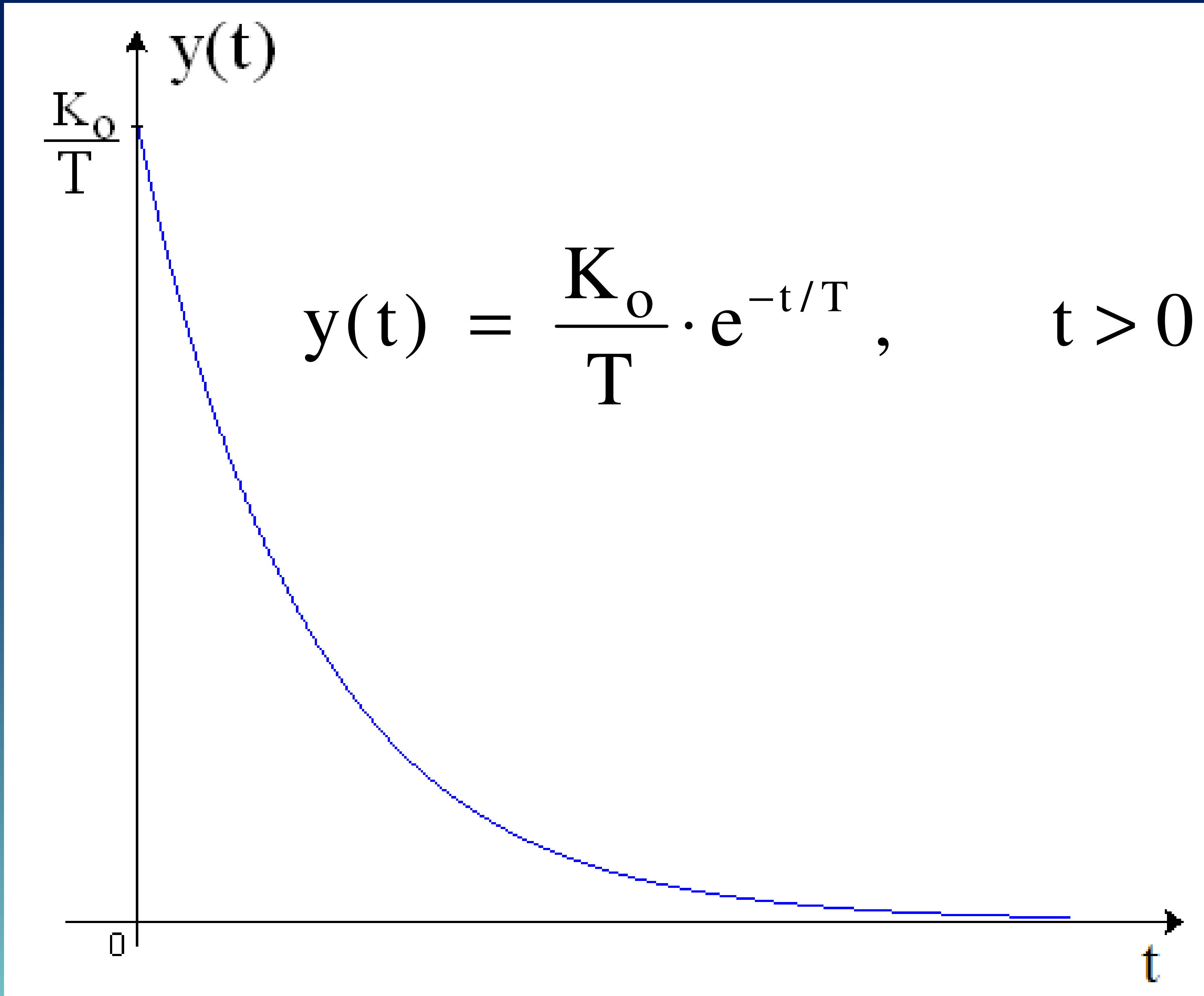
$$y(t) = \frac{K_o}{T} \cdot e^{-t/T}, \qquad t > 0$$



unit impulse input



the unit impulse response is:



$$y(t) = \frac{K_o}{T} \cdot e^{-t/T}, \qquad t > 0$$

Observe that, for the impulse response:

$$y(t) = \frac{K_o}{T} \cdot e^{-t/T}, \qquad t > 0$$

If $t = T \Rightarrow y(T) = (K_o / T) \cdot e^{-1} = 0,368 \cdot (K_o / T)$

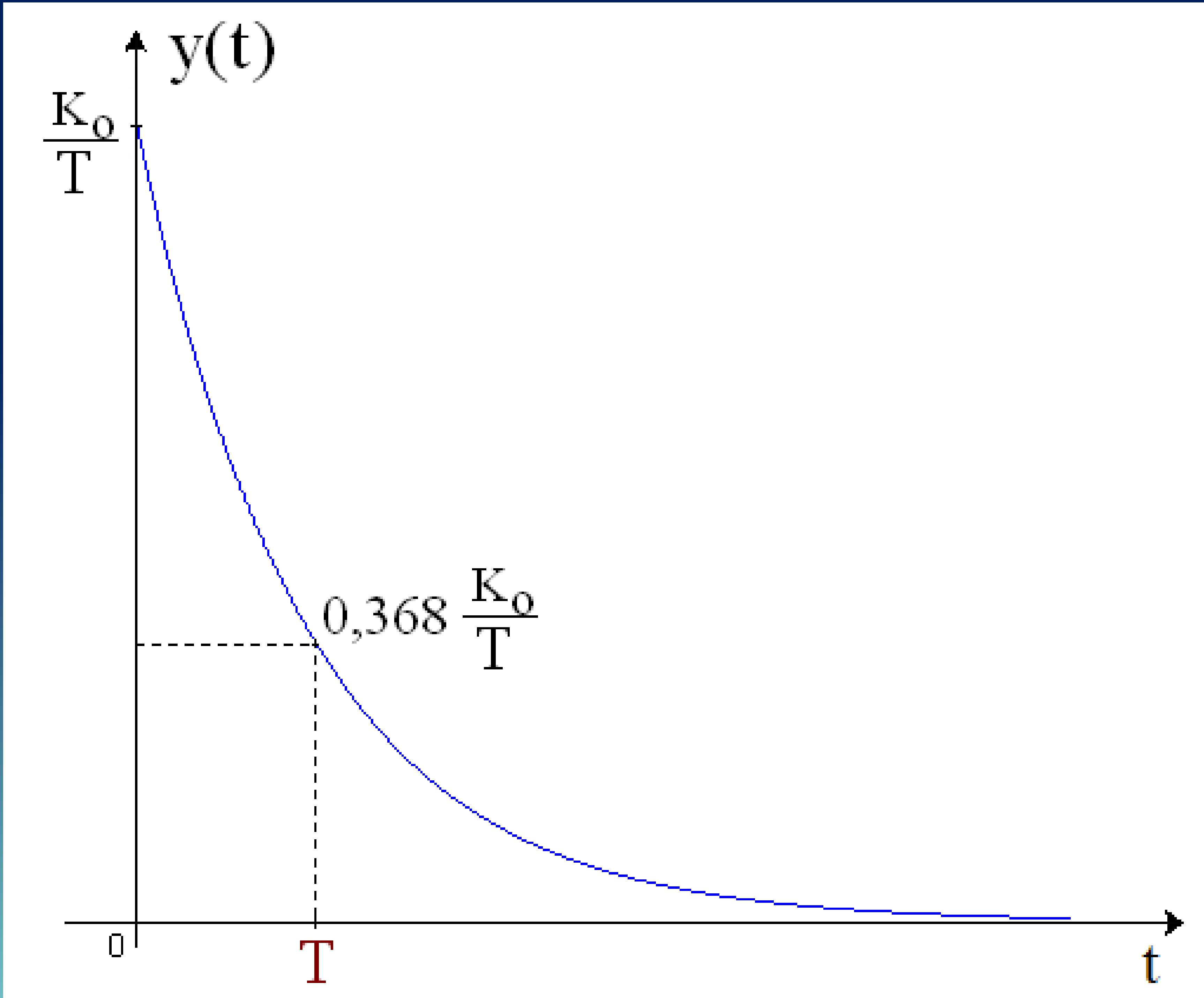
If $t = 2T \Rightarrow y(2T) = (K_o / T) \cdot e^{-2} = 0,135 \cdot (K_o / T)$

If $t = 3T \Rightarrow y(3T) = (K_o / T) \cdot e^{-3} = 0,05 \cdot (K_o / T)$

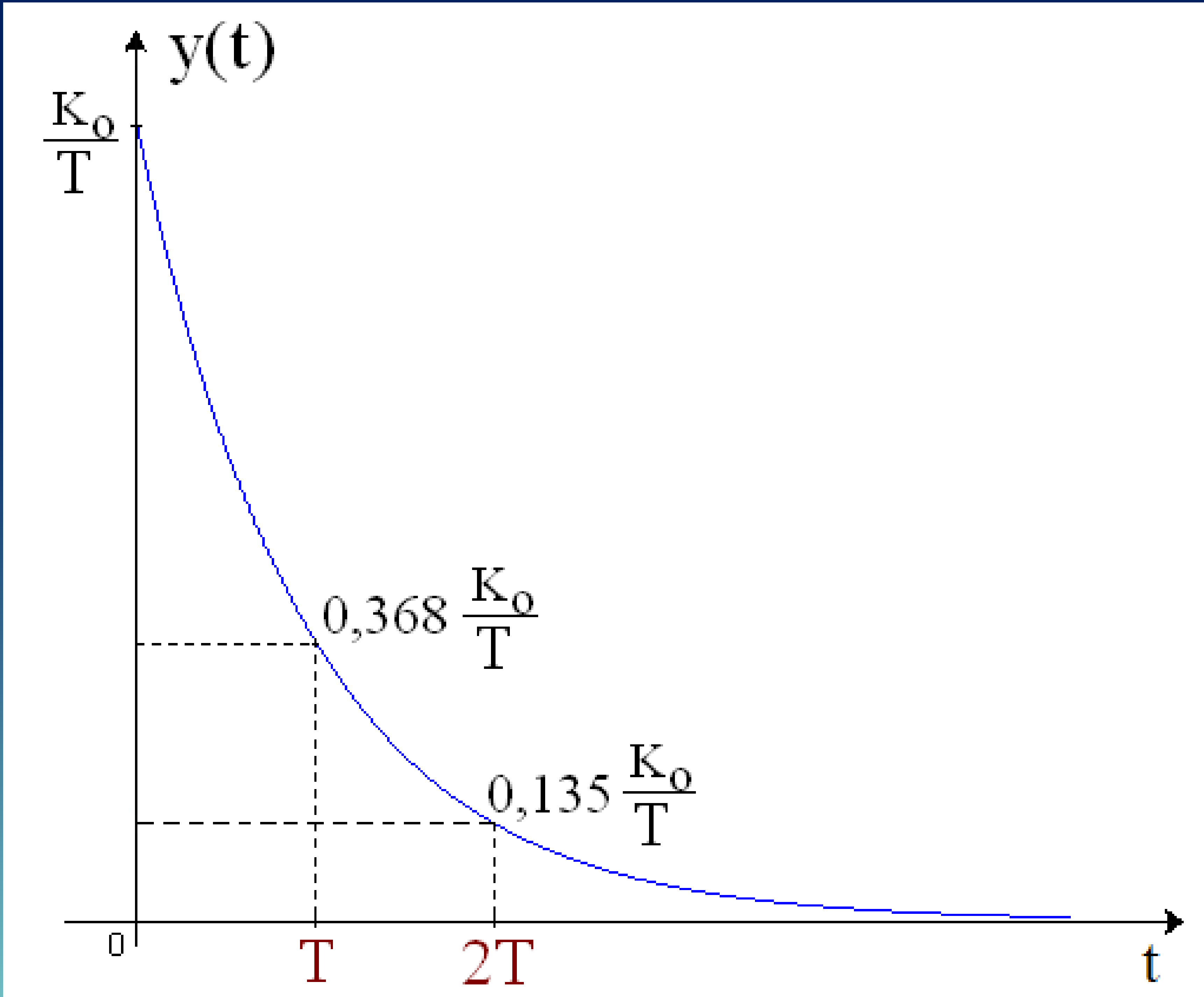
If $t = 4T \Rightarrow y(4T) = (K_o / T) \cdot e^{-4} = 0,02 \cdot (K_o / T)$

If $t = 5T \Rightarrow y(5T) = (K_o / T) \cdot e^{-5} = 0,007 \cdot (K_o / T)$

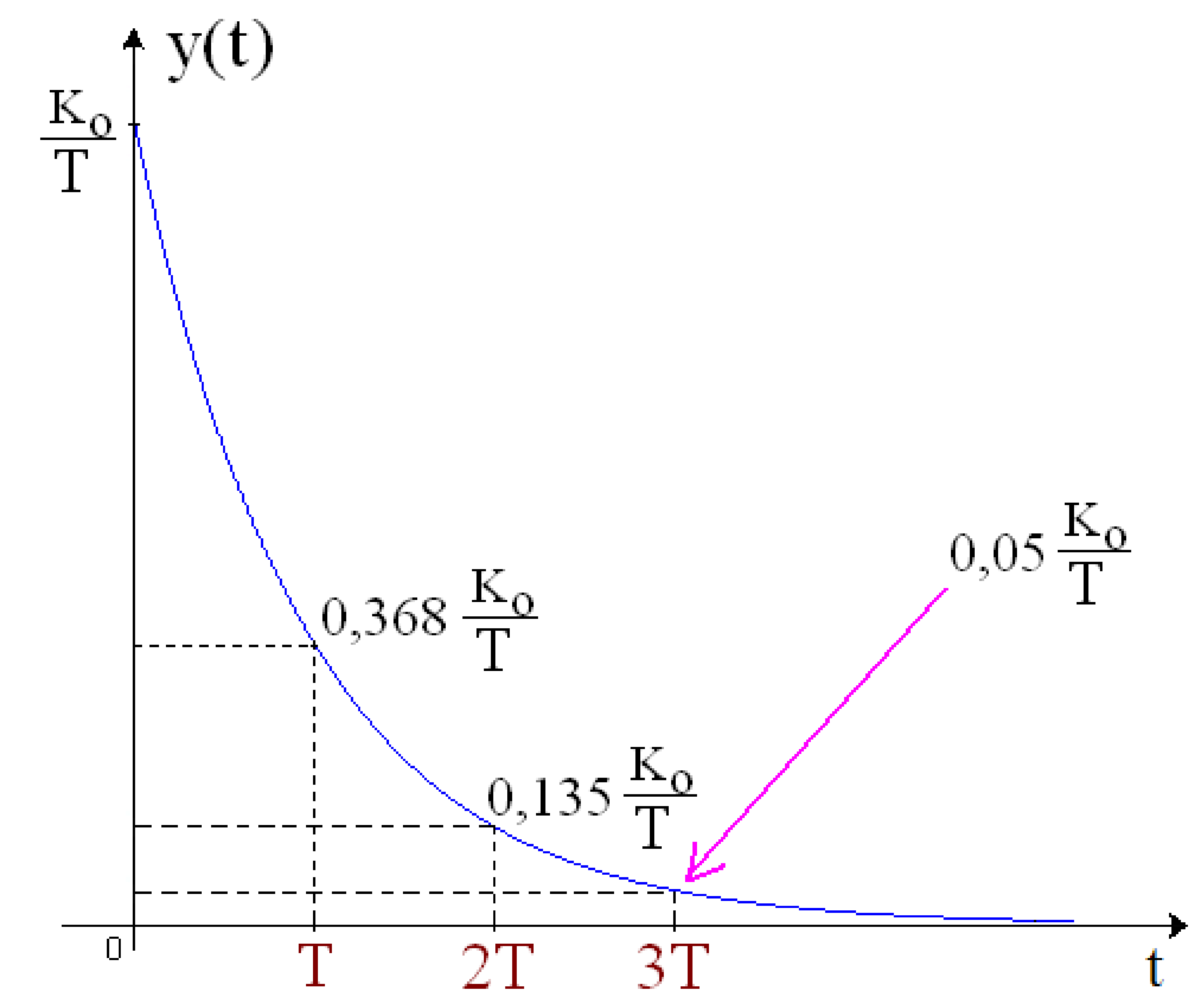
Time domain analysis - 1st order systems



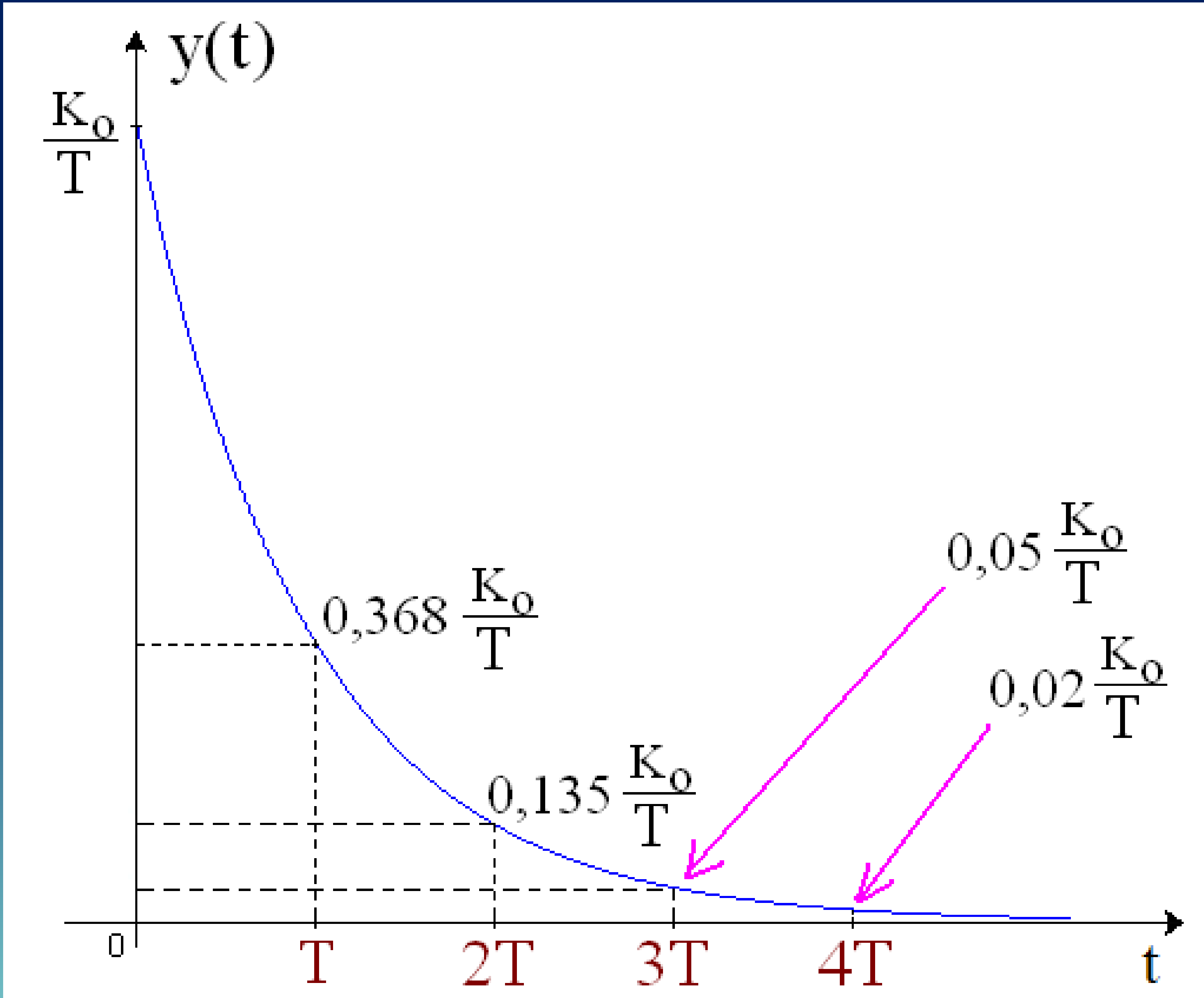
Time domain analysis - 1st order systems



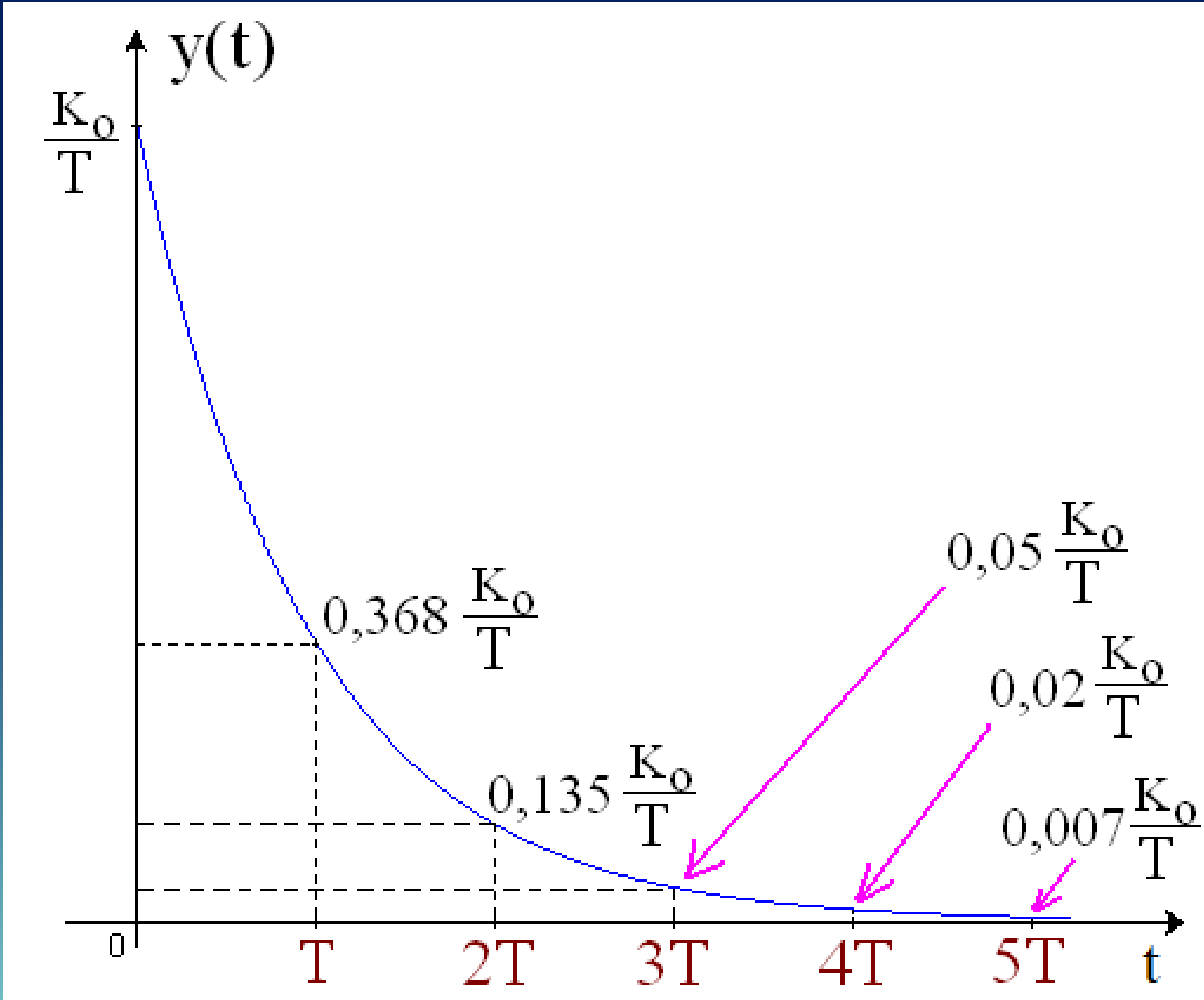
Time domain analysis - 1st order systems

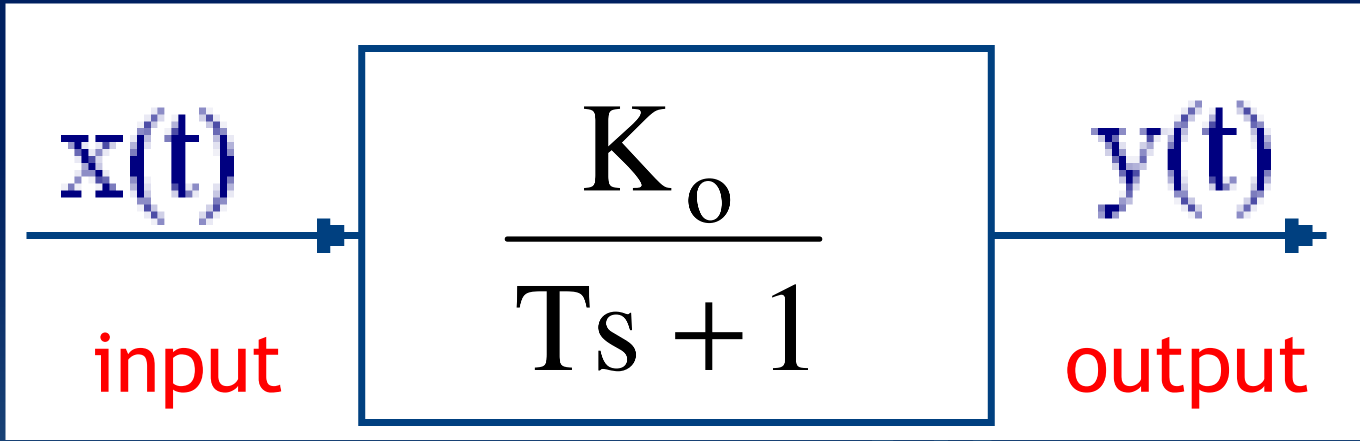


Time domain analysis - 1st order systems

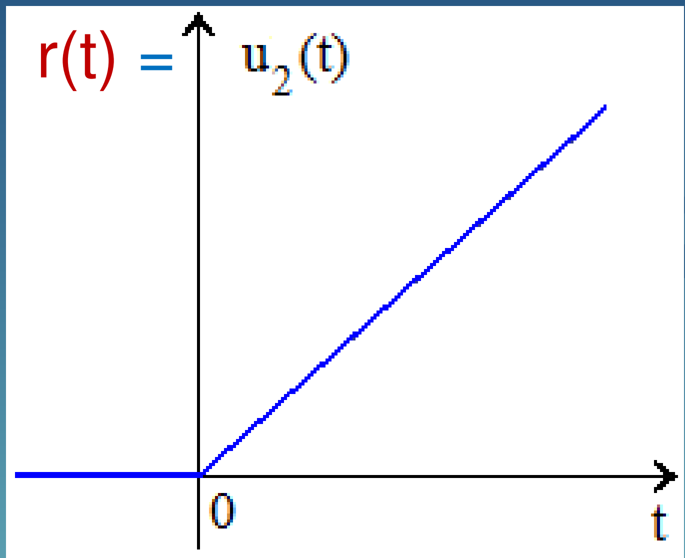


Time domain analysis - 1st order systems

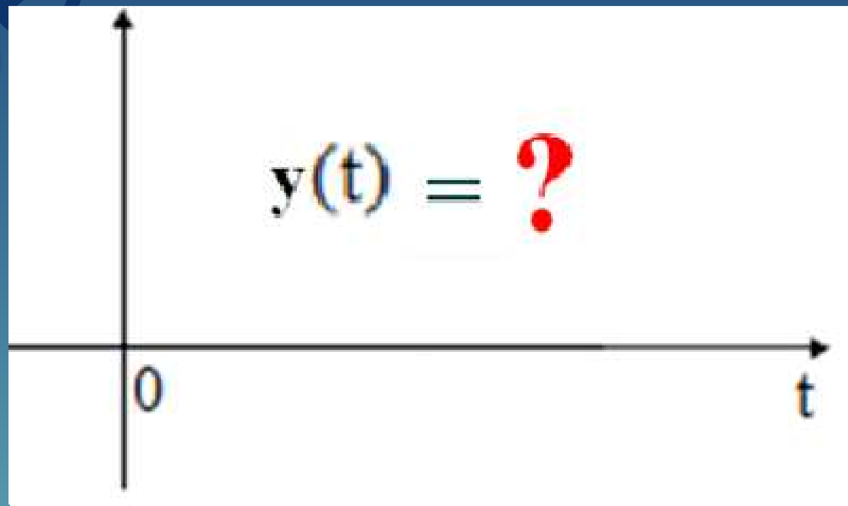




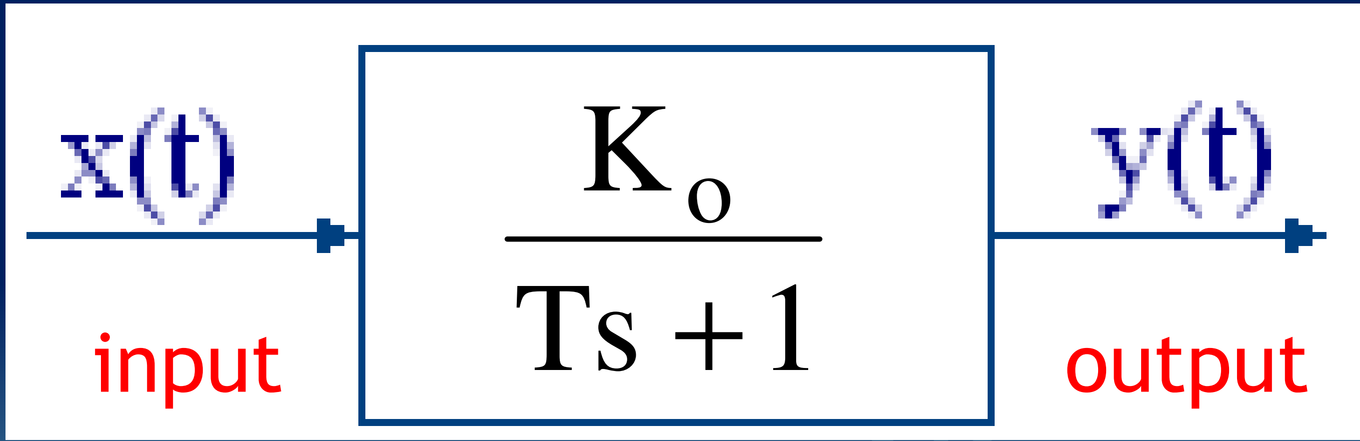
1st order systems



unit ramp input



What is the output?
(*ramp response*)

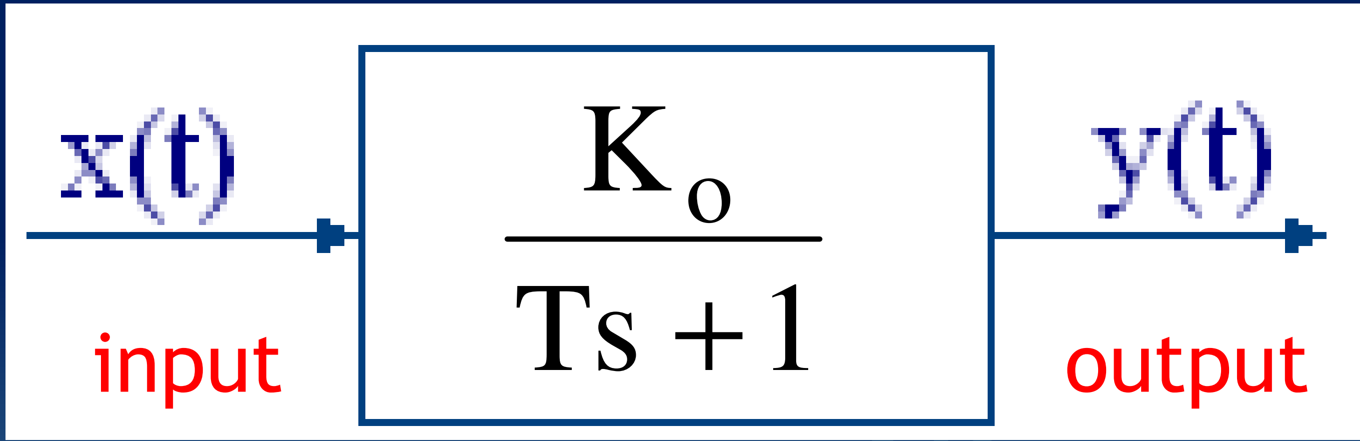


1st order systems

in order to calculate:

$$Y(s) = \frac{K_o}{Ts+1} \cdot R(s)$$

$$Y(s) = \frac{K_o}{(Ts+1)} \cdot \frac{1}{s^2} = \frac{K_o}{s^2} - \frac{K_o T}{s} + \frac{K_o T^2}{(Ts+1)}$$



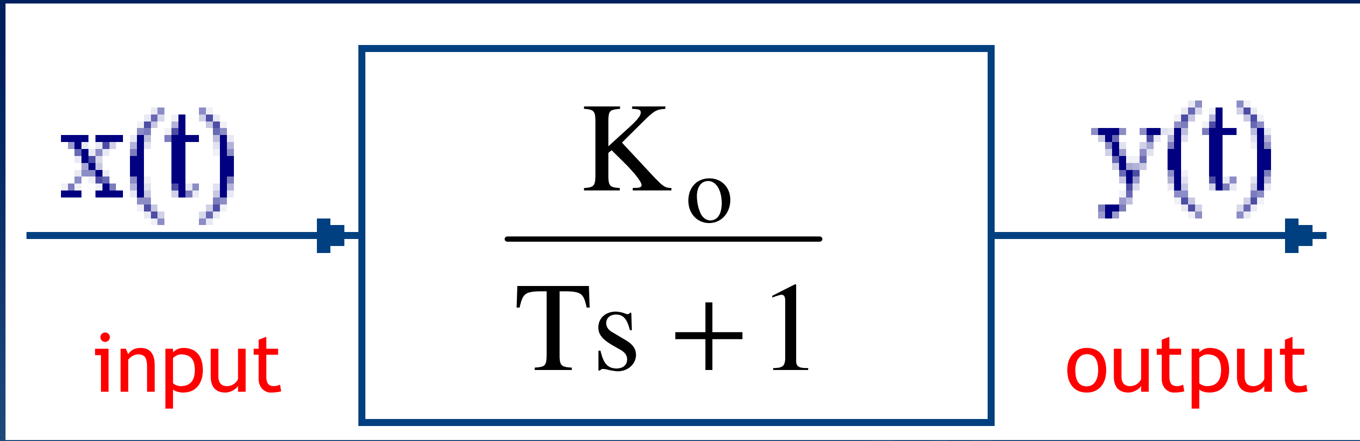
1st order systems

$$y(t) = \mathscr{L}^{-1}\left[Y(s)\right]$$

hence, the unit ramp response is:

$$y(t) = K_o(t - T + T \cdot e^{-t/T}), \qquad t > 0$$

Time domain analysis - 1st order systems

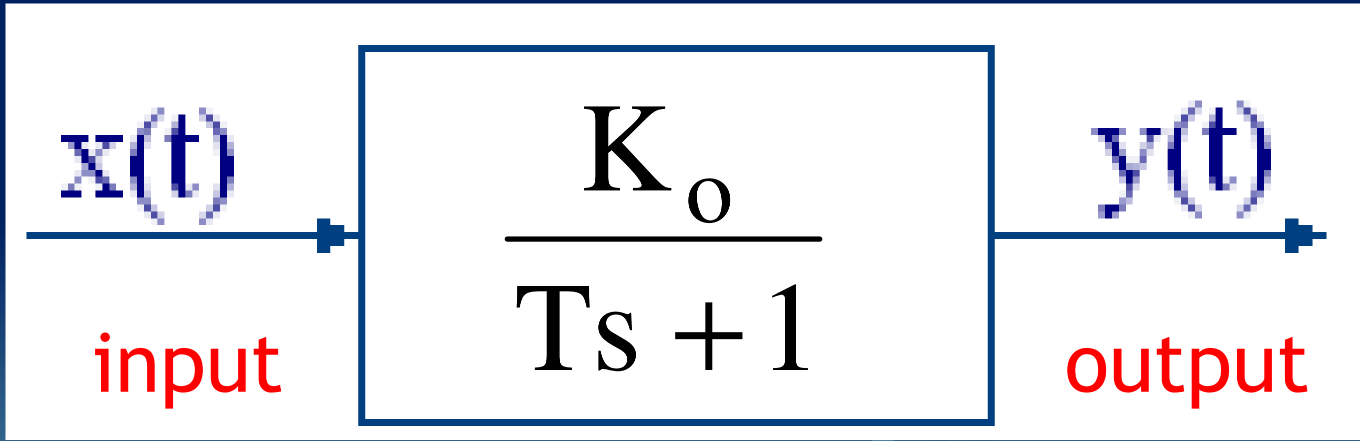


1st order systems

If $K_o = 1$, the unit ramp response is:

$$y(t) = t - T + T \cdot e^{-t/T}, \qquad t > 0$$

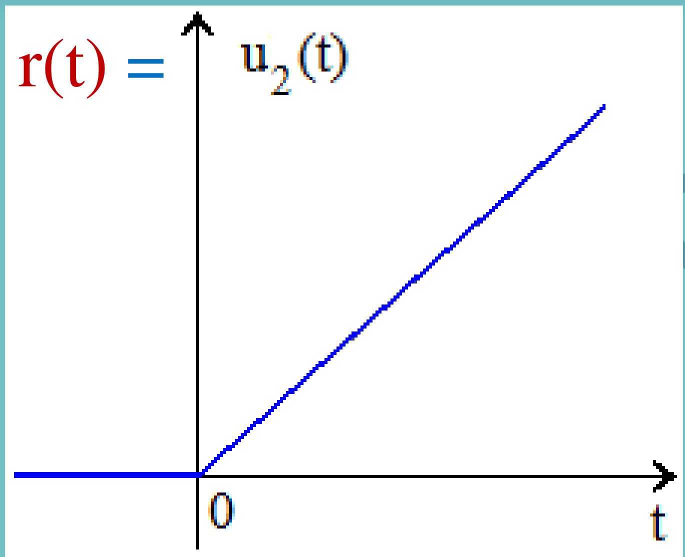
Time domain analysis - 1st order systems



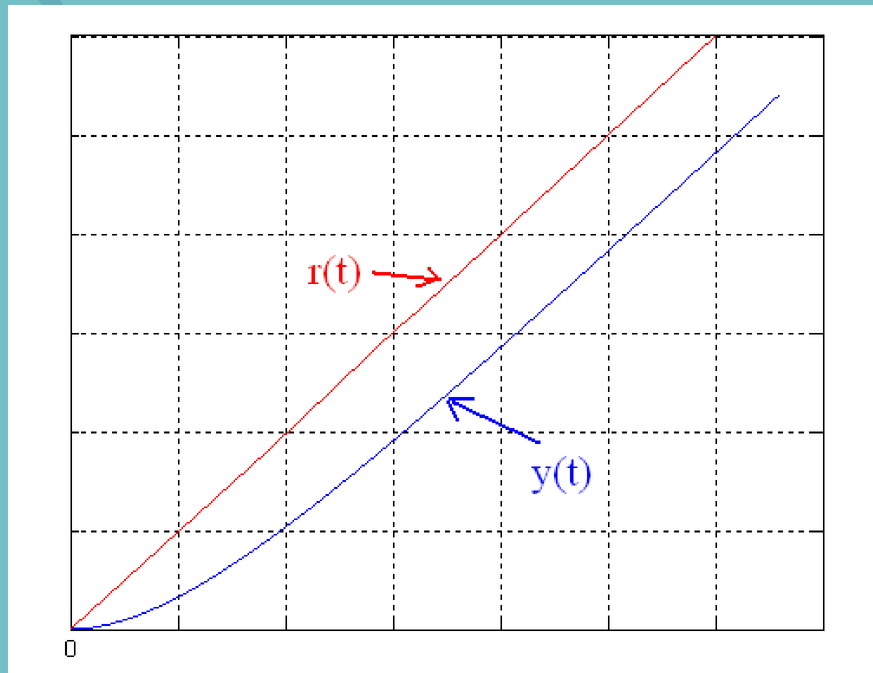
1st order systems

the unit ramp response for $K_o = 1$:

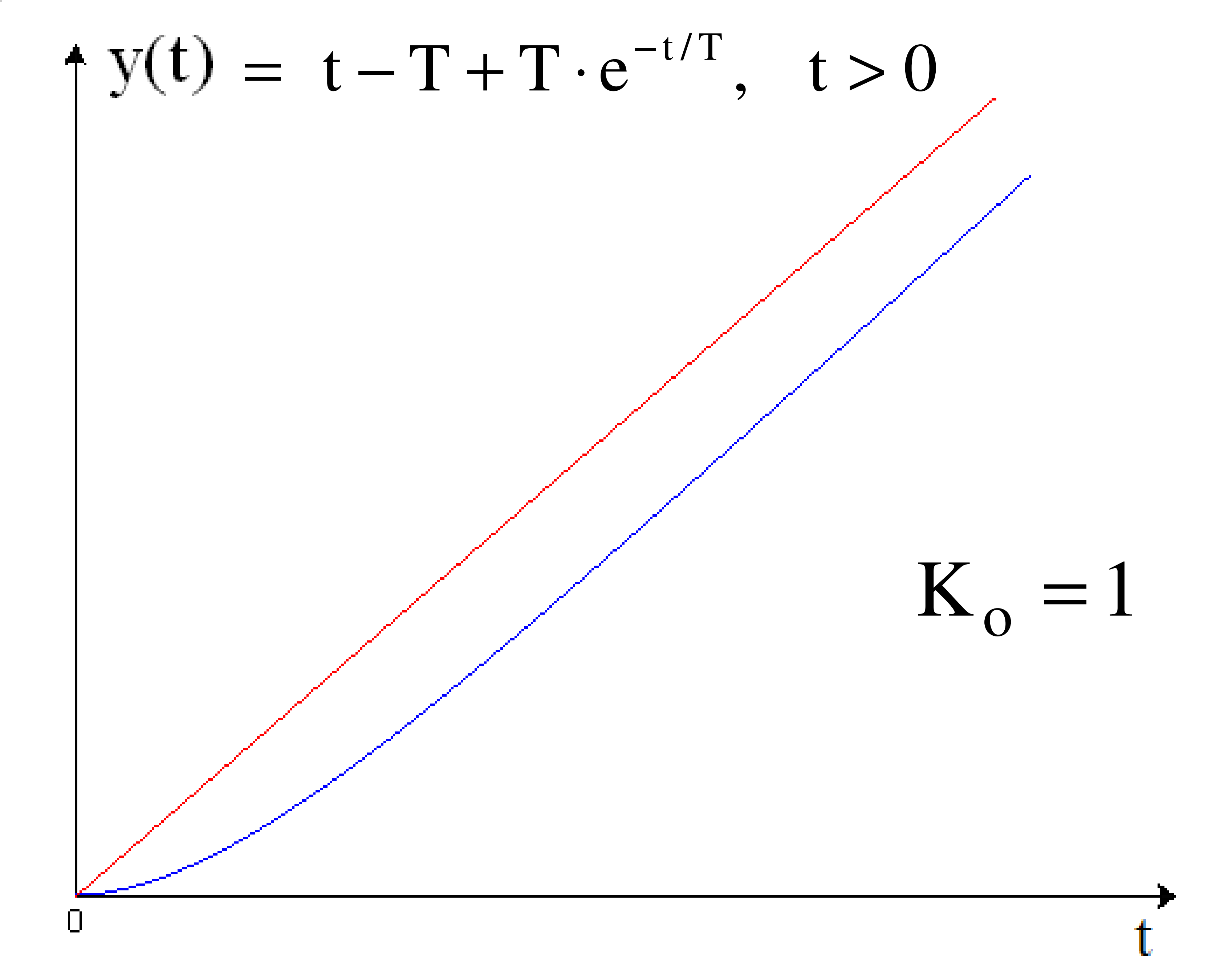
$$y(t) = t - T + T \cdot e^{-t/T}, \quad t > 0$$



unit ramp input



the unit ramp response is:



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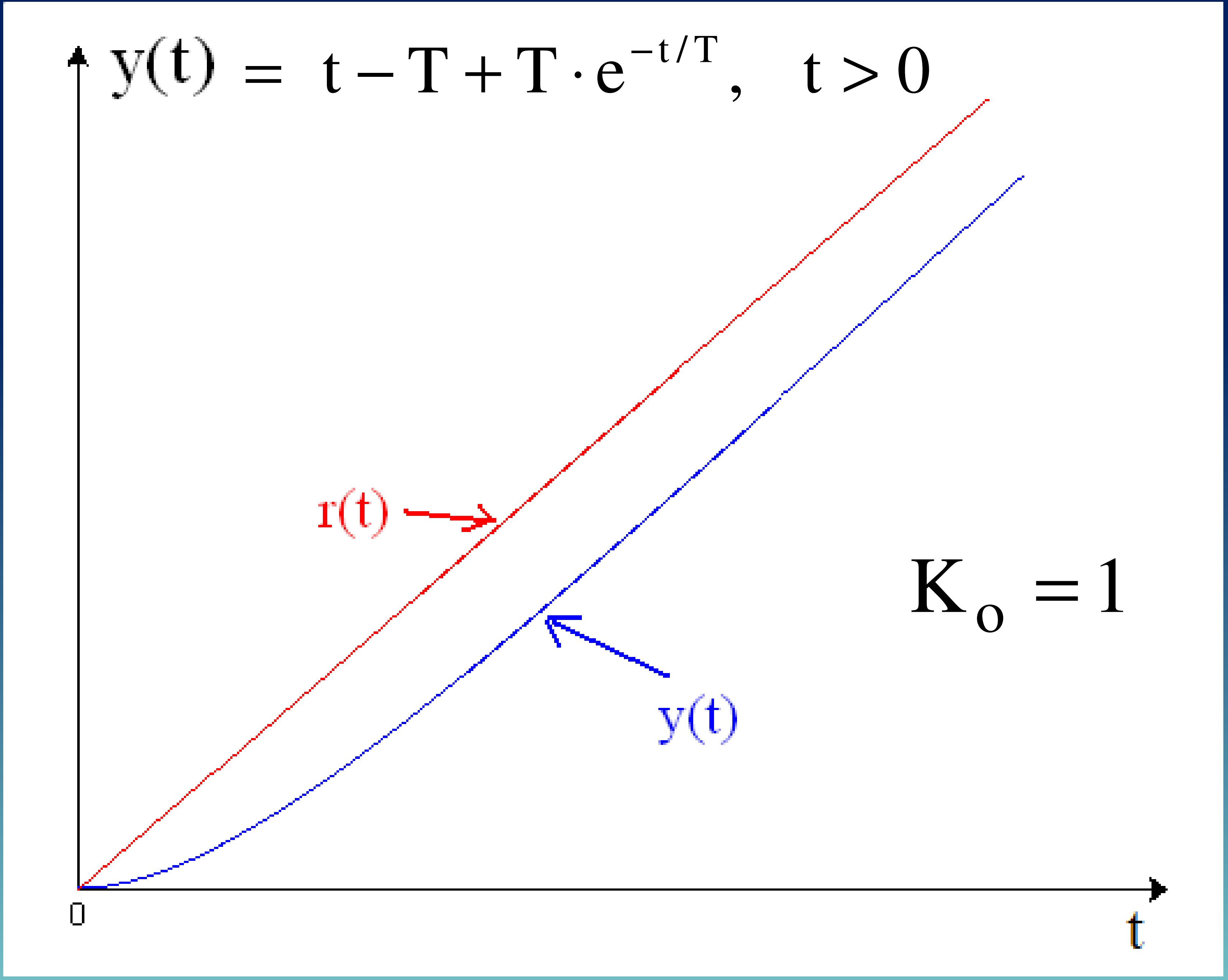
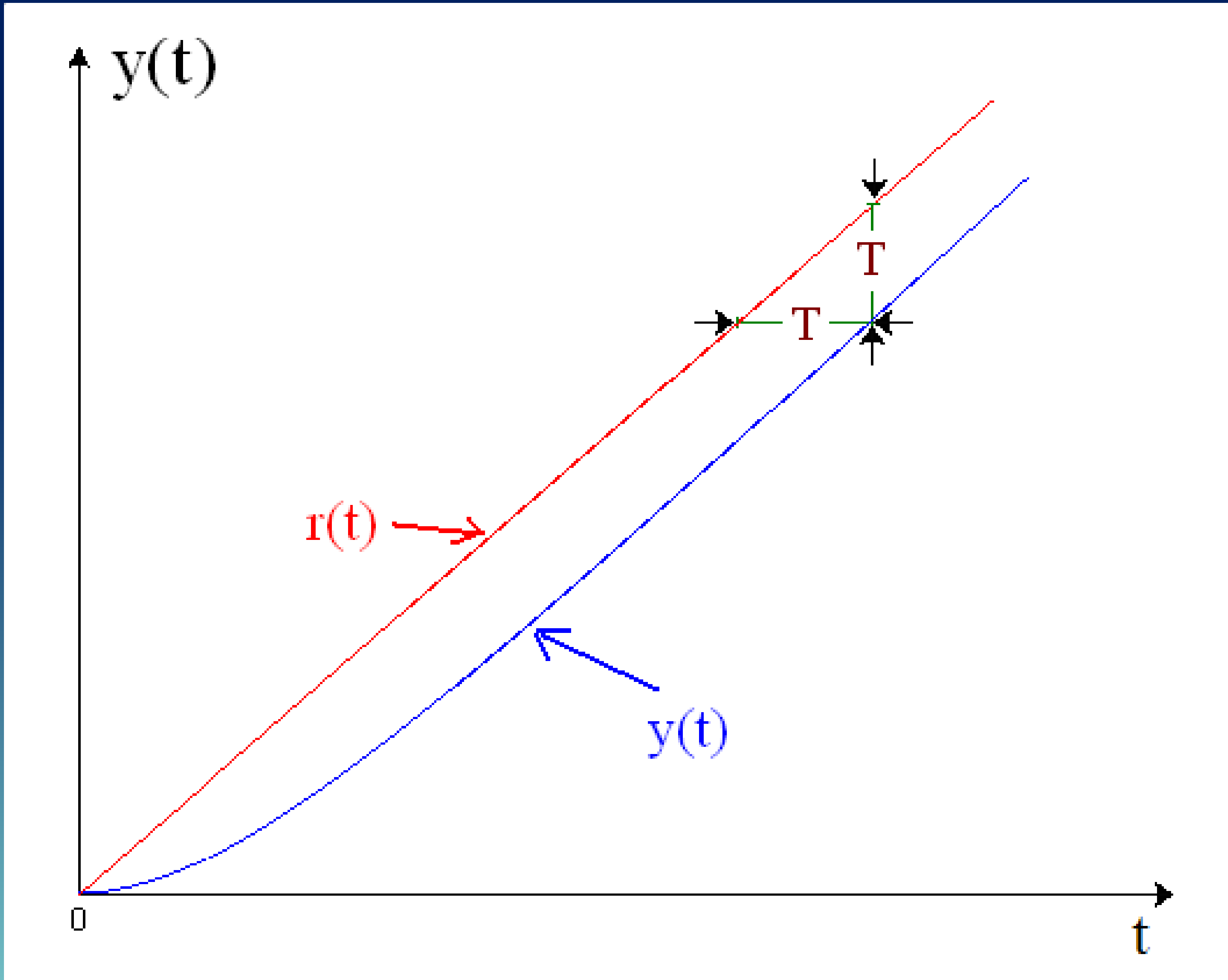
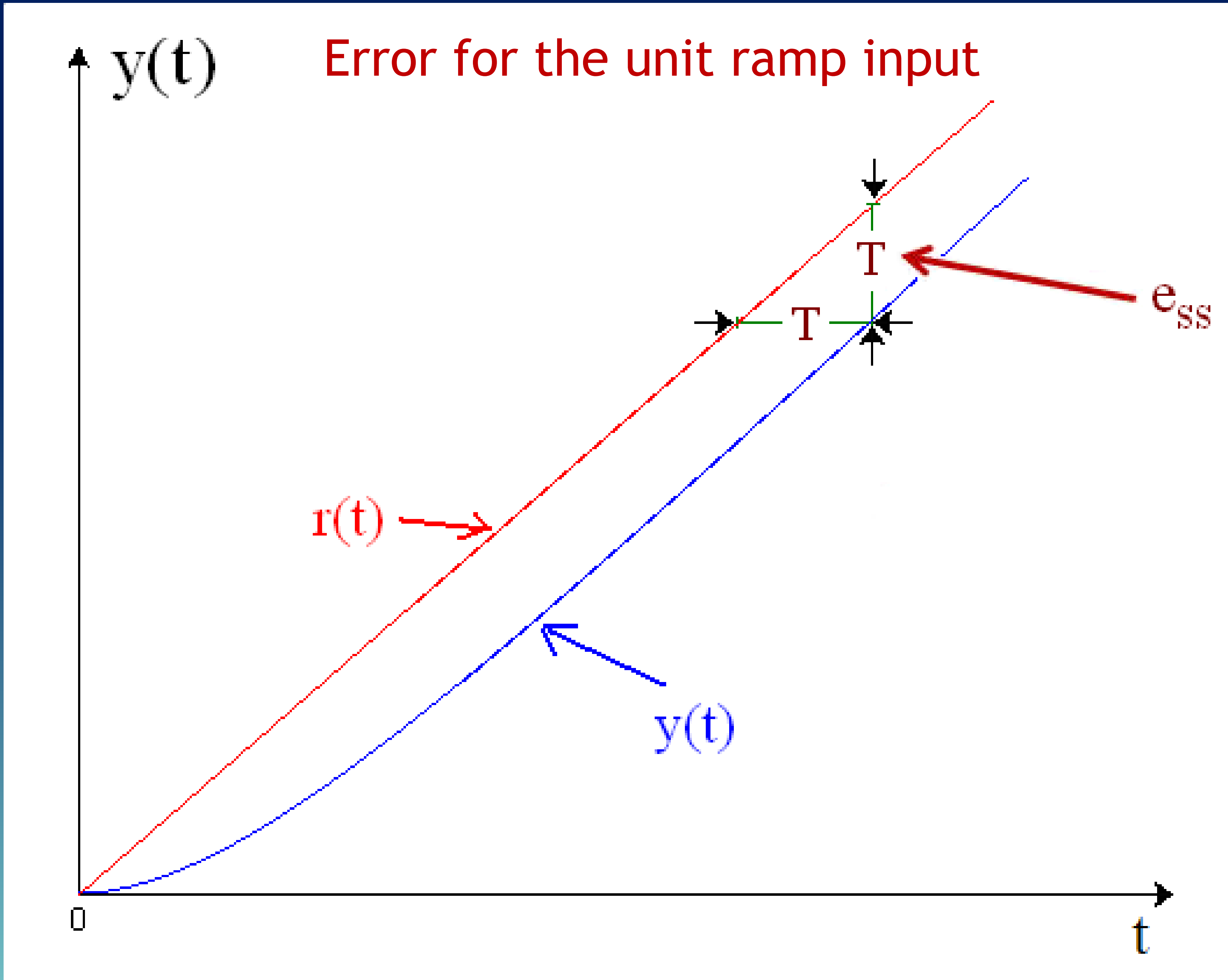
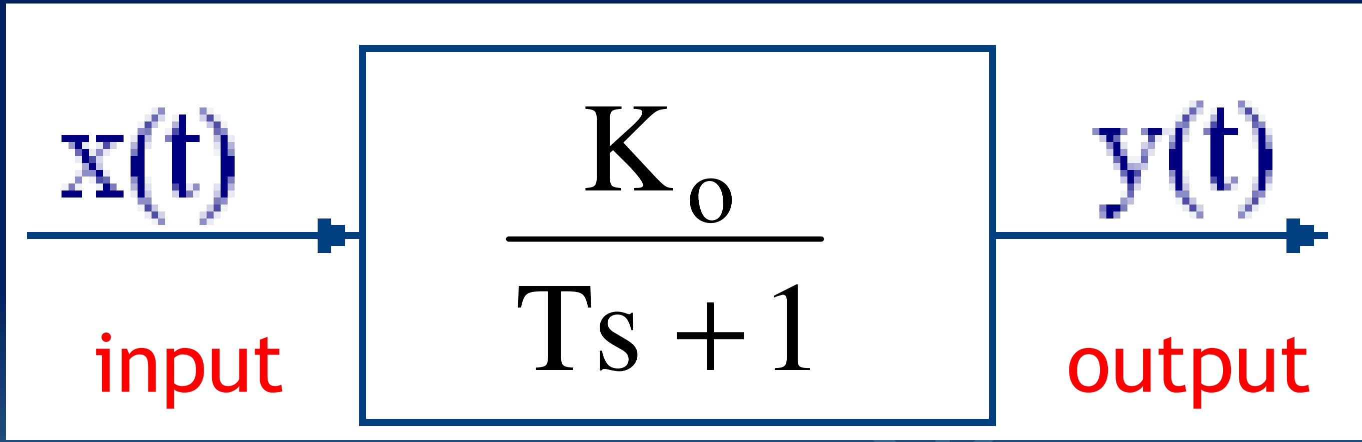


Time domain analysis - 1st order systems



Time domain analysis - 1st order systems



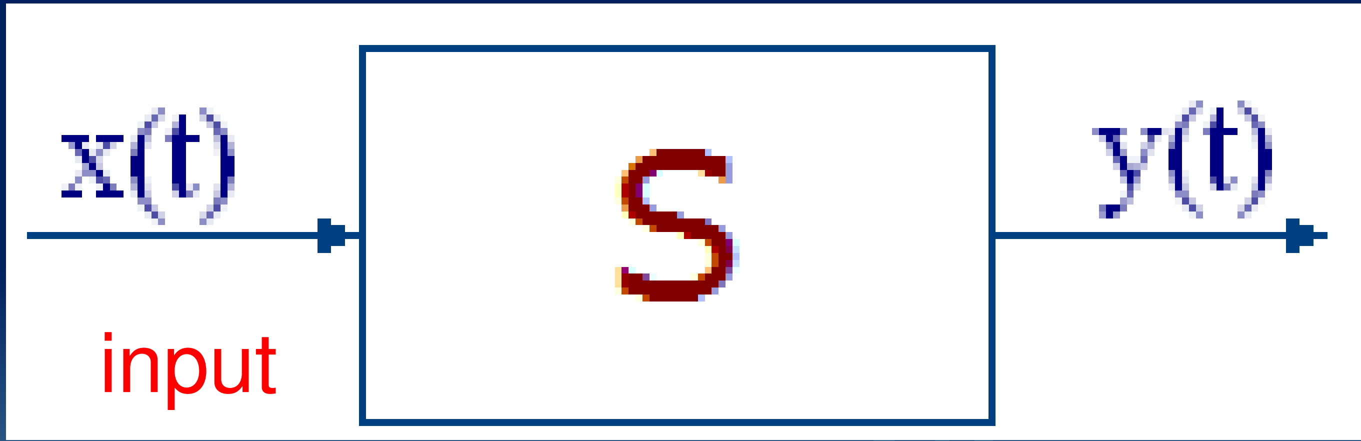


1st order systems

Error for the unit ramp input, $K_o = 1$:

$$E(s) = \frac{1}{s^2} - \frac{1}{Ts+1}\cdot\frac{1}{s^2} = \frac{1}{s^2}\cdot\left(1 - \frac{1}{Ts+1}\right) =$$

$$= \frac{1}{s^2}\cdot\frac{(Ts+1-1)}{Ts+1} = \frac{1}{s^2}\cdot\frac{Ts}{Ts+1}$$



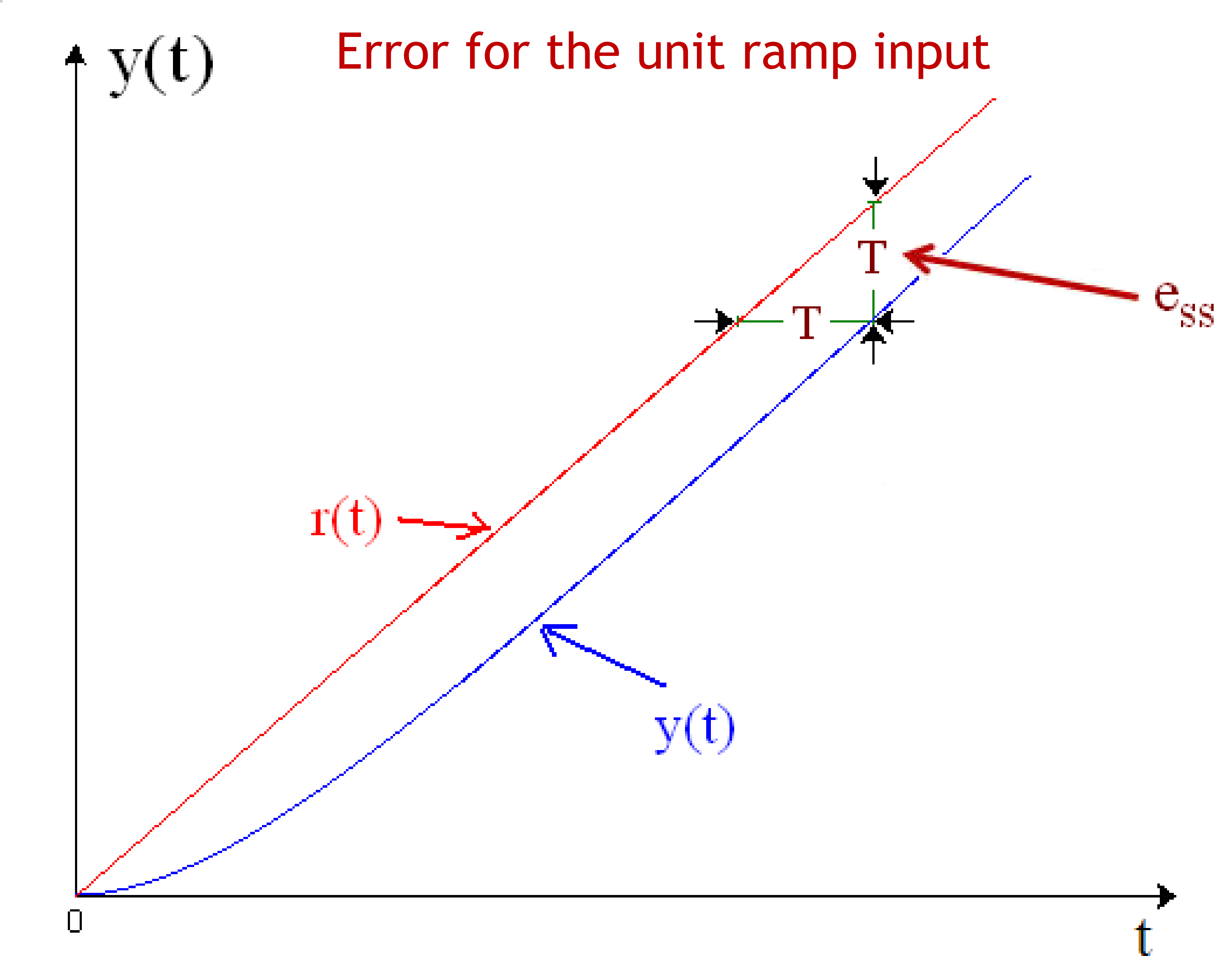
1st order systems

Steady state error:

$$e_{ss} = \lim_{s \to 0} s \cdot E(s) = \lim_{s \to 0} s \cdot \frac{1}{s^2} \cdot \frac{Ts}{Ts+1} =$$

$$= \lim_{s \to 0} \frac{T}{Ts+1} = T$$

$$e_{ss} = T$$


Error for the unit ramp input
y(t)
T
T
e_{ss}
r(t)
y(t)
0
t

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Departamento de
Engenharia Eletromecânica

Thank you!

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