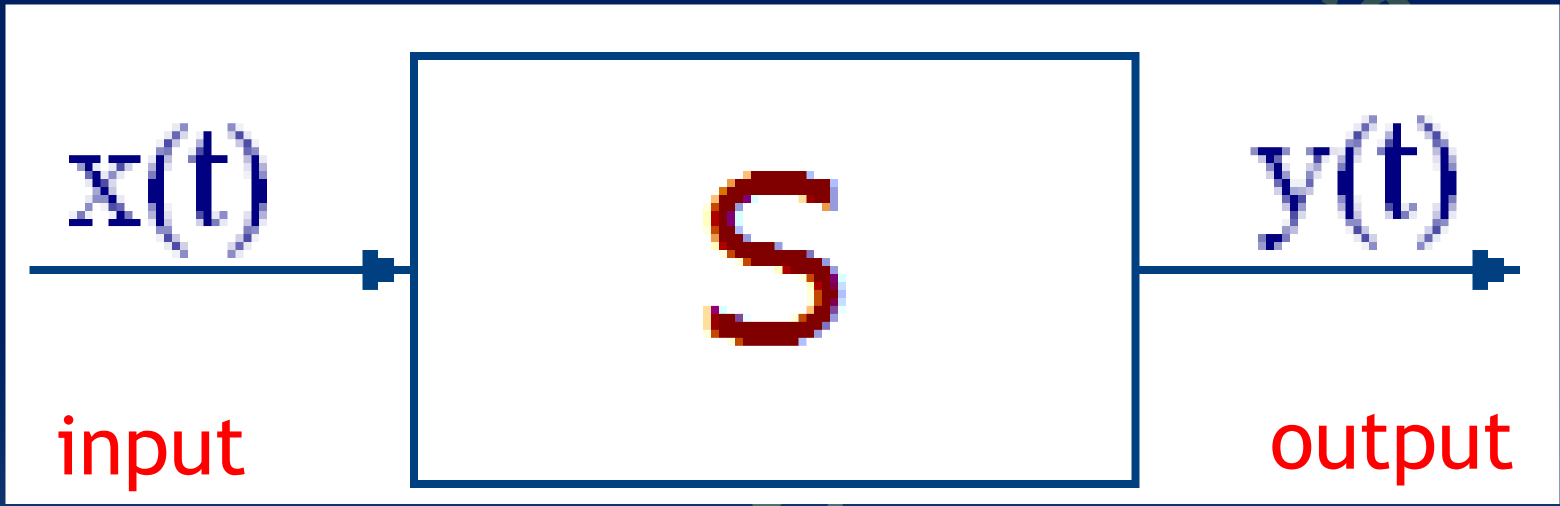
Control Systems

5

"Block Diagram, error and PID controllers"

J. A. M. Felippe de Souza

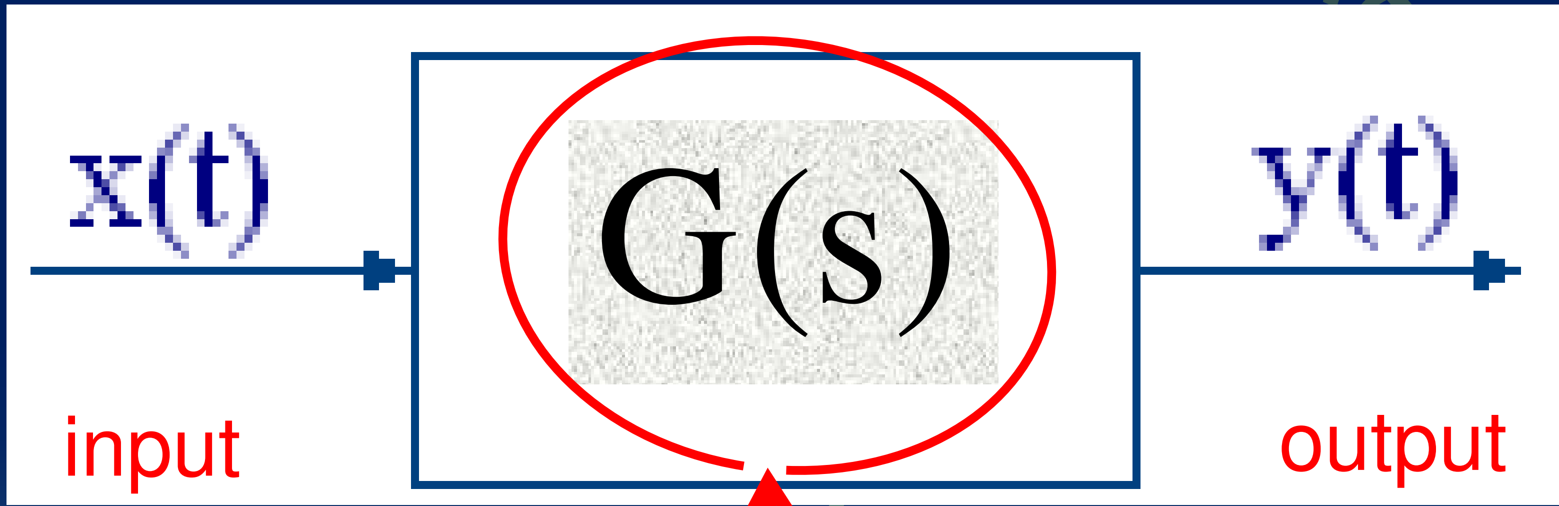
Block Diagrams & error



Single block

Black box

Black box



Transfer Function:

$$\boxed{G(s) = \frac{Y(s)}{X(s)}}$$

or:

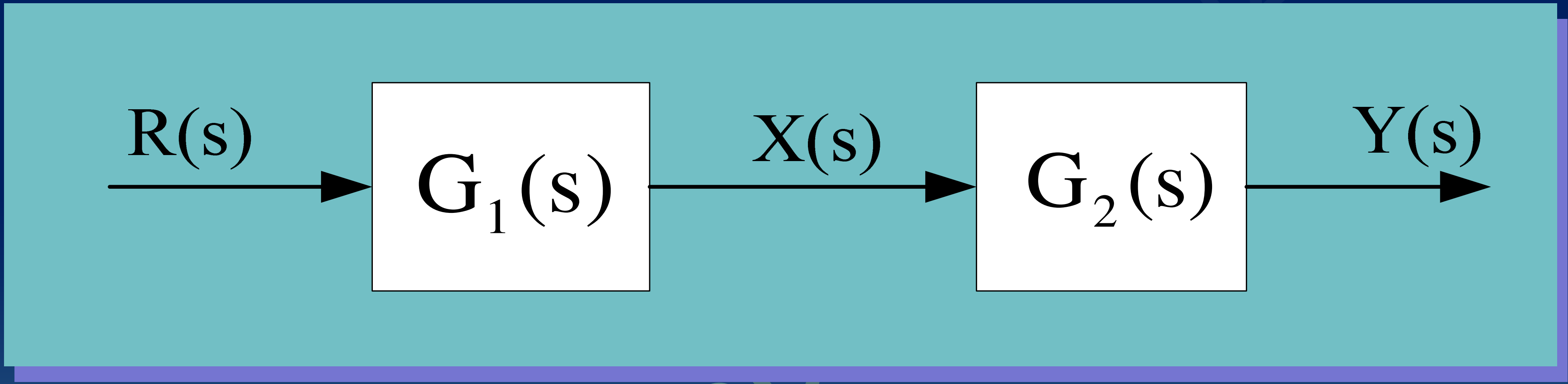
$$Y(s) = G(s) \cdot X(s)$$

that is,

$$\boxed{\text{OUTPUT} = \text{T. F.} \times \text{INPUT}}$$

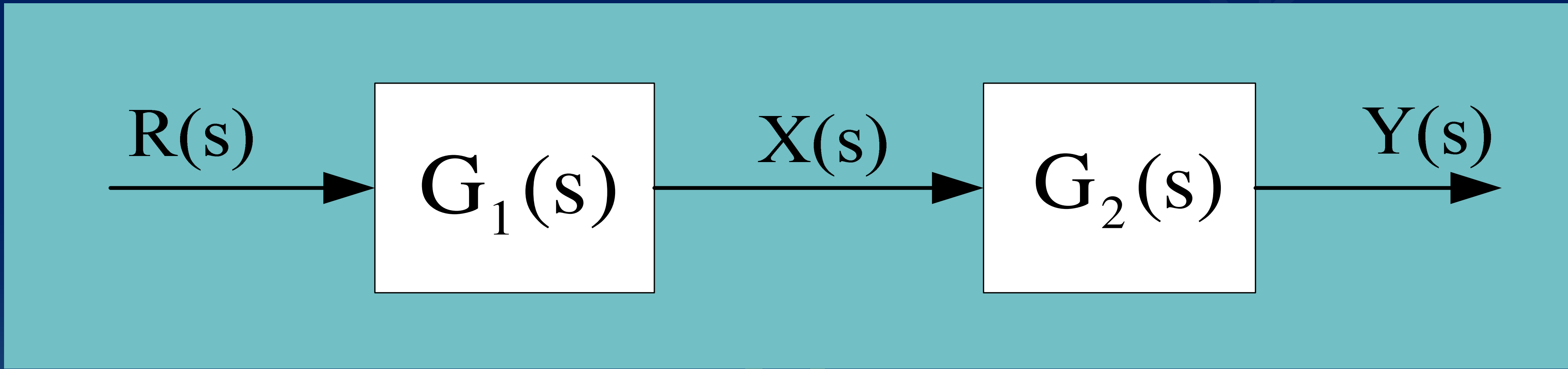
Block combination

blocks in cascade



the output $X(s)$ of the first block is the input of the second block.

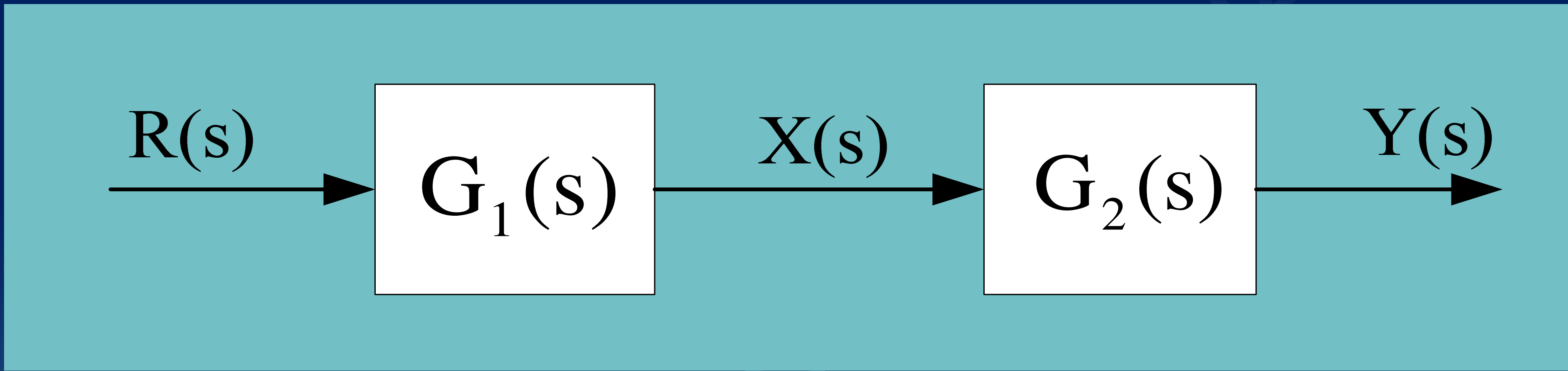
blocks in cascade



hence,

$$G_1(s) = \frac{X(s)}{R(s)} \qquad G_2(s) = \frac{Y(s)}{X(s)}$$

blocks in cascade



OUTPUT = T.F. x INPUT

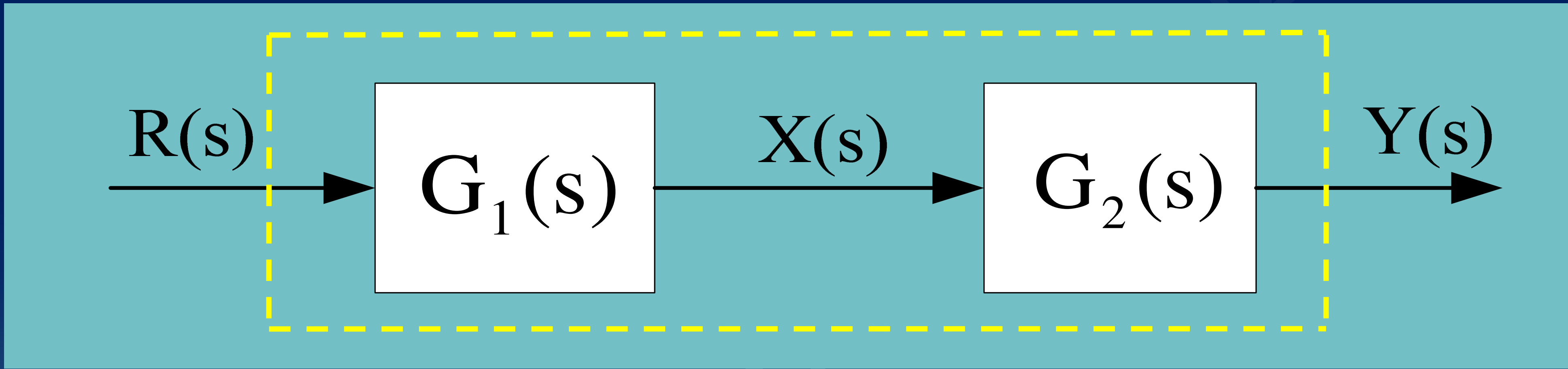
that is,

$$\begin{cases} X(s) = G_1(s) \cdot R(s) & \text{for the 1º block} \\ Y(s) = G_2(s) \cdot X(s) & \text{for the 2º block} \end{cases}$$

OUTPUT = T.F. x INPUT

Block Diagrams & error

blocks in cascade



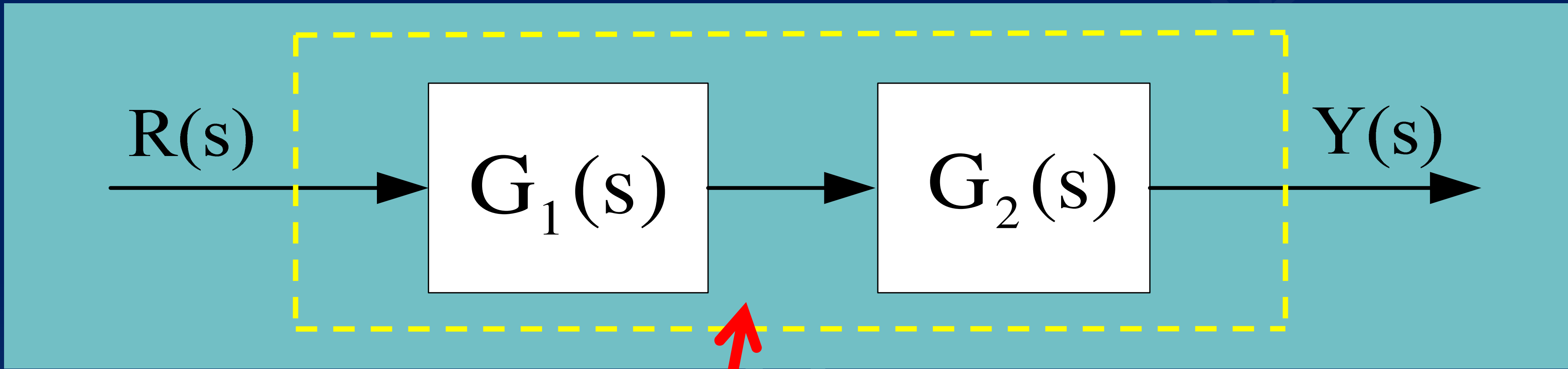
hence:

$$Y(s) = G_1(s) \cdot G_2(s) \cdot R(s)$$

or,

$$\frac{Y(s)}{R(s)} = G_1(s) \cdot G_2(s)$$

blocks in cascade

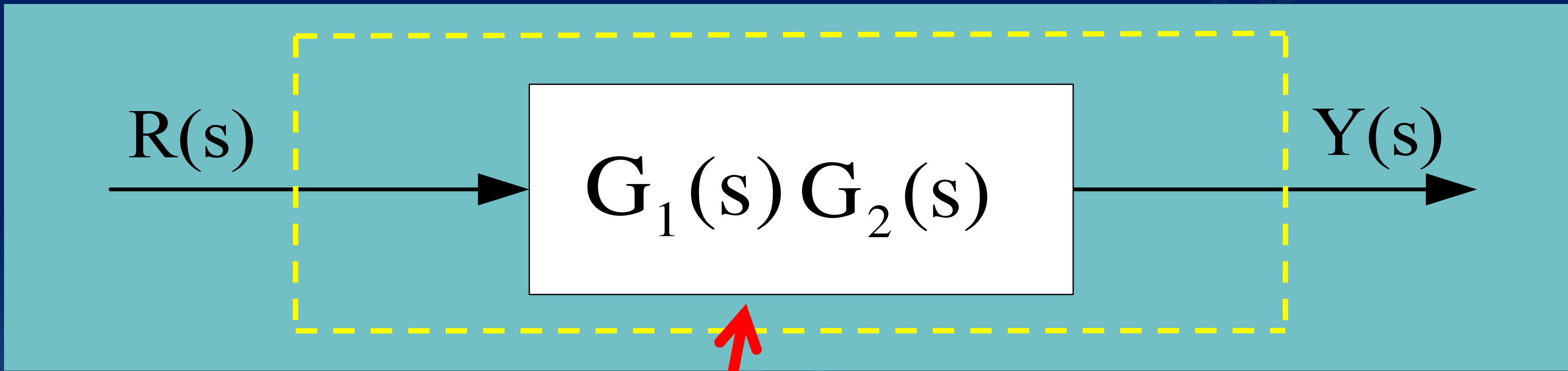


thus:

$$G_1(s) \cdot G_2(s)$$

Block Diagrams & error

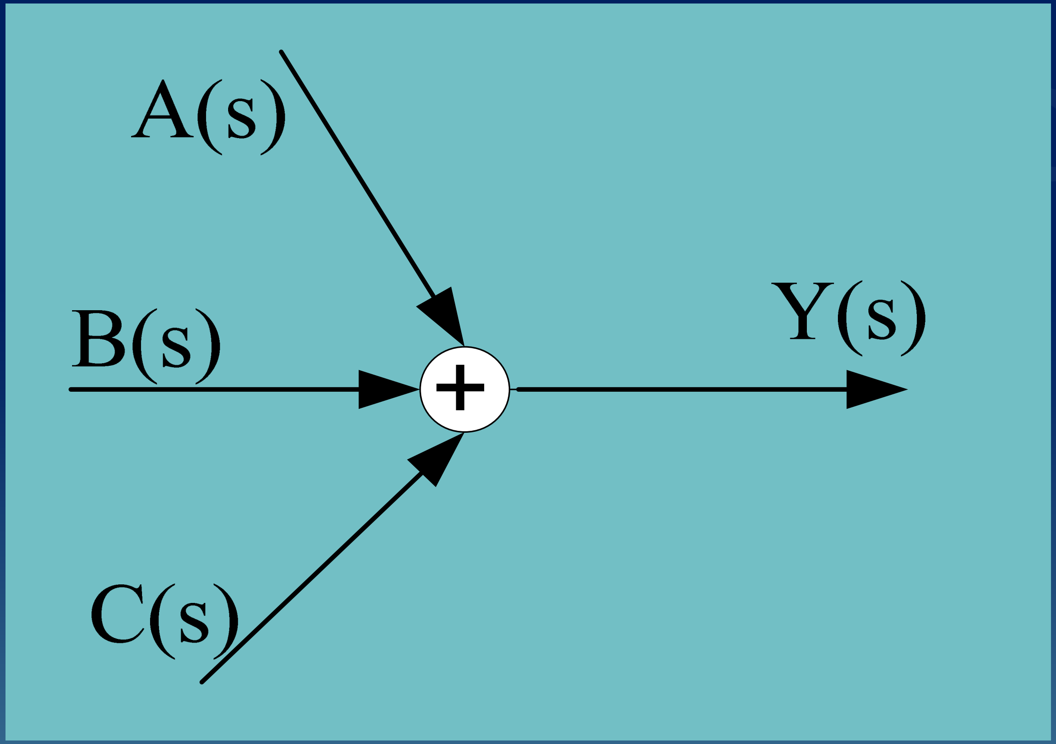
blocks in cascade



and hence:

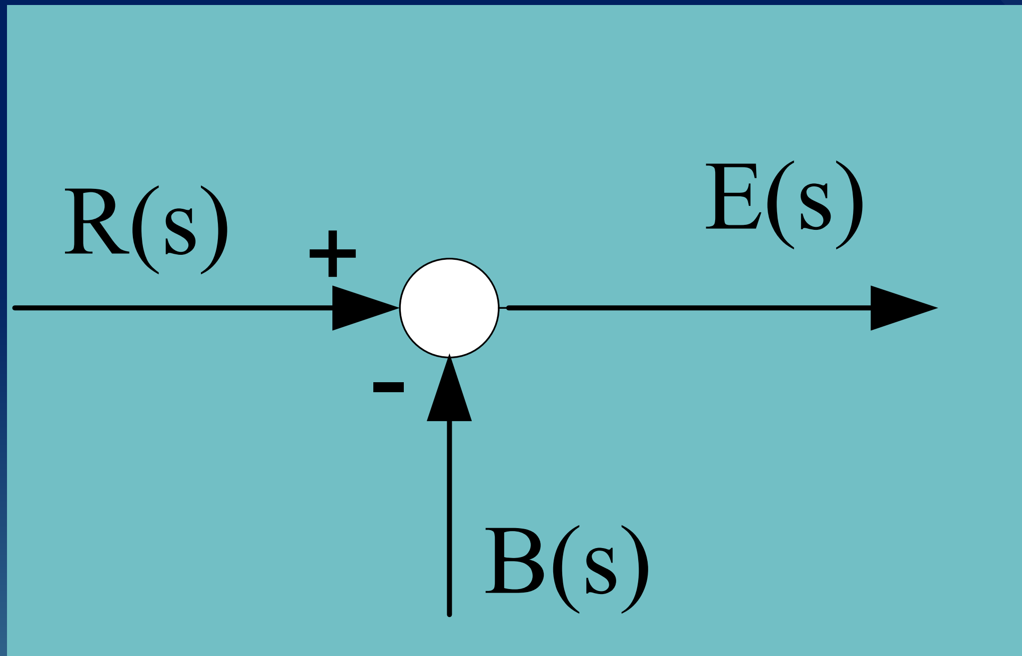
$$G_1(s) \cdot G_2(s)$$

summing point



$$Y(s) = A(s) + B(s) + C(s)$$

error detector

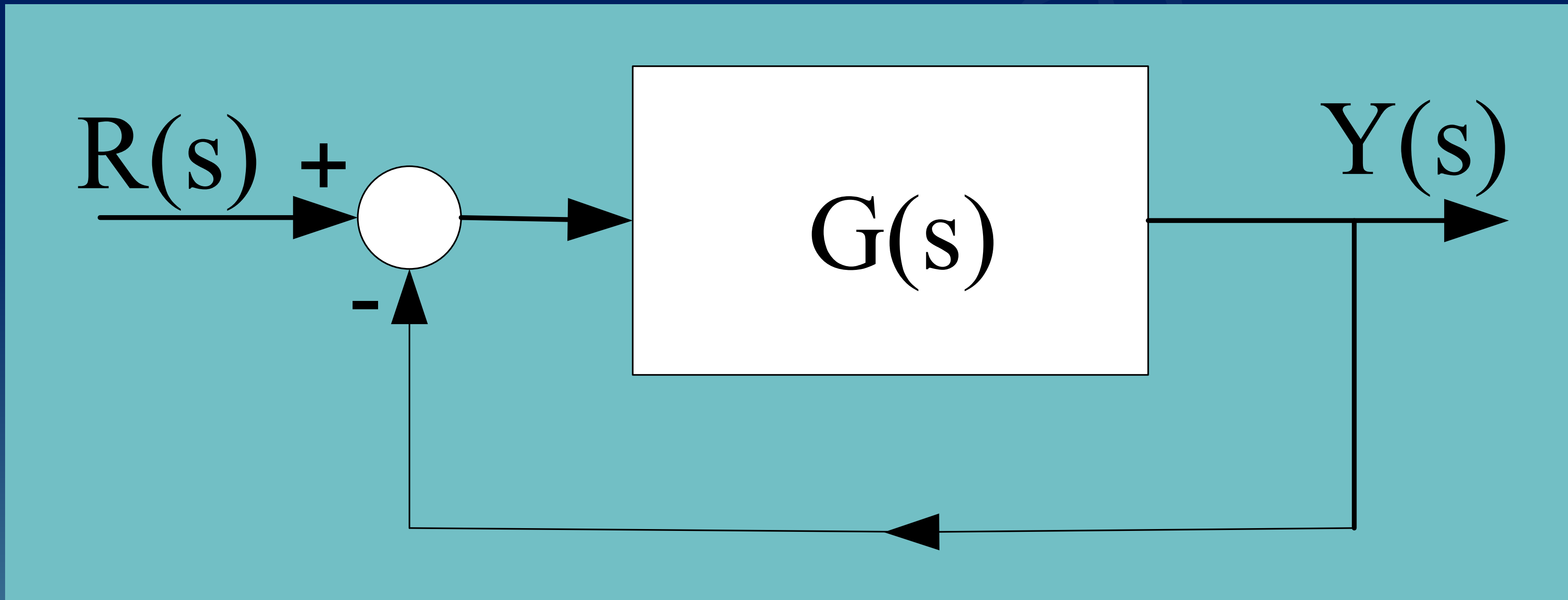


$$E(s) = R(s) - B(s)$$

Error

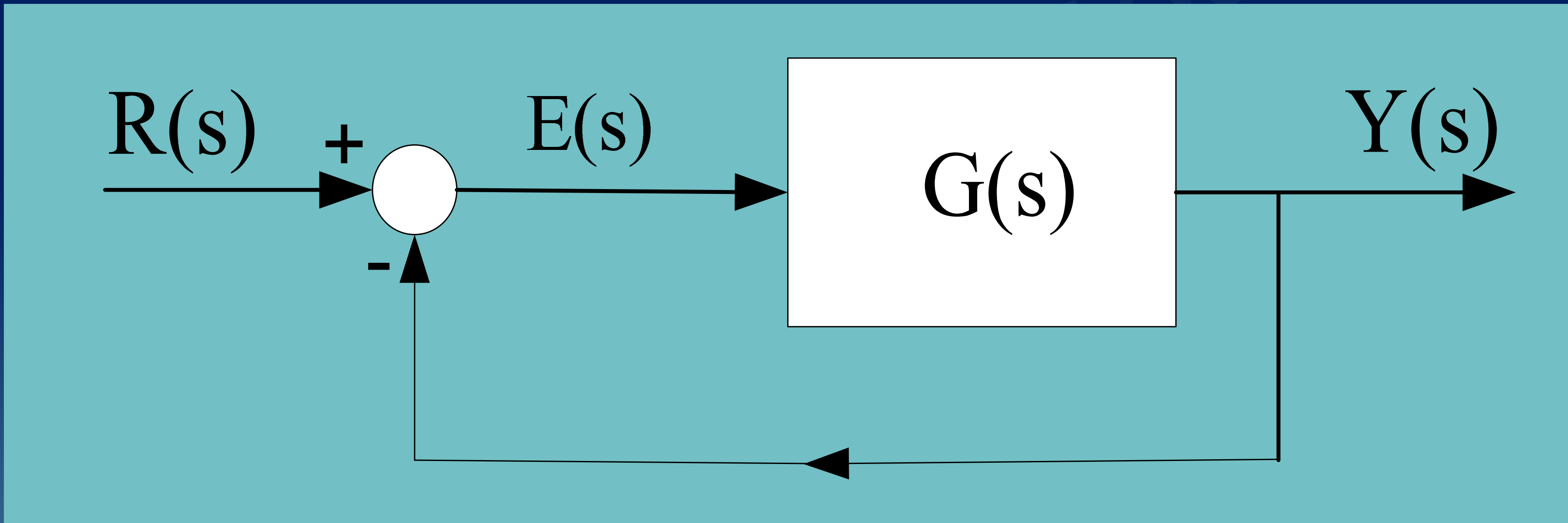
feedback

feedback



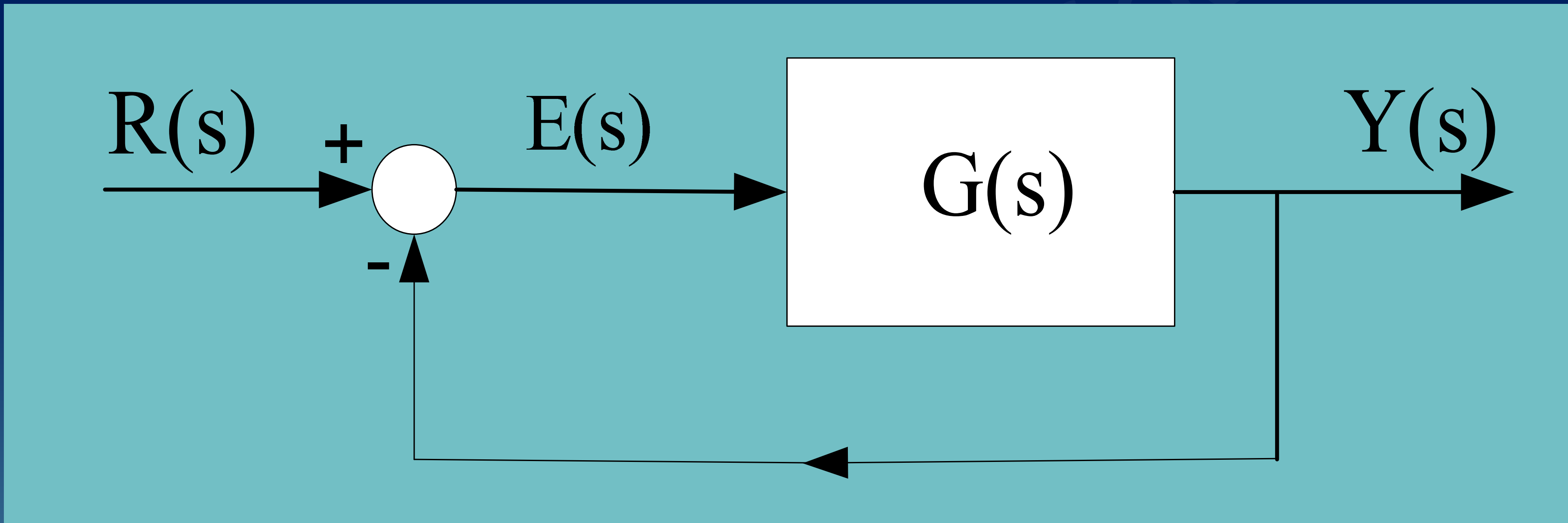
information from the output Y(s) is reintroduced in the input, after comparing with the reference signal R(s).

unit feedback



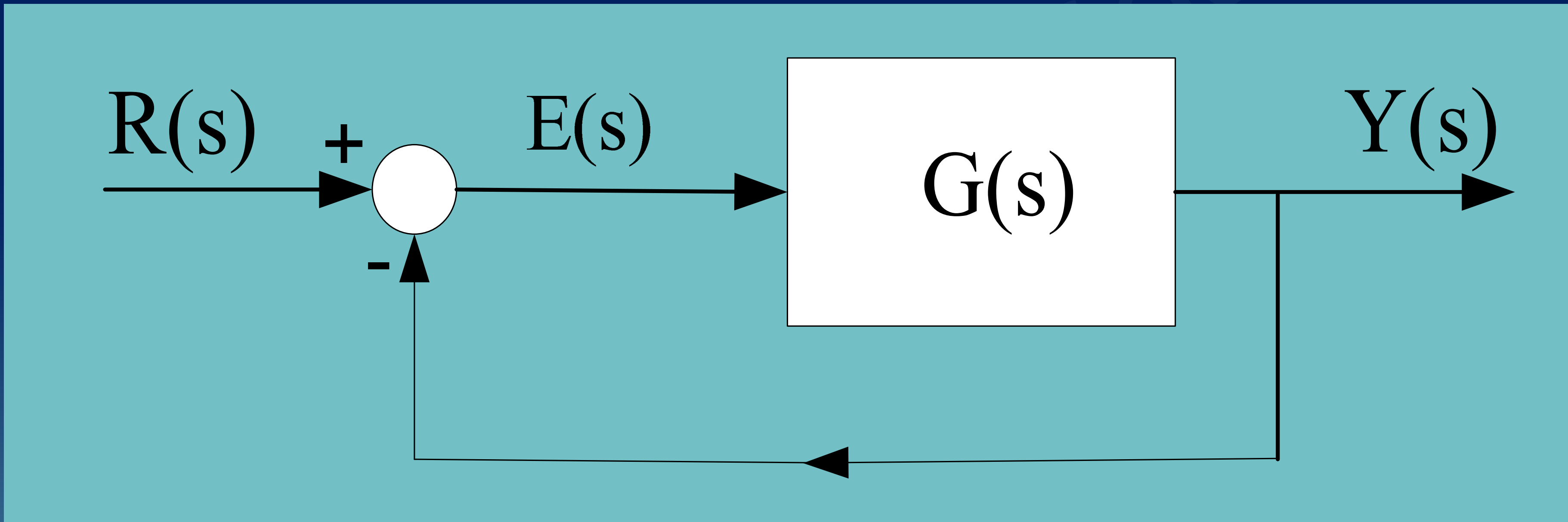
$$\begin{cases} E(s) = R(s) - Y(s) \\ Y(s) = G(s) \cdot E(s) \end{cases}$$

unit feedback



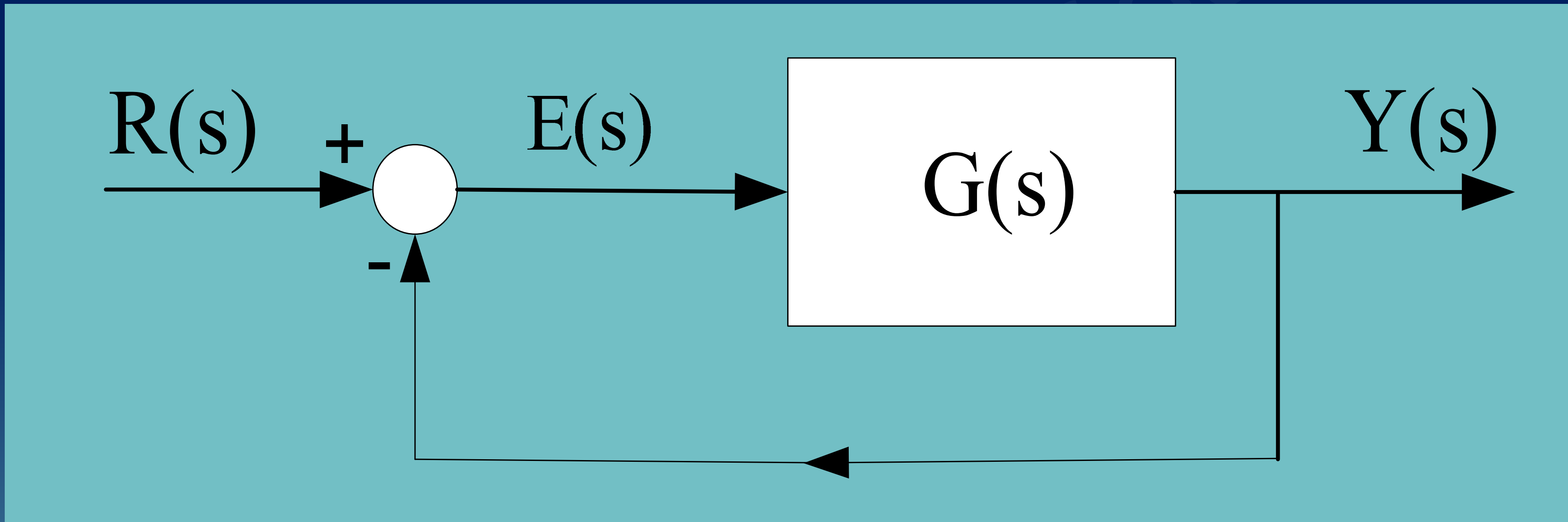
$$Y(s) = G(s) \cdot \underbrace{\left[R(s) - Y(s)\right]}_{E(s)}$$

unit feedback



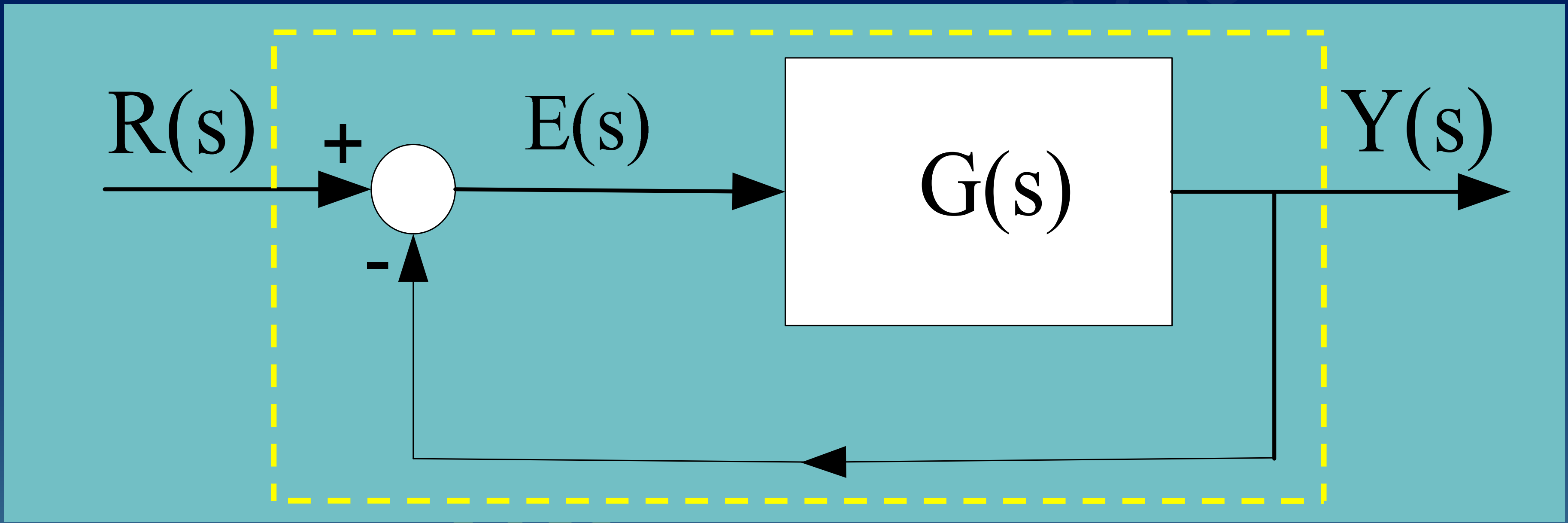
$$Y(s) = G(s) \cdot R(s) - G(s)Y(s)$$

unit feedback



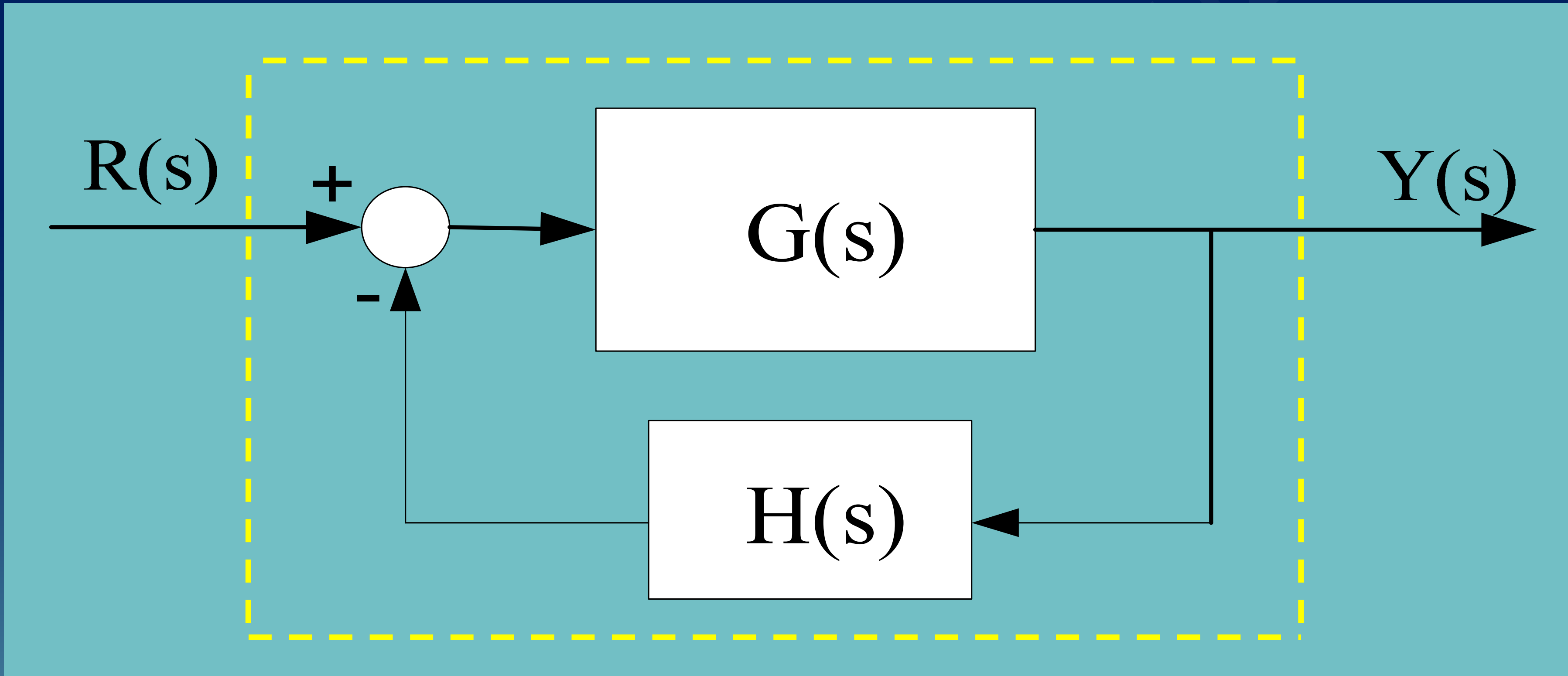
$$Y(s)\cdot\left[1+G(s)\right] = R(s)G(s)$$

unit feedback



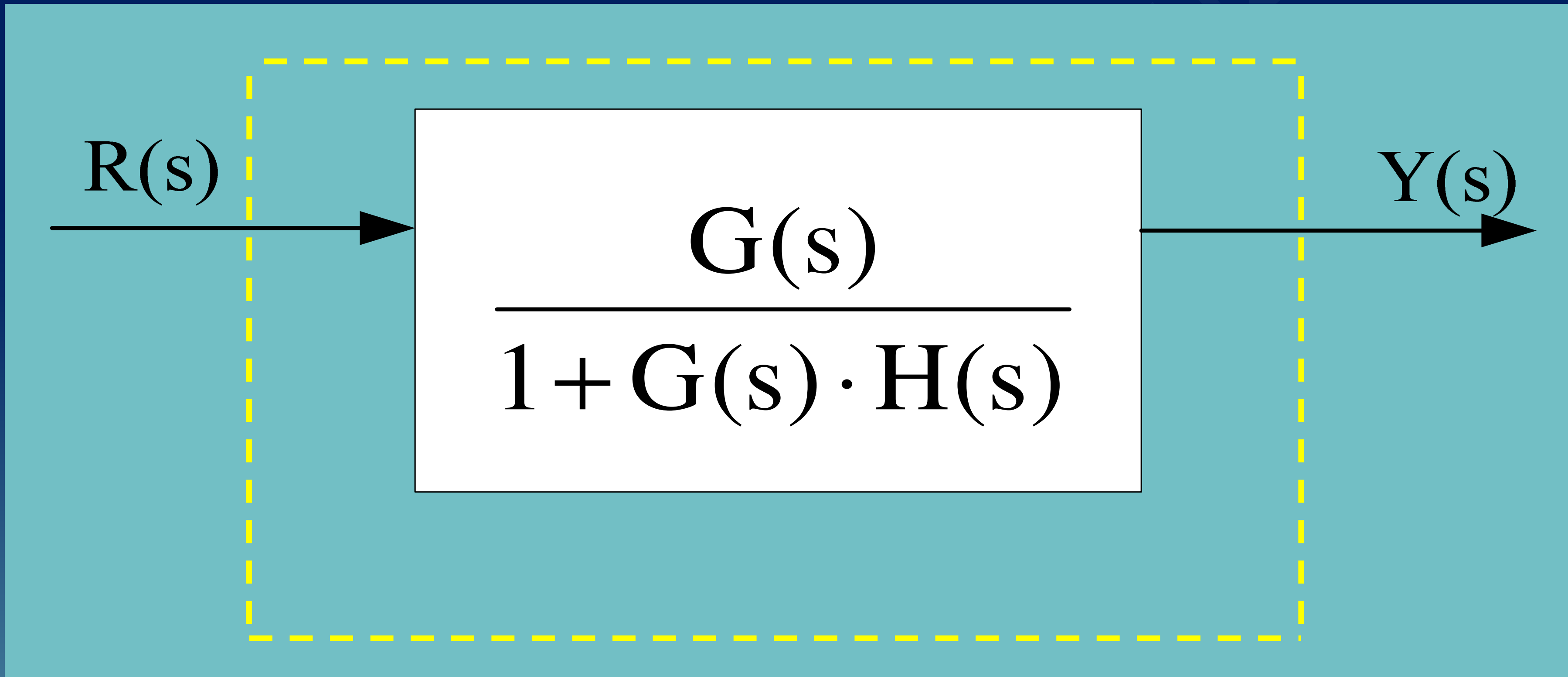
$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1+G(s)}$$

non unit feedback



$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1+G(s)\cdot H(s)}$$

non unit feedback



$$\boxed{\frac{Y(s)}{R(s)} = \frac{G(s)}{1+G(s)\cdot H(s)}}$$

Note that

the unit feedback

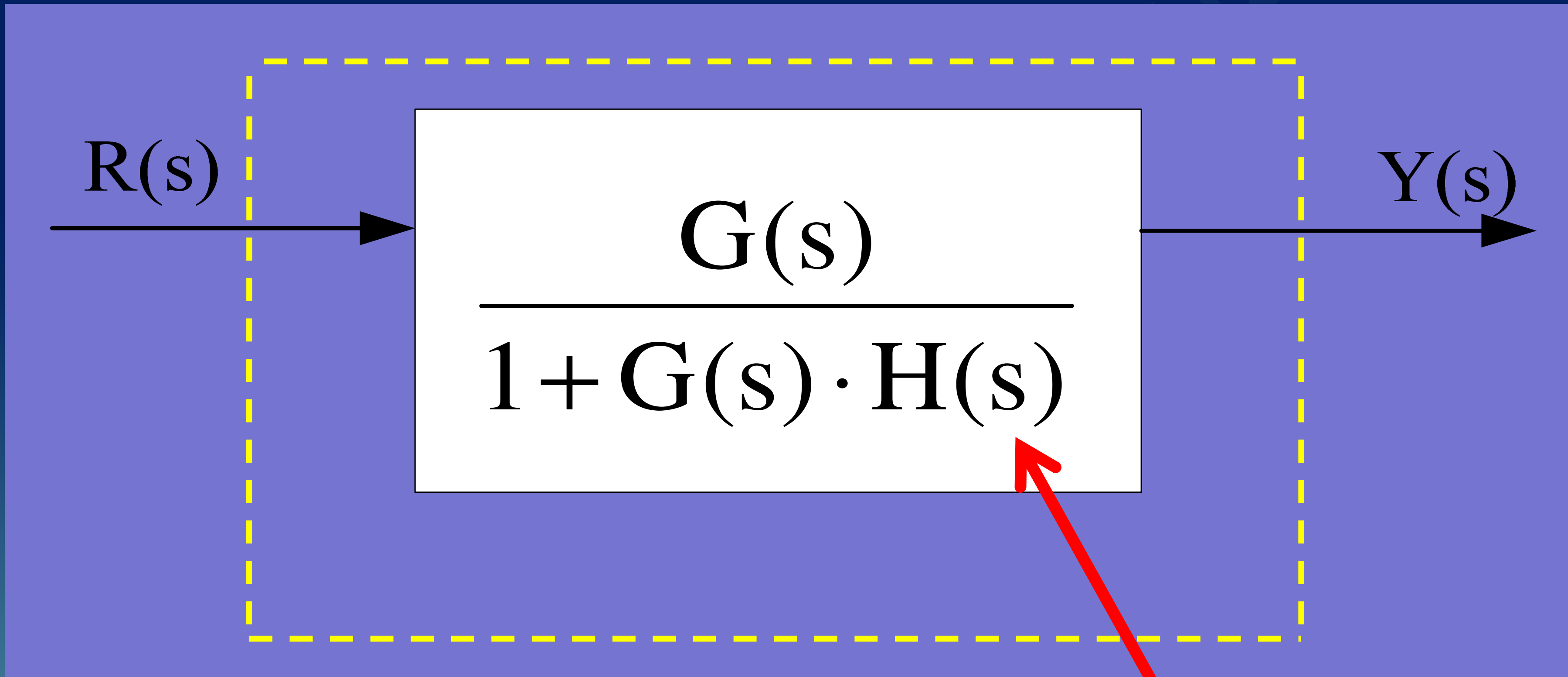
corresponds to

non unit feedback

with

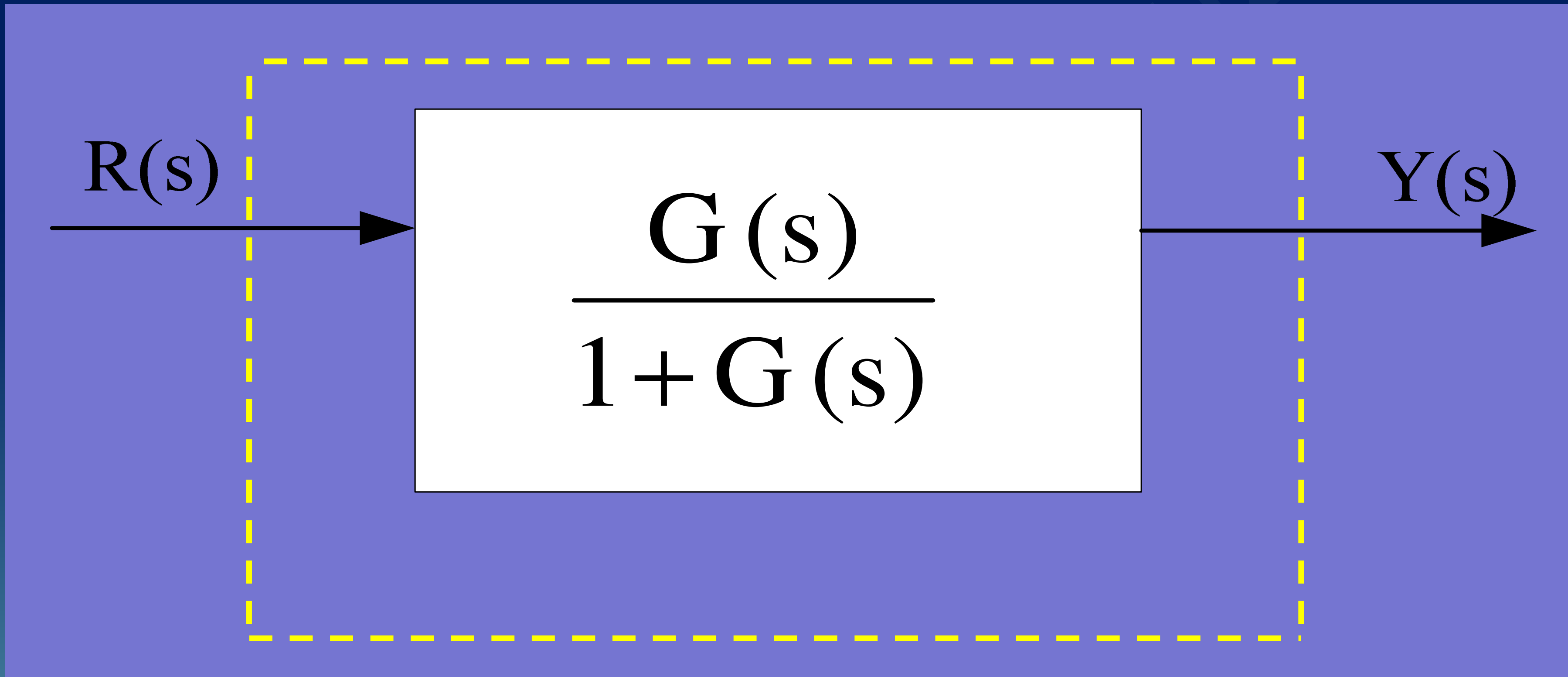
$$H(s) = 1$$

unit feedback



$$H(s)=1$$

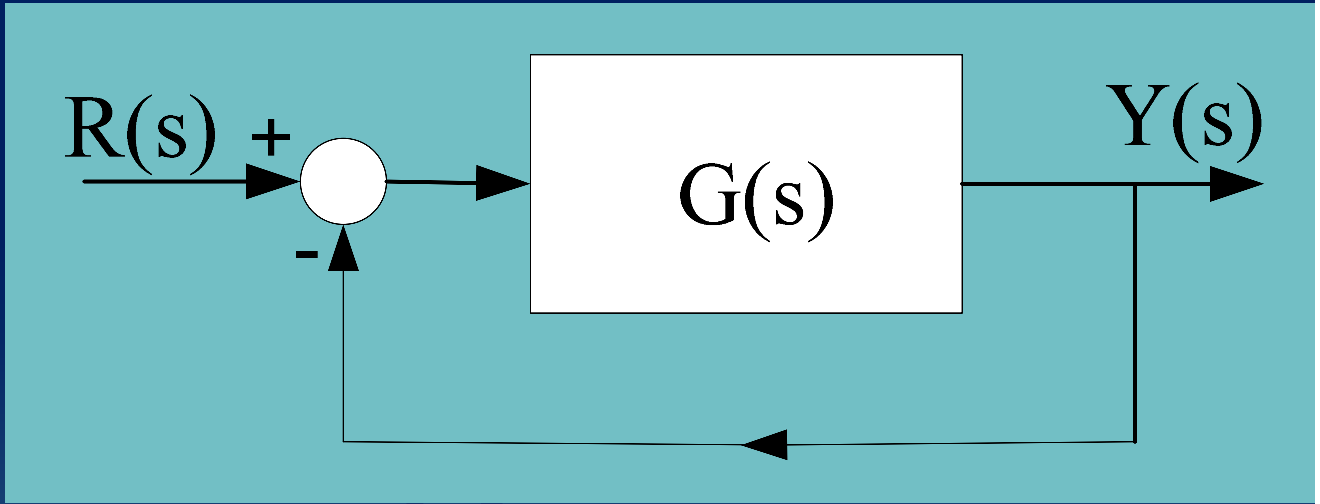
unit feedback



$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1+G(s)}$$

Example 1:

$$G(s) = \frac{5}{s(s+4)}$$



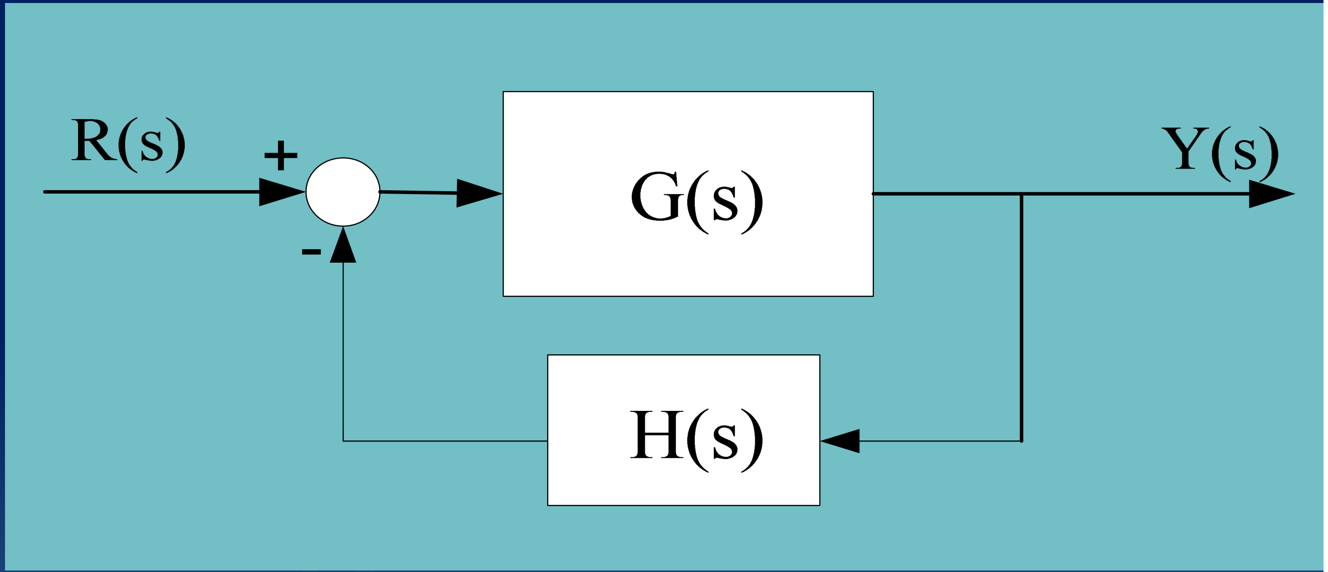
$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{\frac{5}{s(s+4)}}{1+\frac{5}{s(s+4)}} = \frac{5}{s^2+4s+5}$$

Block Diagrams & error

Example 2:

$$G(s) = \frac{5}{s(s+4)}$$

$$H(s) = \frac{1}{(s+3)}$$



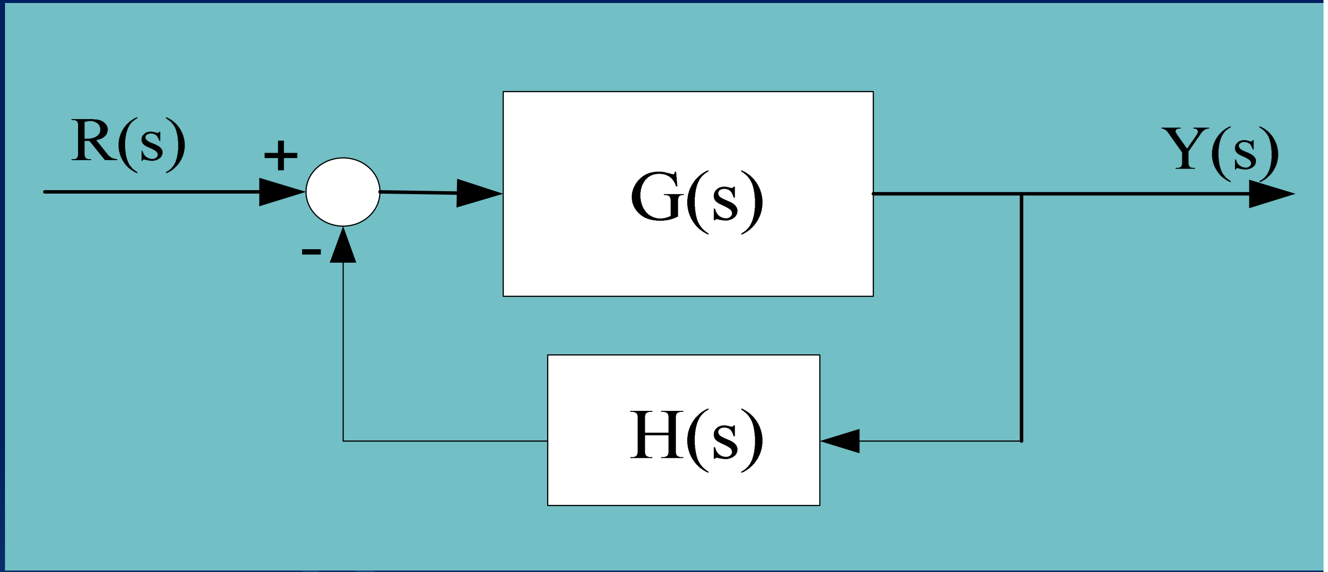
$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{\dfrac{5}{s(s+4)}}{1+\dfrac{5}{s(s+4)}\cdot\dfrac{1}{(s+3)}}$$

Block Diagrams & error

Example 2:

$$G(s) = \frac{5}{s(s+4)}$$

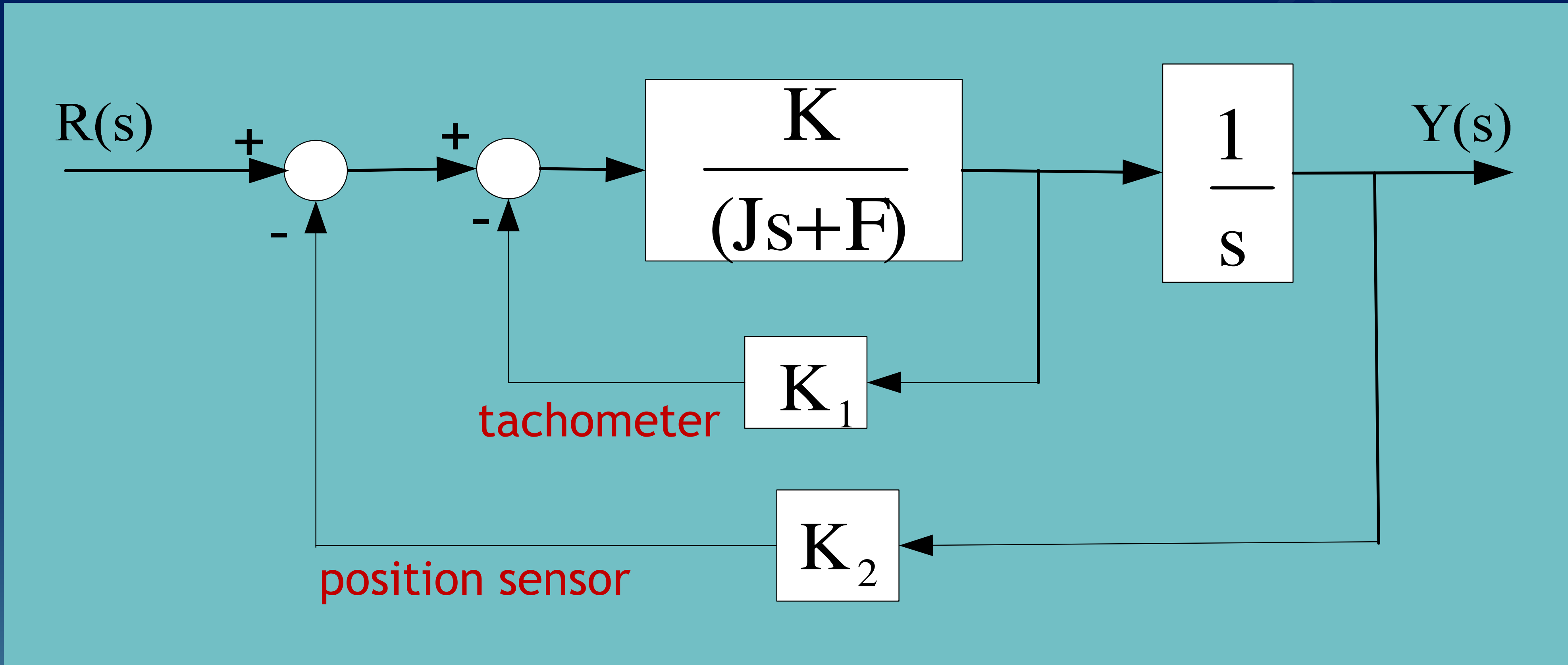
$$H(s) = \frac{1}{(s+3)}$$



$$\frac{Y(s)}{R(s)} = \frac{5(s+3)}{(s^3+7s^2+12s+5)}$$

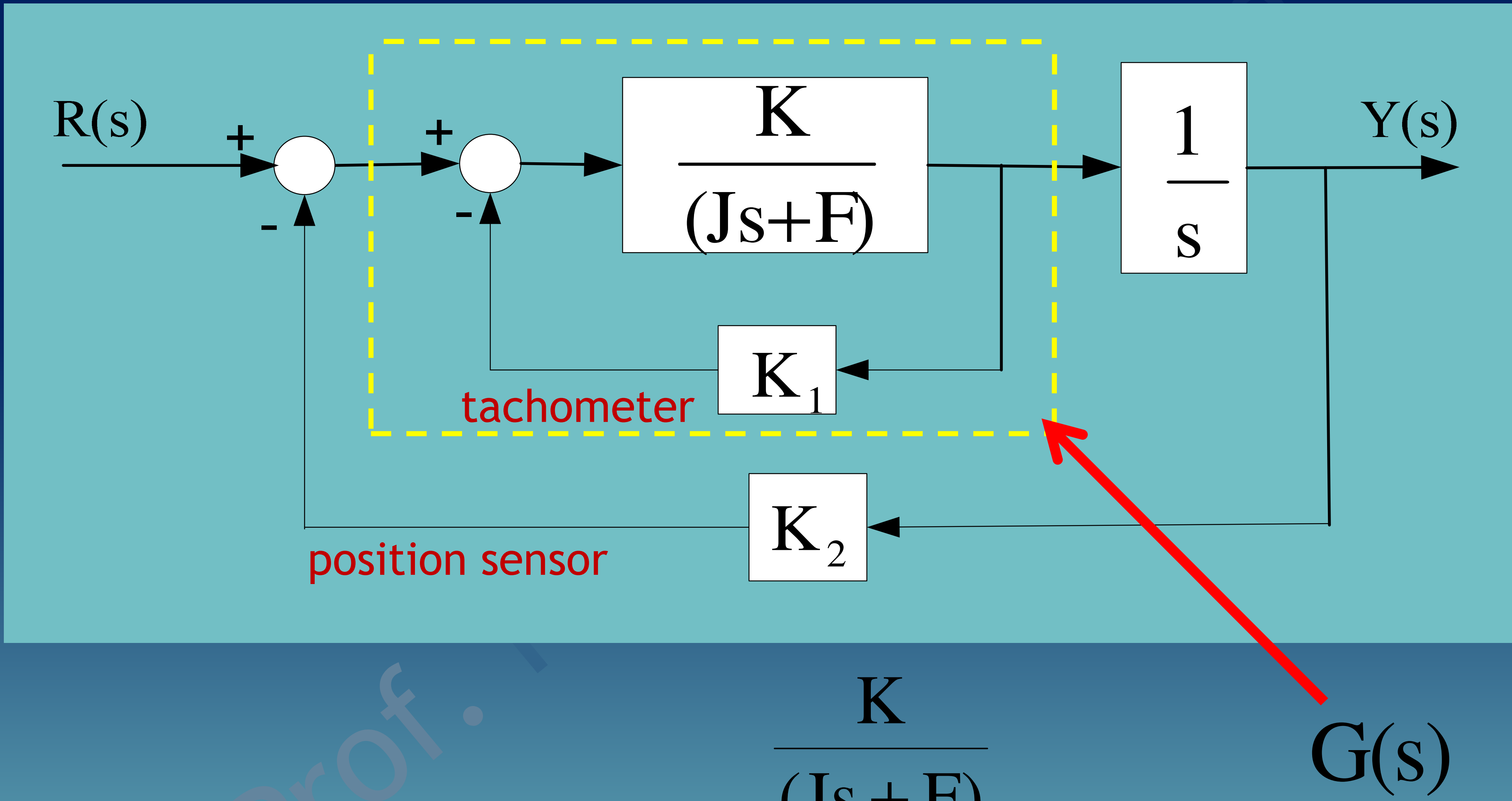
feedback with tachometers

feedback with tachometers



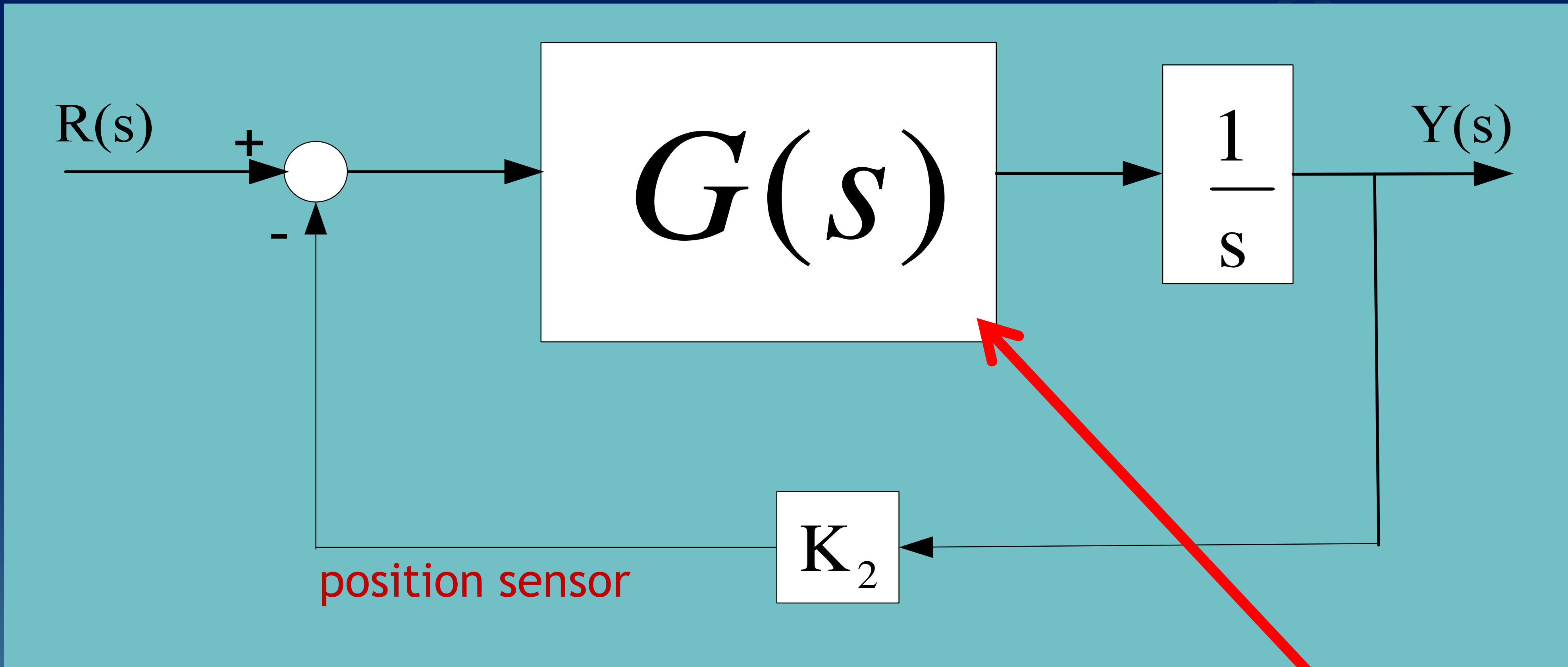
for servo systems with velocity feedback

feedback with tachometers



$$G(s) = \frac{\dfrac{K}{(Js+F)}}{1+\dfrac{KK_1}{(Js+F)}}$$

feedback with tachometers

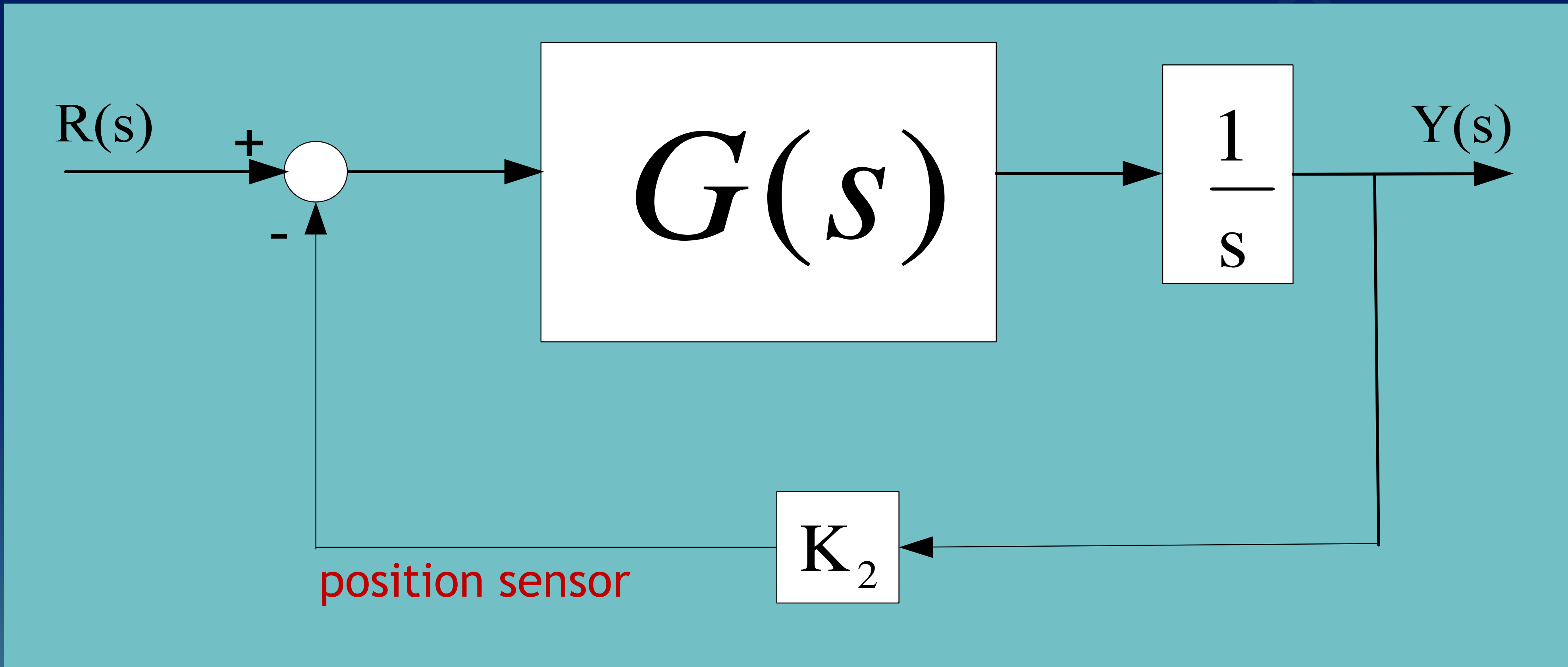


G(s)

$$G(s) = \frac{\dfrac{K}{(Js+F)}}{1+\dfrac{KK_1}{(Js+F)}}$$

Block Diagrams & error

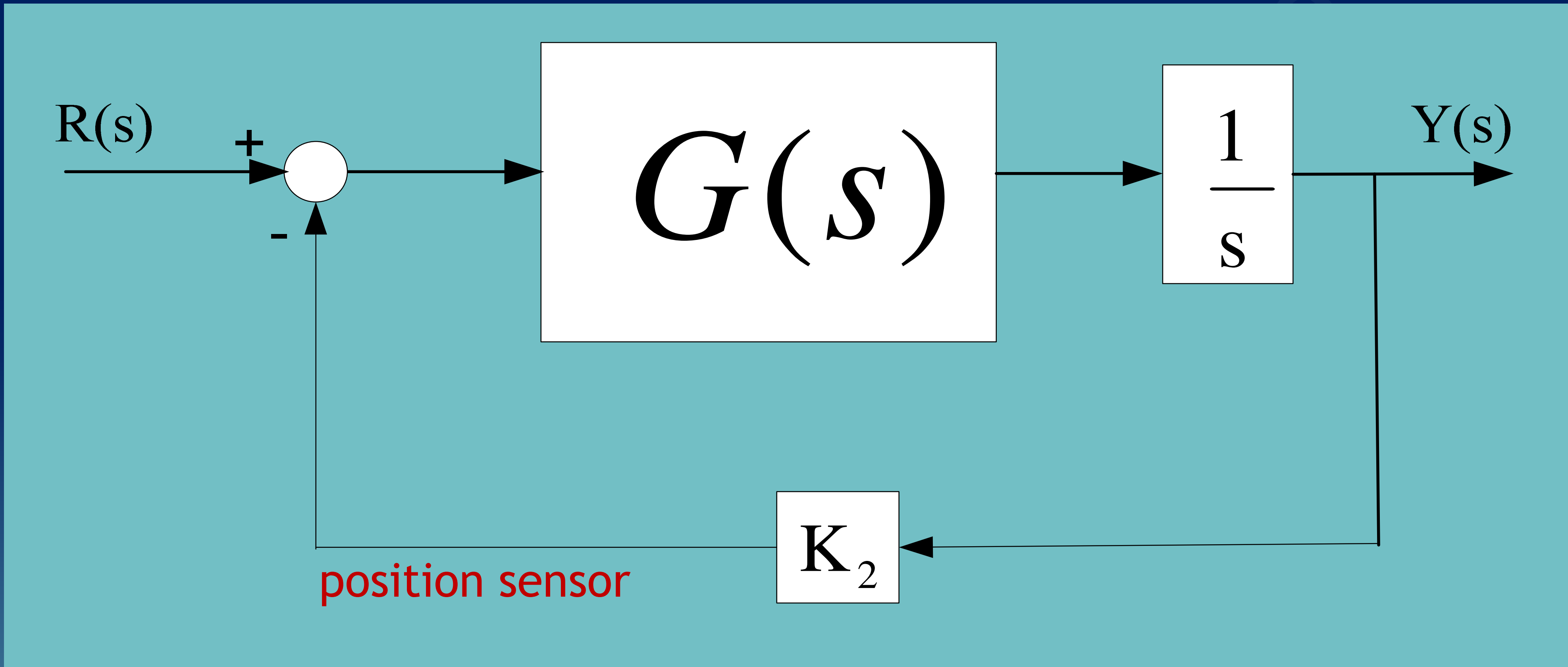
feedback with tachometers



$$G(s) = \frac{\dfrac{K}{(Js+F)}}{1+\dfrac{KK_1}{(Js+F)}}$$

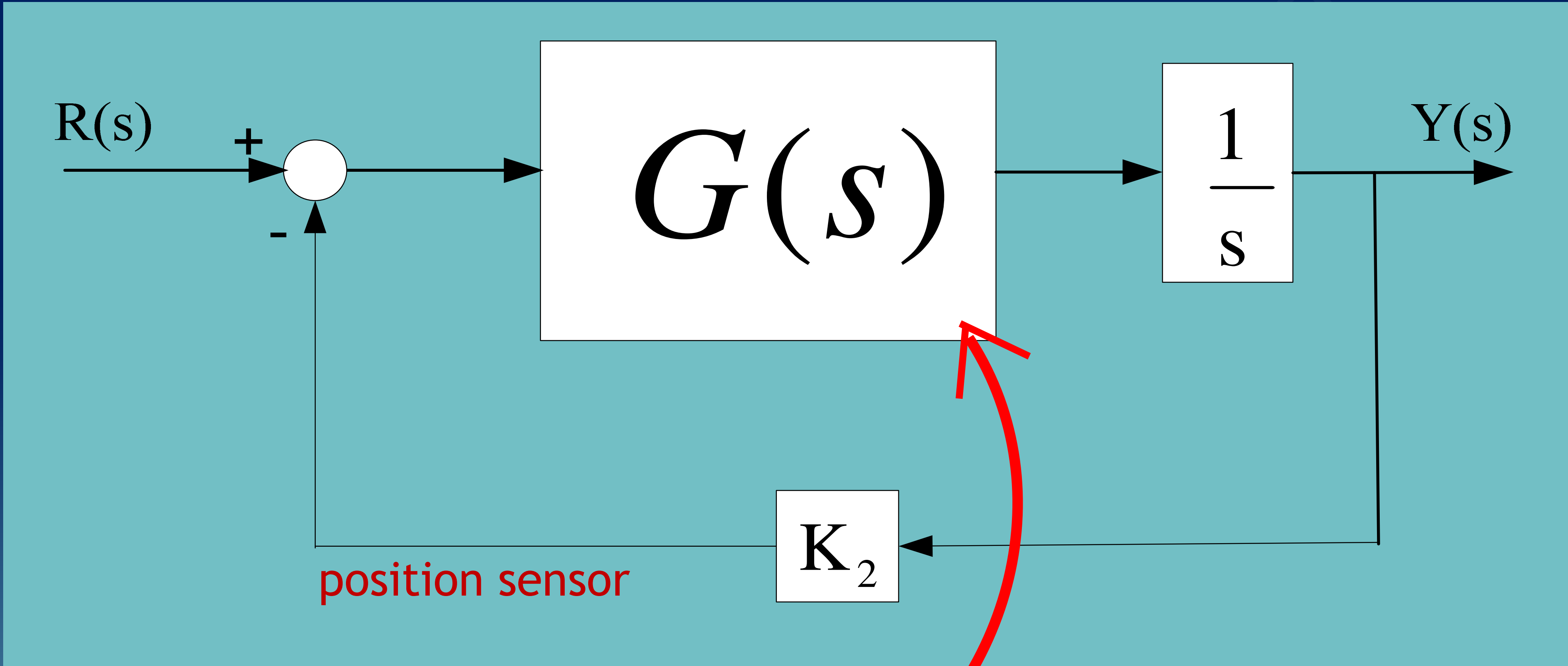
Block Diagrams & error

feedback with tachometers



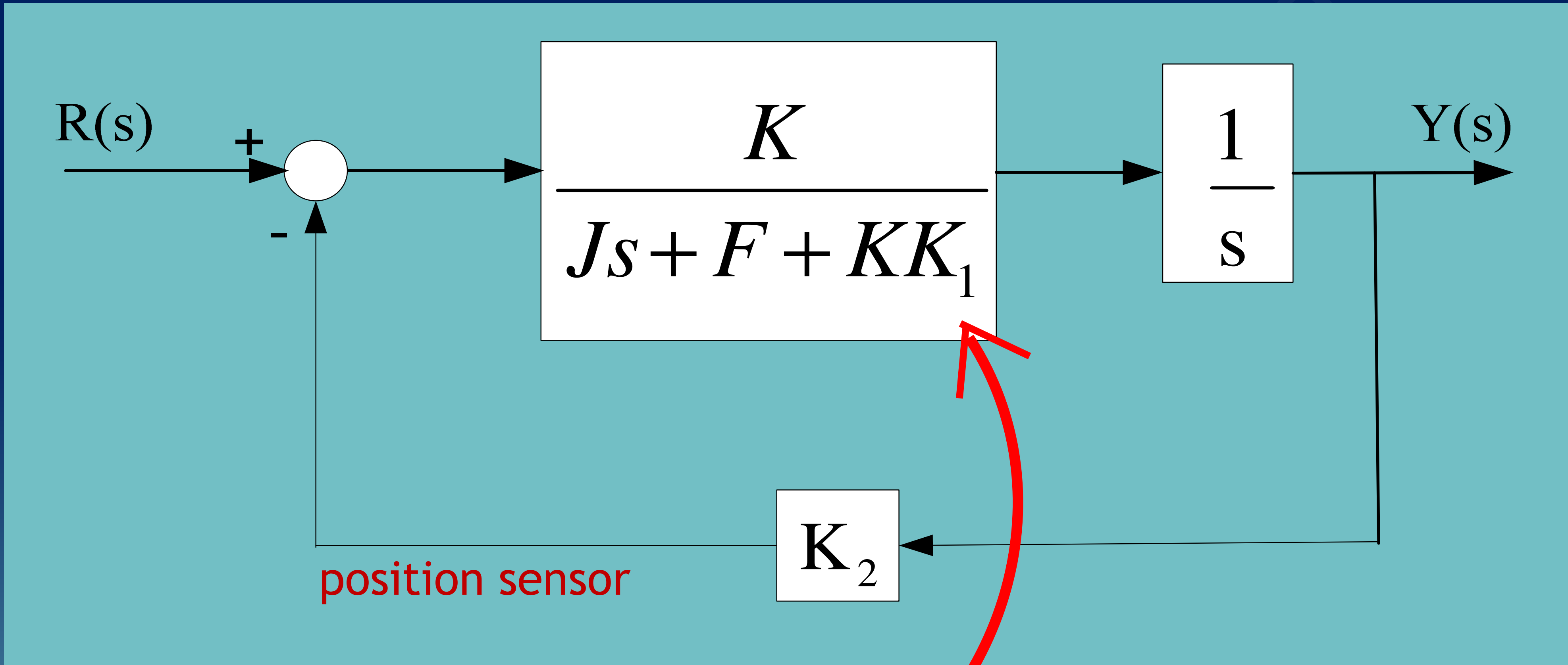
$$G(s) = \frac{\dfrac{K}{(Js+F)}}{1+\dfrac{KK_1}{(Js+F)}} = \frac{K}{Js+F+KK_1}$$

feedback with tachometers



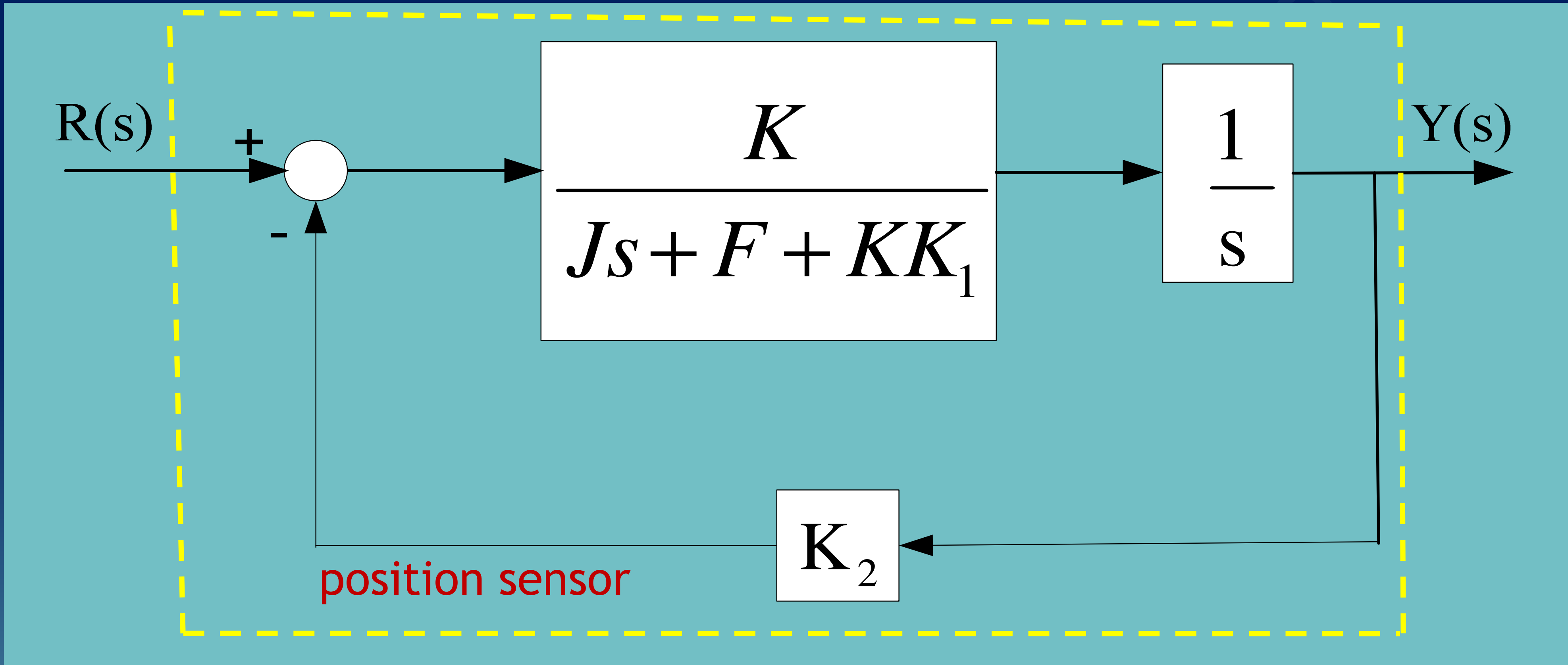
$$G(s) = \frac{K}{Js + F + KK_1}$$

feedback with tachometers



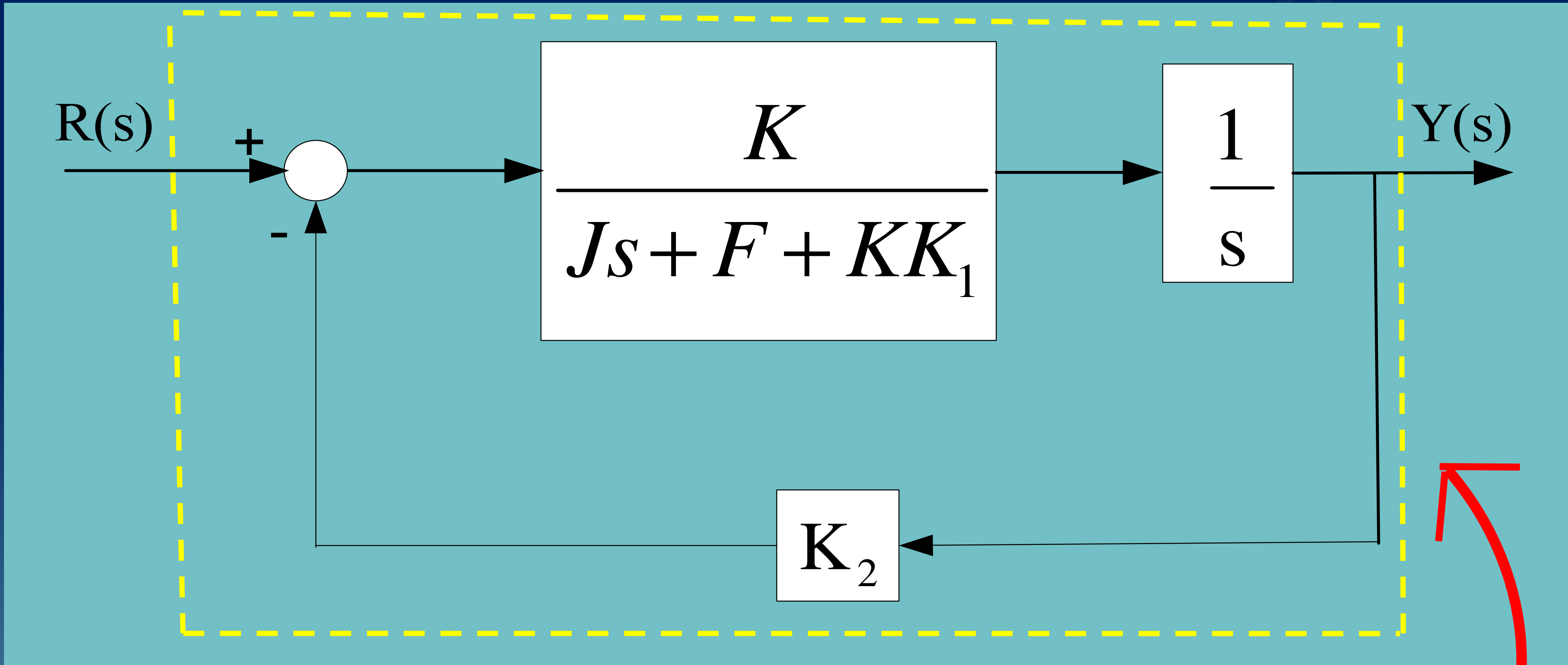
$$G(s) = \frac{K}{Js+F+KK_1}$$

feedback with tachometers



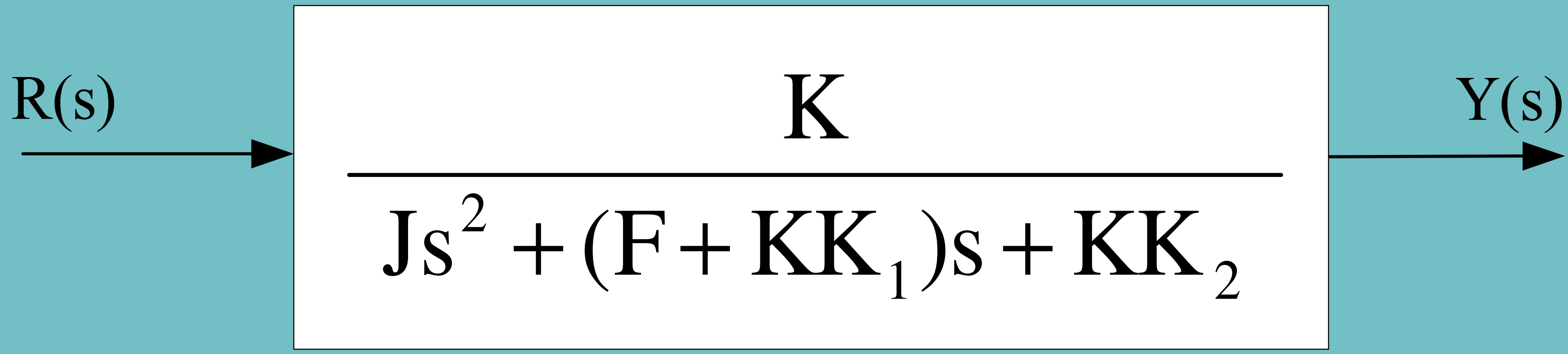
$$\frac{Y(s)}{R(s)} = \frac{\frac{K}{Js+(F+KK_1)}\cdot\frac{1}{s}}{1+\frac{K}{Js+(F+KK_1)}\cdot\frac{1}{s}\cdot K_2}$$

feedback with tachometers



$$\frac{Y(s)}{R(s)}=\frac{K}{Js^2+(F+KK_1)s+KK_2}$$

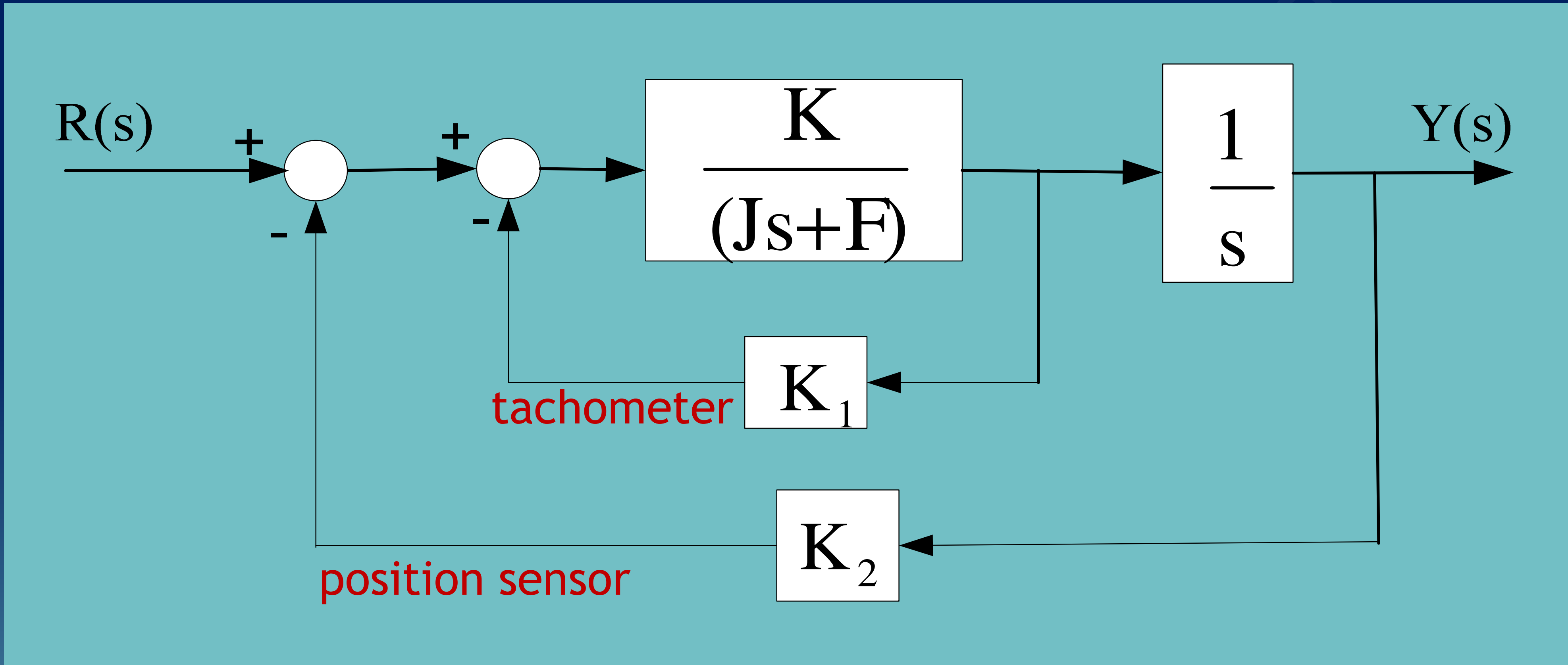
feedback with tachometers



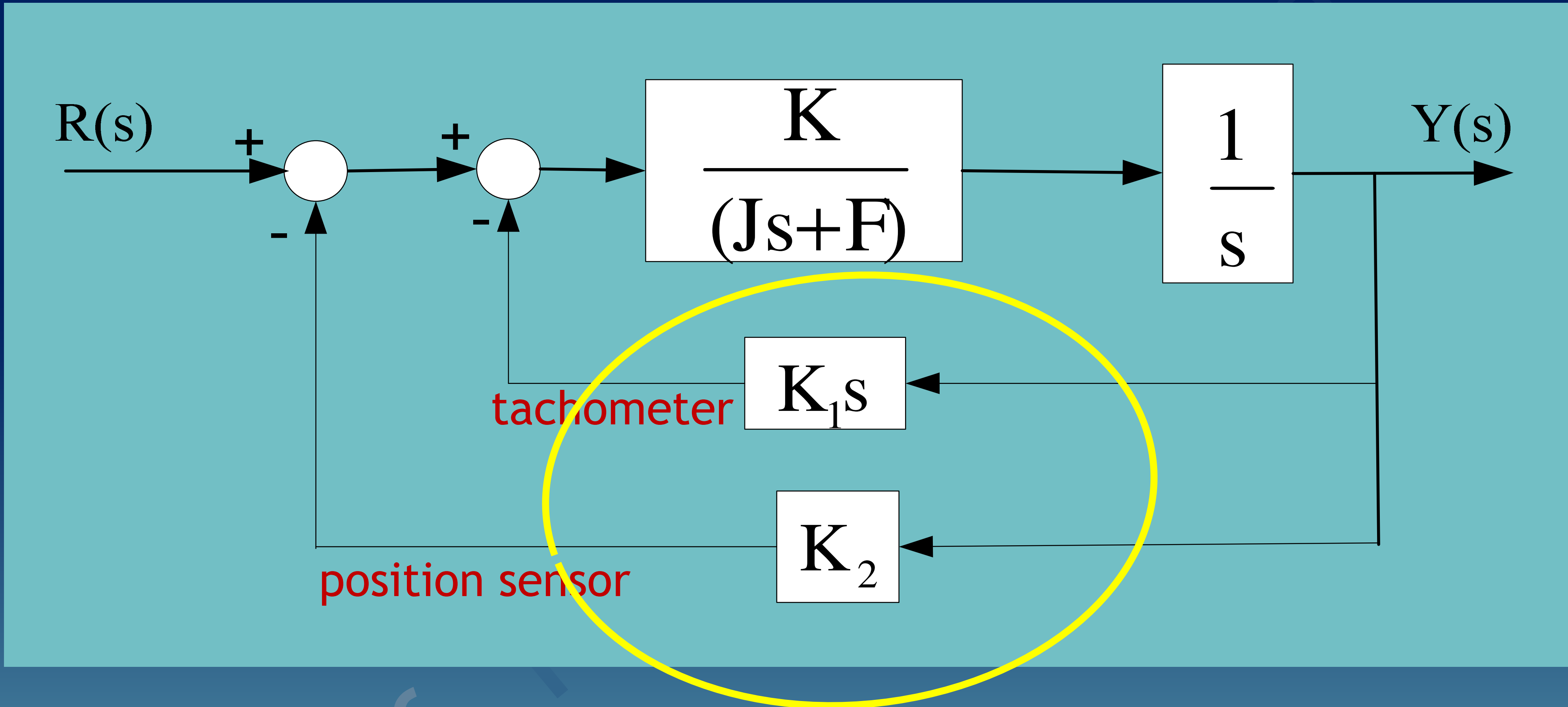
feedback with tachometers

(another approach)

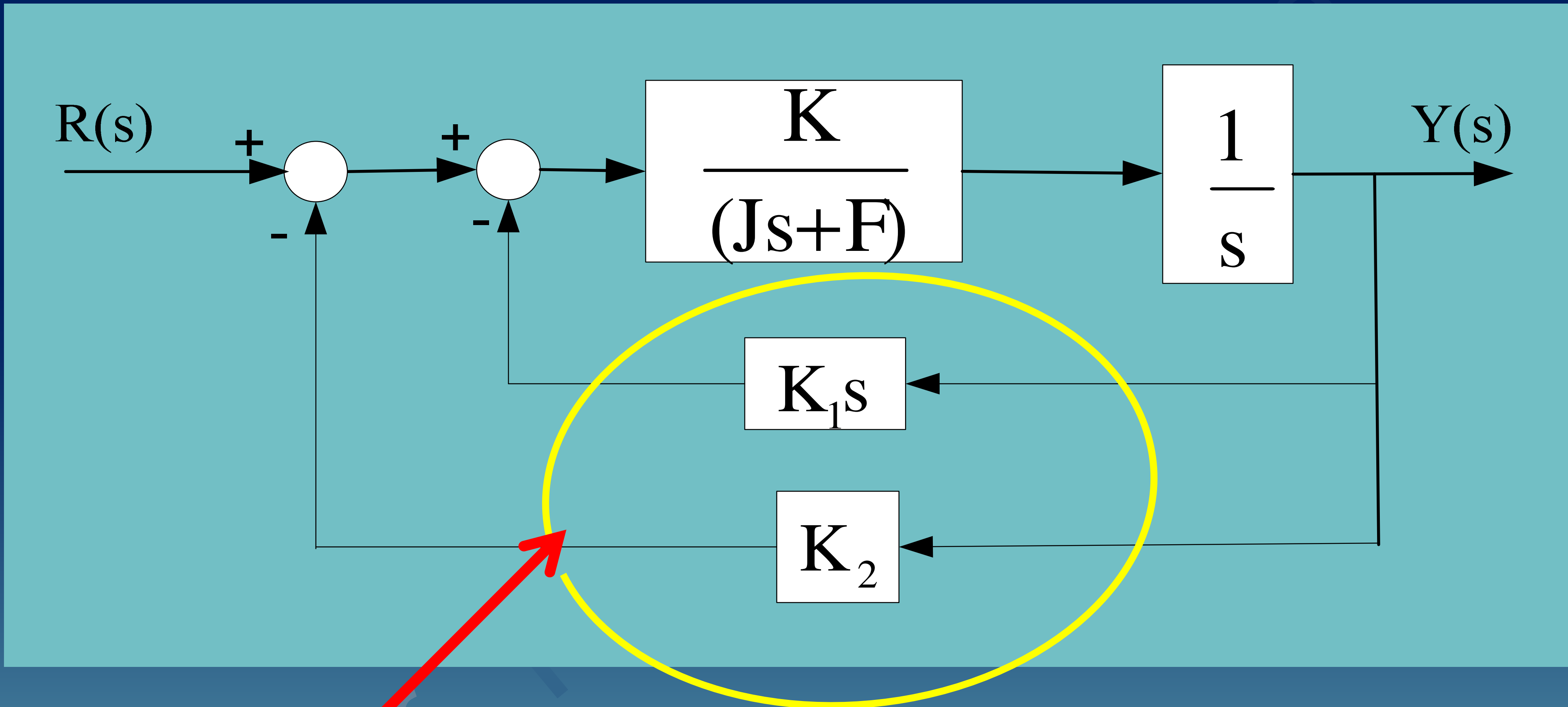
feedback with tachometers



feedback with tachometers



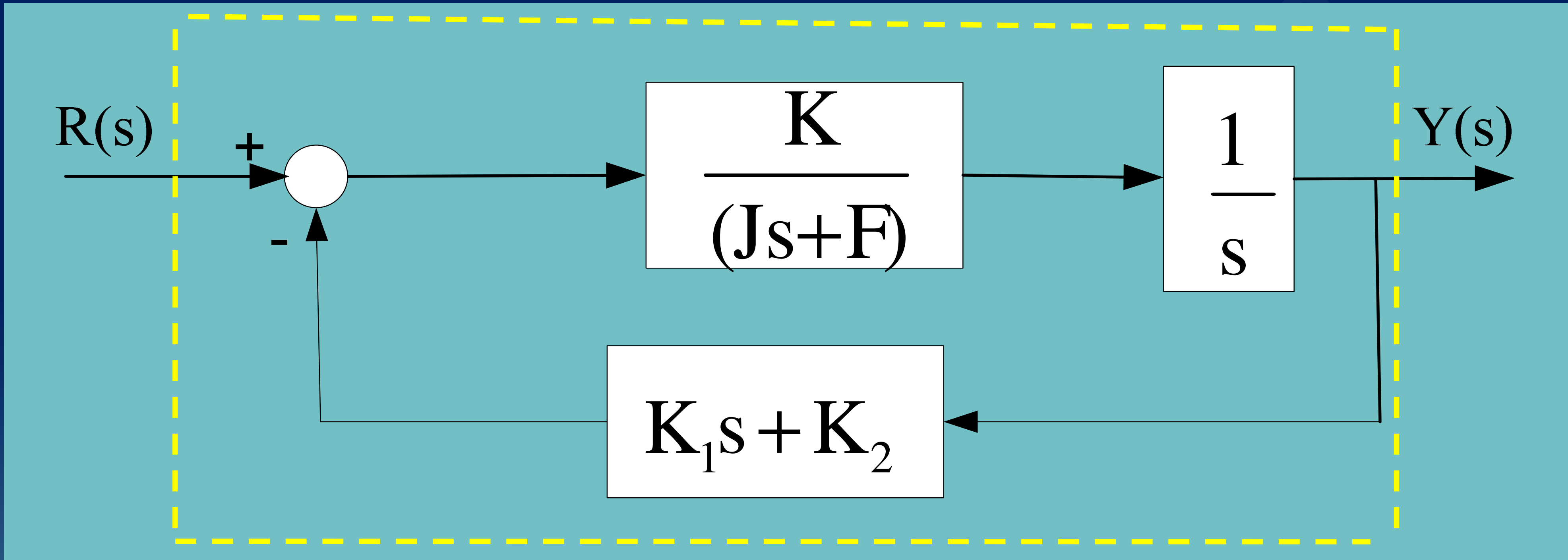
feedback with tachometers



$$K_1 s + K_2$$

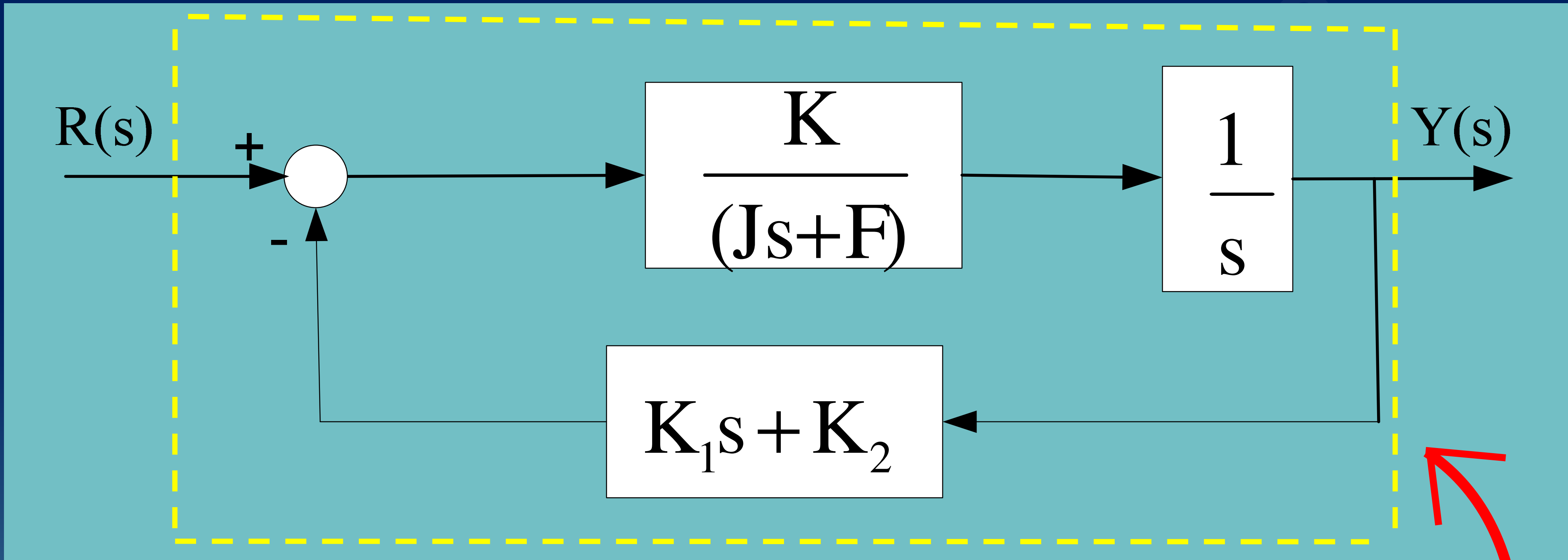
Block Diagrams & error

feedback with tachometers



$$\frac{Y(s)}{R(s)} = \frac{\dfrac{K}{(Js+F)}\cdot\dfrac{1}{s}}{1+\dfrac{K(K_1 s+K_2)}{(Js+F)\,s}}$$

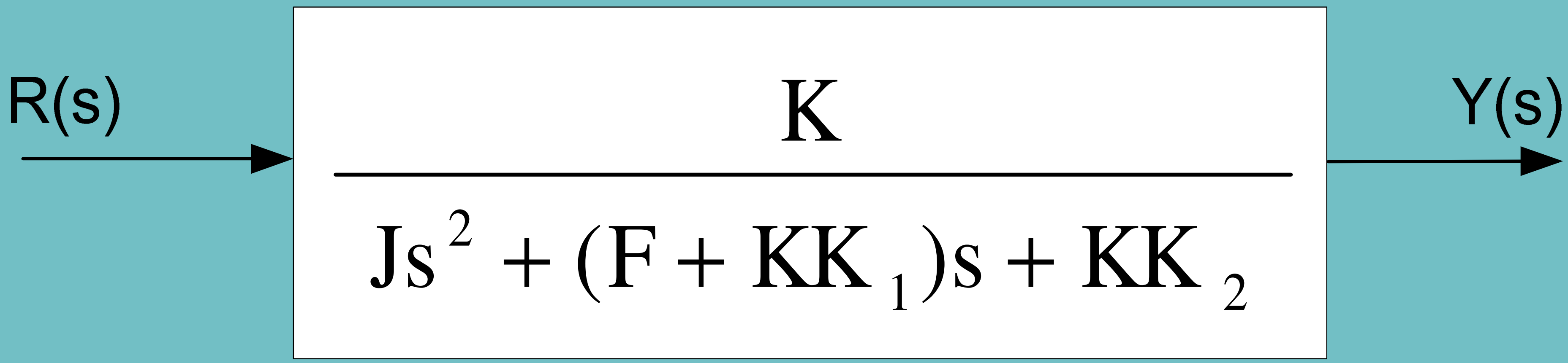
feedback with tachometers



$$\frac{Y(s)}{R(s)} = \frac{K}{Js^2 + (F + KK_1)s + KK_2}$$

which is the same result obtained earlier, with the first approach

feedback with tachometers



$$\frac{Y(s)}{R(s)} = \frac{K}{Js^2 + (F + KK_1)s + KK_2}$$

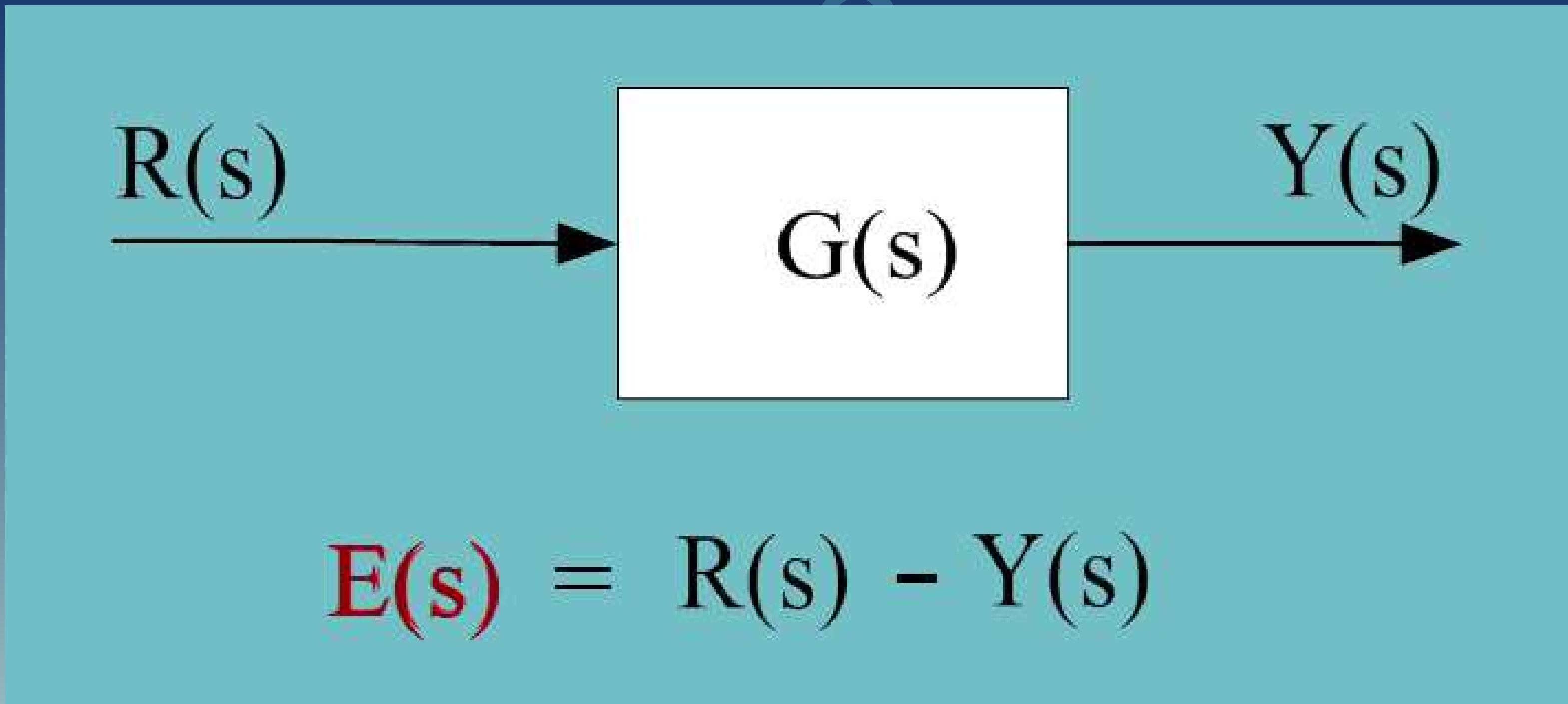
which is the same result obtained earlier, with the first approach

Error

Error

For an open loop system (that is, with no feedback) the definition of error is:

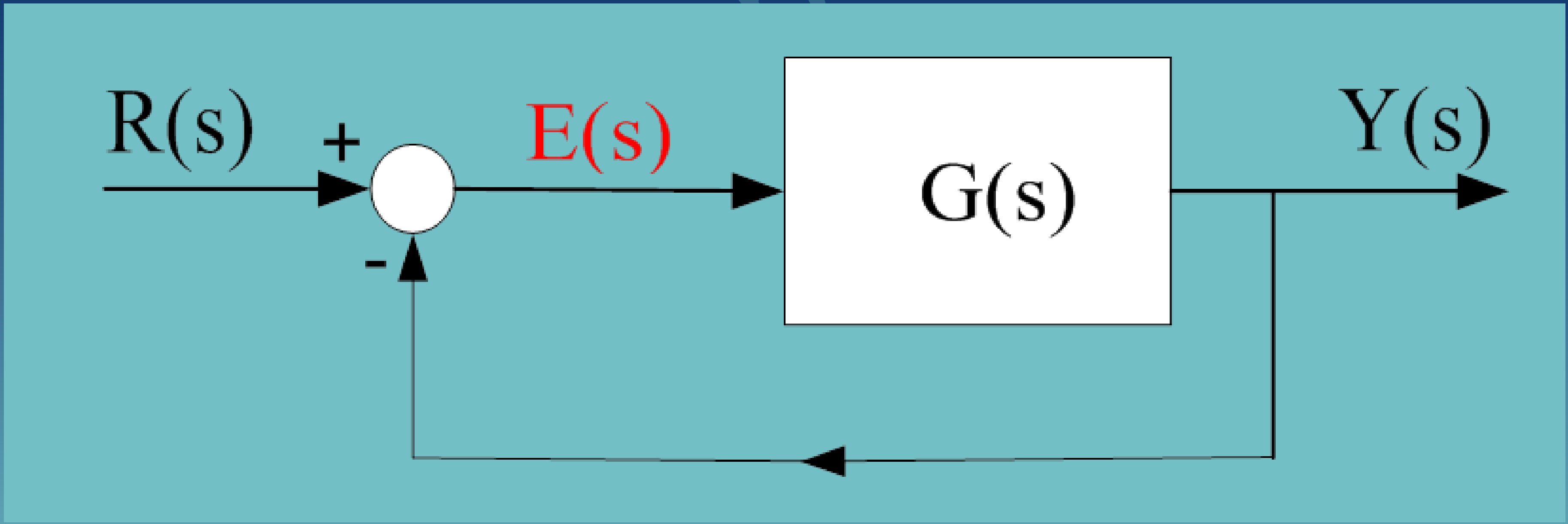
$$E(s) = R(s) - Y(s)$$



Error

unit feedback

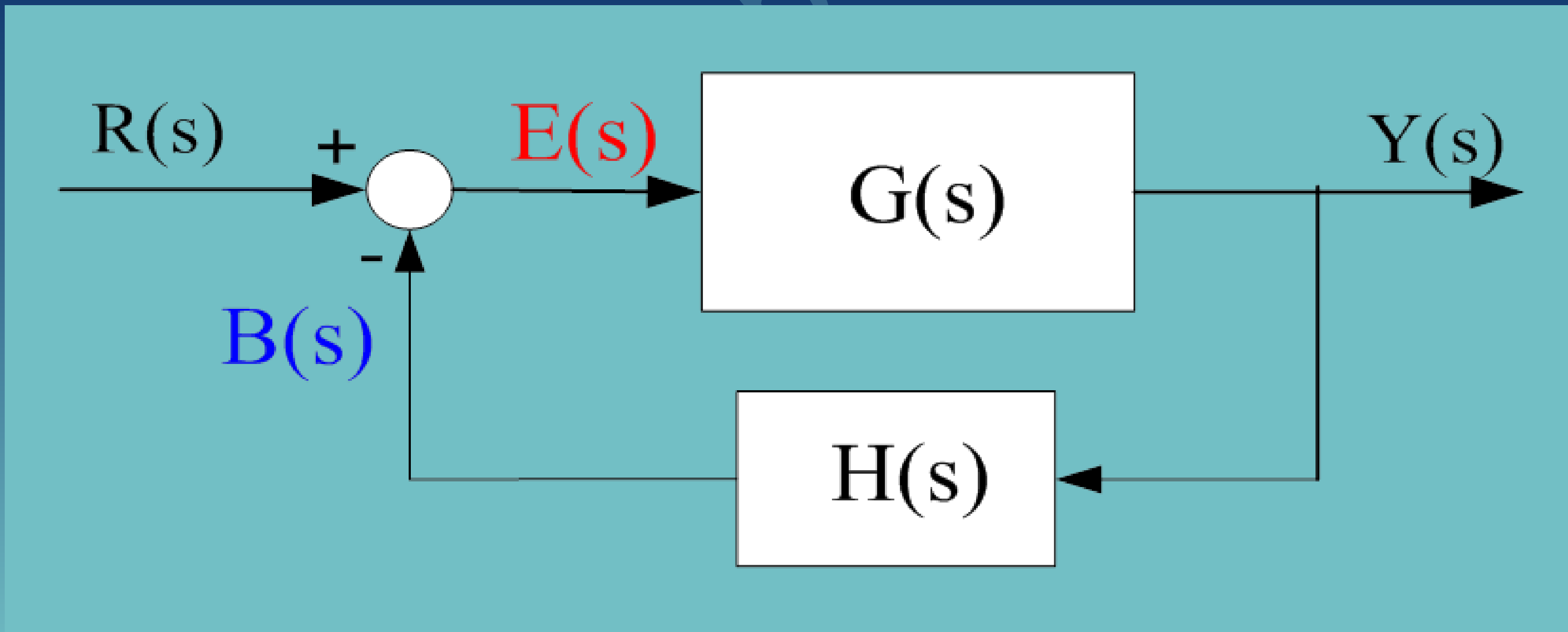
here, the error is also: $$E(s) = R(s) - Y(s)$$



Error

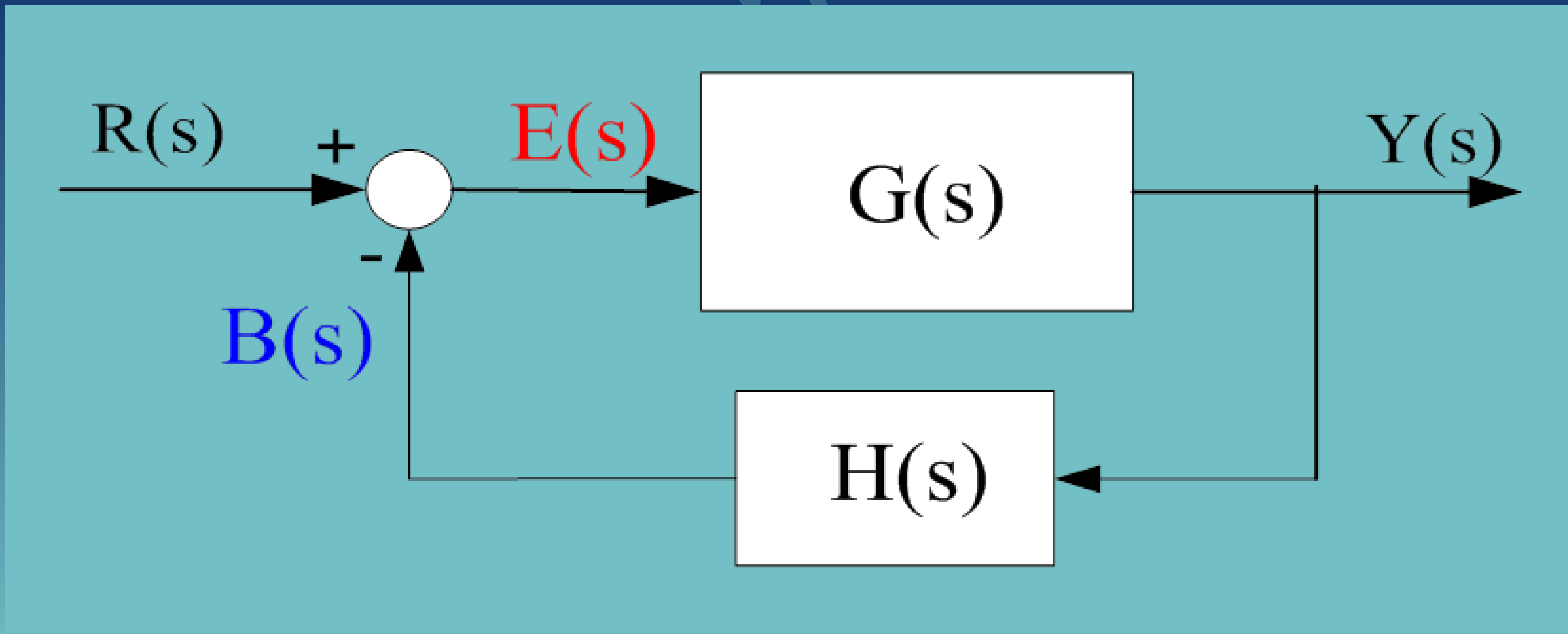
non unit feedback

the error becomes: $$E(s) = R(s) - B(s)$$



that is:

error: $$E(s) = R(s) - \overbrace{Y(s)H(s)}^{B(s)}$$

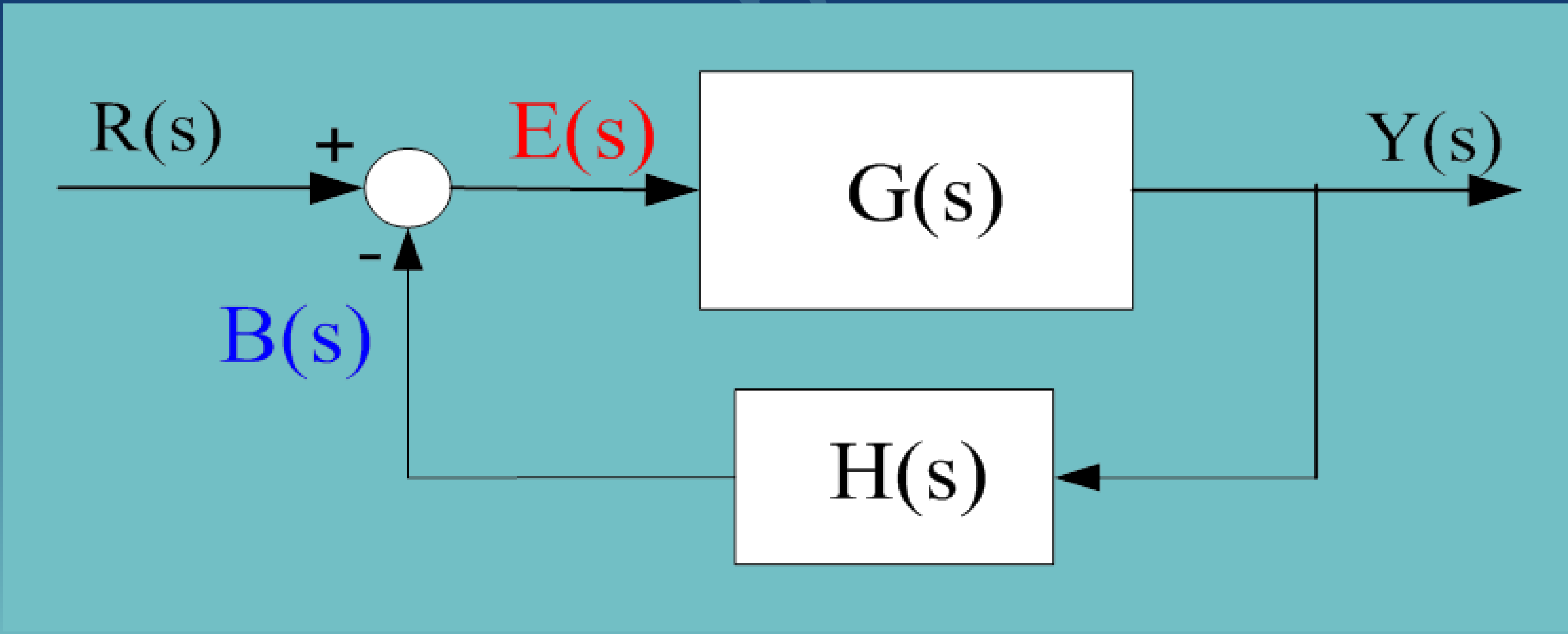


but:

$$\boxed{Y(s) = G(s) \cdot E(s)}$$

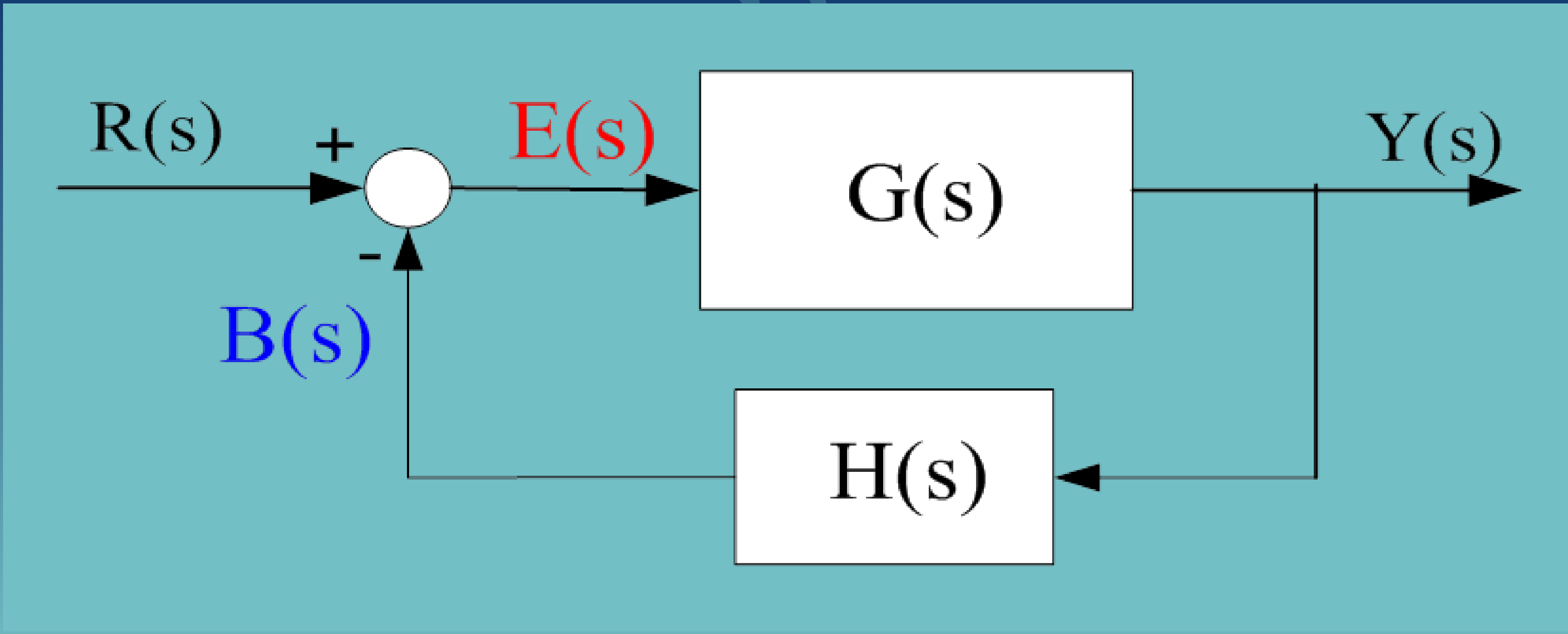
thus:

$$\boxed{E(s) = R(s) - \underbrace{G(s)E(s)}_{Y(s)}H(s)}$$



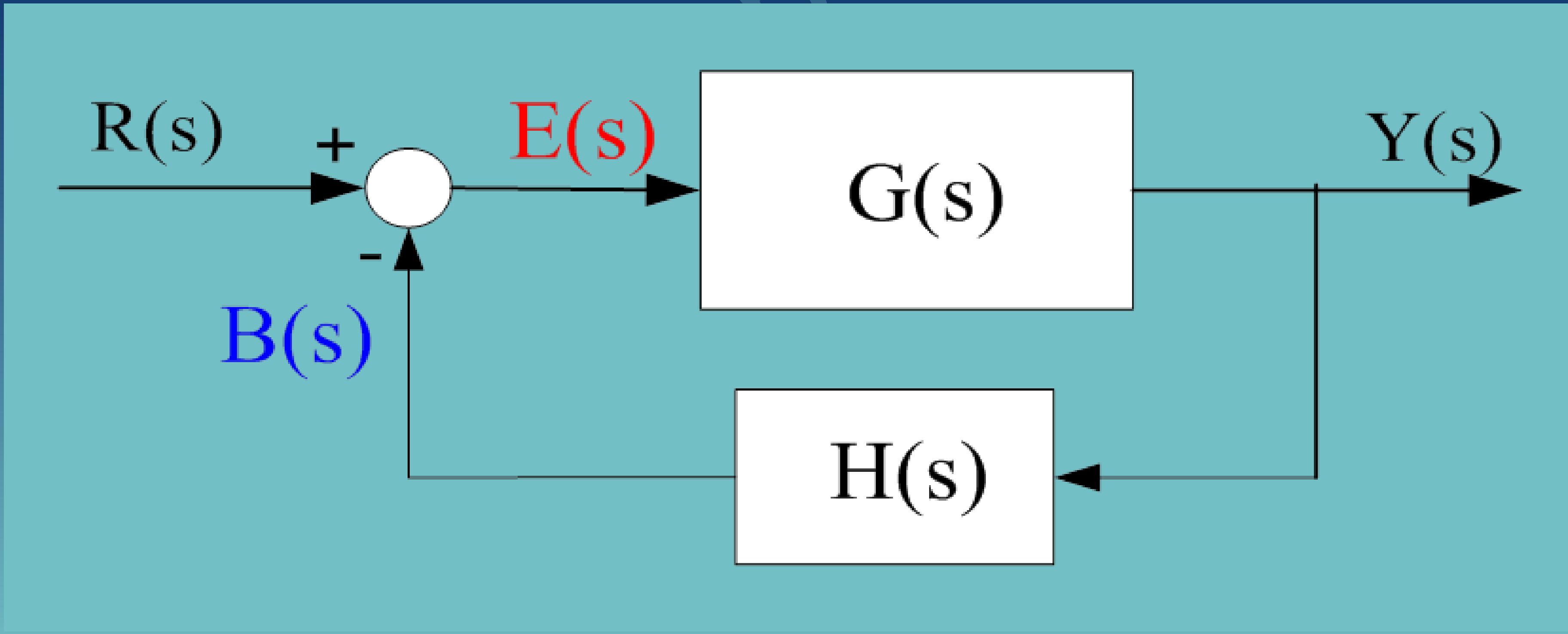
hence:

$$E(s)\left[1+G(s)H(s)\right]=R(s)$$



and the expression for the 'error' is:

$$E(s) = \frac{R(s)}{[1 + G(s)H(s)]}$$



Steady state error

Steady state output

Initial Value Theorem:

$$x(0) = \lim_{t \to 0} x(t) = \lim_{s \to \infty} s \cdot X(s) \quad \text{(IVT)}$$

Final Value Theorem:

$$x(\infty) = \lim_{t \to \infty} x(t) = \lim_{s \to 0} s \cdot X(s) \quad \text{(FVT)}$$

Initial Value Theorem:

$$x(0) = \lim_{s \to \infty} s \cdot X(s)$$ (IVT)

Final Value Theorem:

$$x(\infty) = \lim_{s \to 0} s \cdot X(s)$$ (FVT)

Example 3:

$$Y(s) = \frac{3s-2}{s(s+5)}$$

IVT

$$y(0) = \lim_{s\to\infty} s\cdot Y(s)$$
$$= \lim_{s\to\infty} \cancel{s}\cdot\frac{3s-2}{\cancel{s}(s+5)}$$
$$= \lim_{s\to\infty} \frac{3s-2}{(s+5)}$$
$$= 3$$

FVT

$$y(\infty) = \lim_{s\to 0} s\cdot Y(s)$$
$$= \lim_{s\to 0} \cancel{s}\cdot\frac{3s-2}{\cancel{s}(s+5)}$$
$$= \lim_{s\to 0} \frac{3s-2}{(s+5)}$$
$$= -2/5$$

Example 4:

$$E(s) = \frac{2s}{s^2 - 4s + 3}$$

IVT

$$e(0) = \lim_{s\to\infty} s \cdot E(s)$$

$$= \lim_{s\to\infty} \frac{2s^2}{s^2 - 4s + 3}$$

$$= 2$$

FVT

$$e(\infty) = \lim_{s\to 0} s \cdot E(s)$$

$$= \lim_{s\to 0} \frac{2s^2}{s^2 - 4s + 3}$$

$$= 0$$

Steady state error:

$$e_{ss} = \lim_{t \to \infty} e(t)$$

$$e_{ss} = \lim_{t \to \infty} e(t) = \lim_{s \to 0} s \cdot E(s) \quad \text{(FVT)}$$

$$e_{ss} = \lim_{s \to 0} s \cdot E(s) = \lim_{s \to 0} \frac{s \cdot R(s)}{[1 + G(s)H(s)]}$$

Steady state error:

$$\boxed{e_{ss} = \lim_{s \to 0} \frac{s \cdot R(s)}{\left[1 + G(s)H(s)\right]}}$$

Steady state output:

$$y_{ss} = \lim_{t\to\infty} y(t)$$

$$\Rightarrow \quad y_{ss} = \lim_{t\to\infty} y(t) = \lim_{s\to 0} s\cdot Y(s) \qquad \text{(FVT)}$$

However, we know that

$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

Steady state output:

hence,
$$Y(s) = \frac{G(s)}{[1+G(s)H(s)]} \cdot R(s)$$

$$y_{ss} = \lim_{s \to 0} s \cdot Y(s) =$$

$$= \lim_{s \to 0} s \cdot \frac{G(s)}{[1+G(s)H(s)]} \cdot R(s)$$

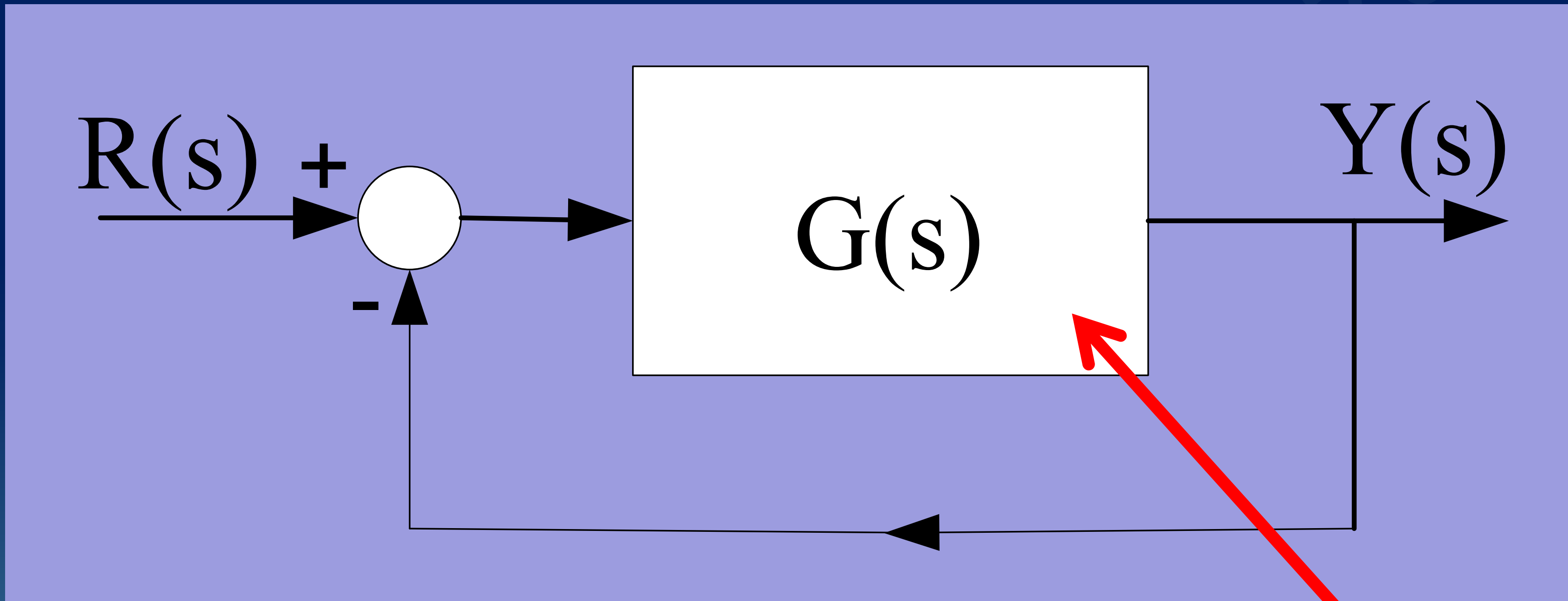
Steady state output:

and then,

$$y_{ss} = \lim_{s\to 0} s\cdot Y(s) = \lim_{s\to 0} \frac{s\,G(s)R(s)}{\left[1+G(s)H(s)\right]}$$

$$\Rightarrow\; y_{ss} = \lim_{s\to 0} \frac{s\,G(s)R(s)}{\left[1+G(s)H(s)\right]}$$

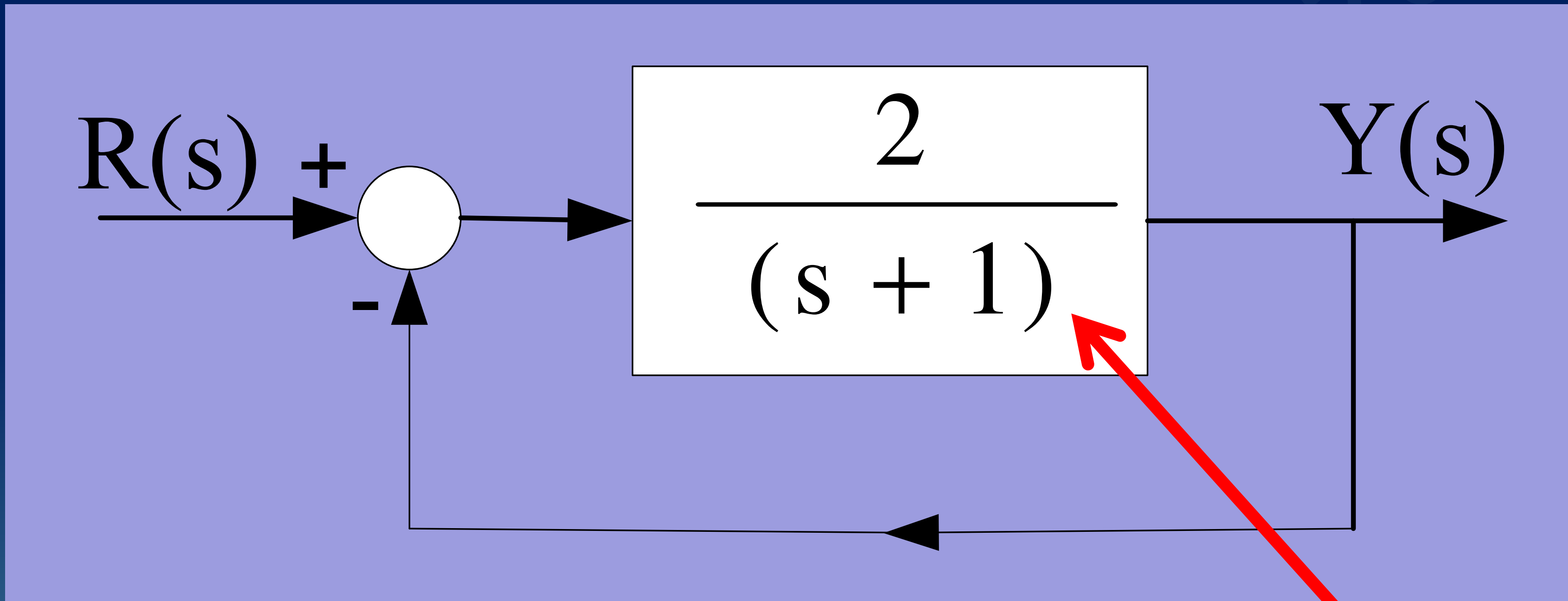
Example 5:



$$G(s) = \frac{2}{(s+1)}$$

Example 5:

(continued)



input r(t):

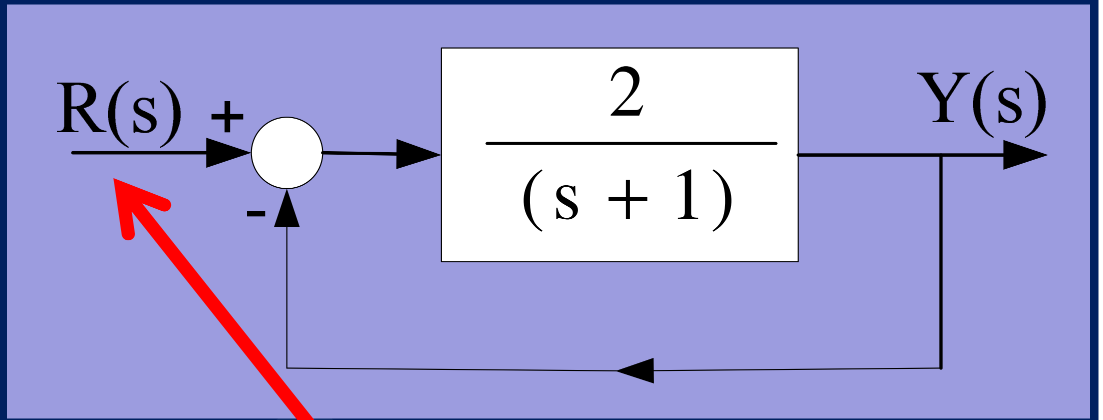
$$r(t) = u_1(t)$$

$$G(s) = \frac{2}{(s+1)}$$

r(t) = unit step

Example 5:

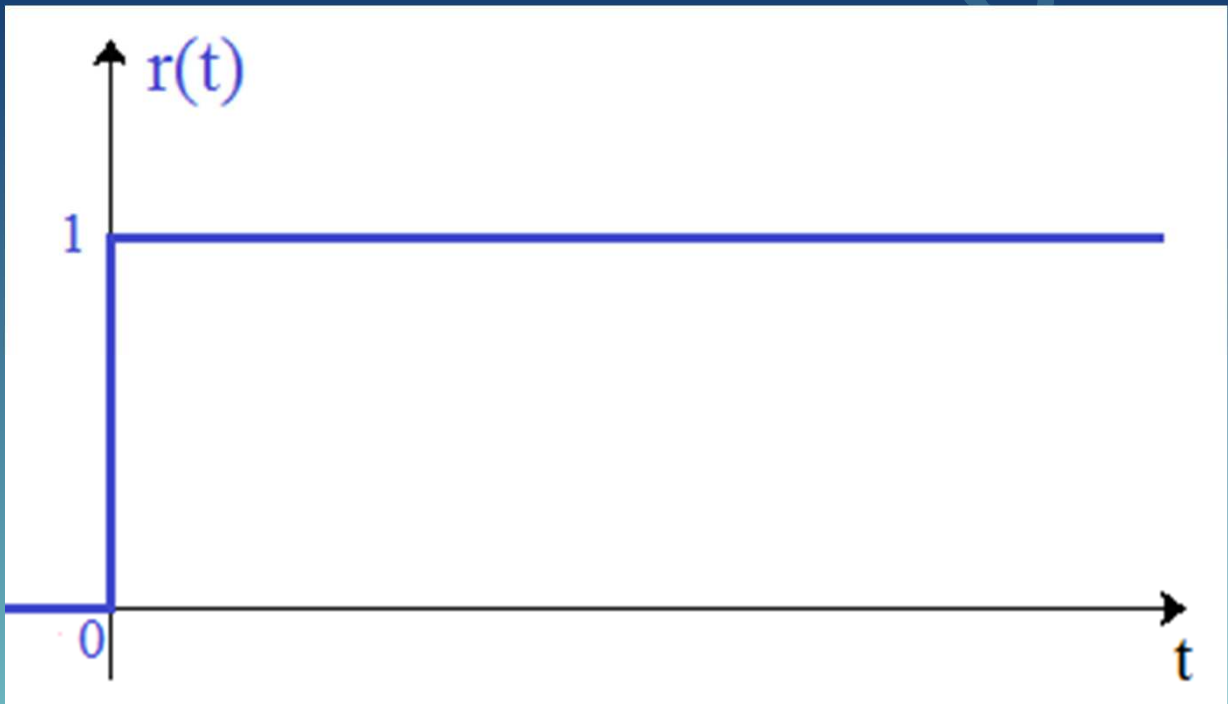
(continued)



input r(t):

$$r(t) = u_1(t)$$

r(t) = unit step

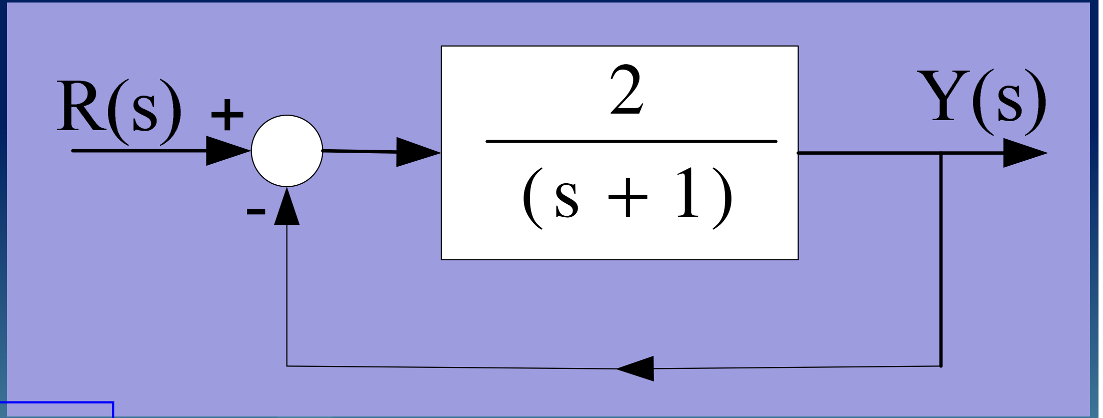


$$R(s) = \frac{1}{s}$$

Example 5:

(continued)

$$R(s) = \frac{1}{s}$$



output $y(t)$

$$Y(s) = \frac{G(s)}{[1+G(s)H(s)]} \cdot R(s)$$

$$= \frac{\frac{2}{(s+1)}}{\left[1+\frac{2}{(s+1)}\right]} \cdot \frac{1}{s}$$

$$= \frac{2}{s(s+3)}$$

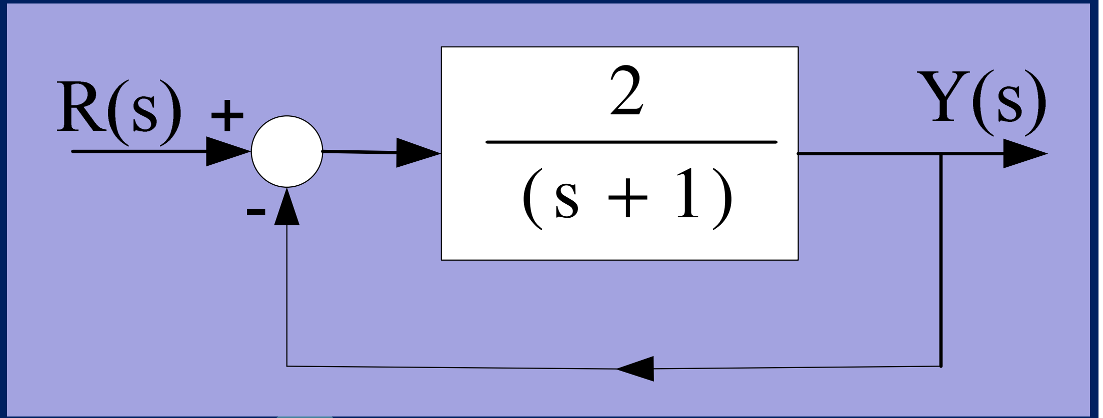
output in steady state y_{ss}

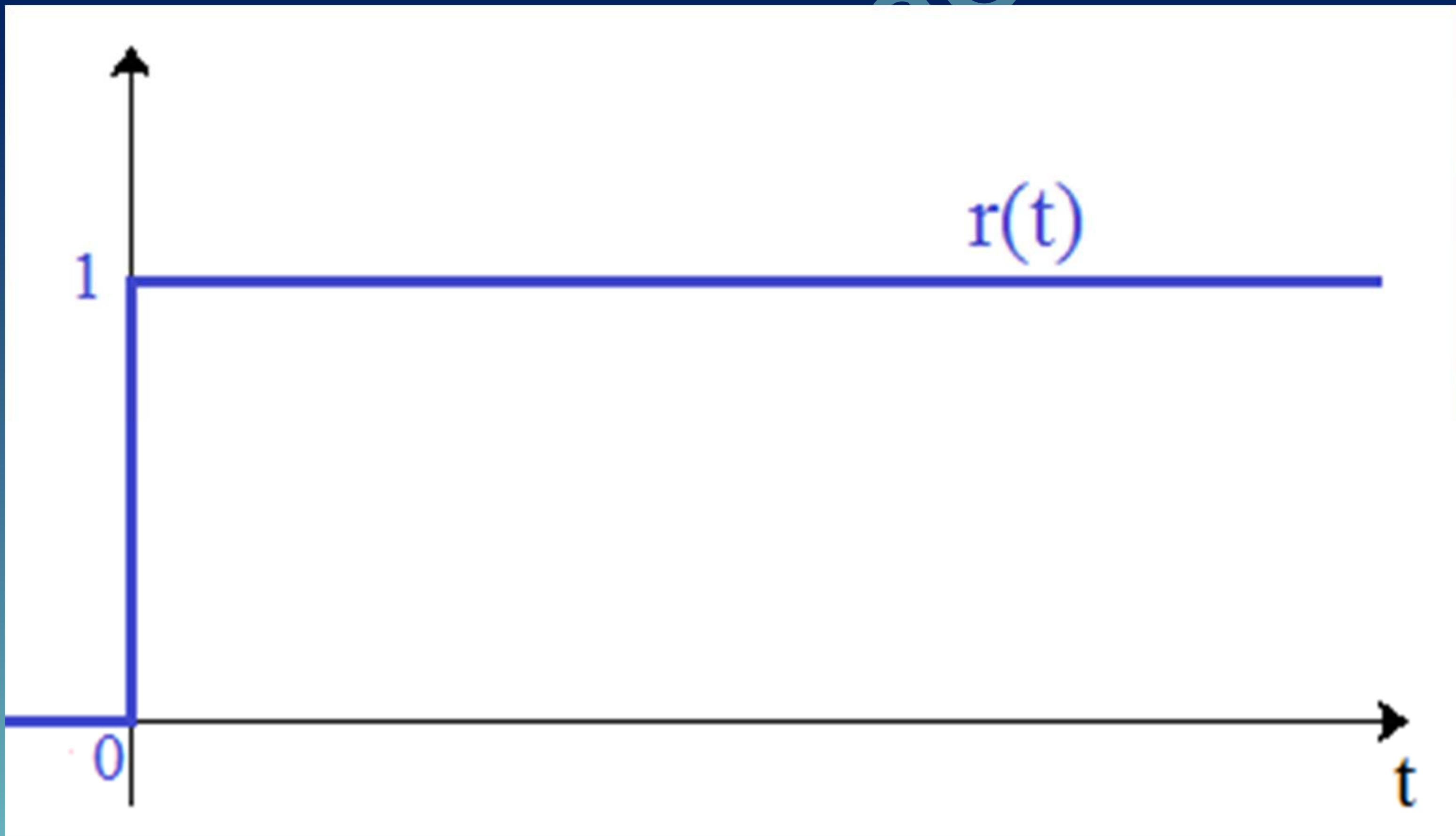
$$y_{ss} = \lim_{s\to 0} s \cdot Y(s) =$$

$$= \lim_{s\to 0} \cancel{s} \cdot \frac{2}{\cancel{s}(s+3)}$$

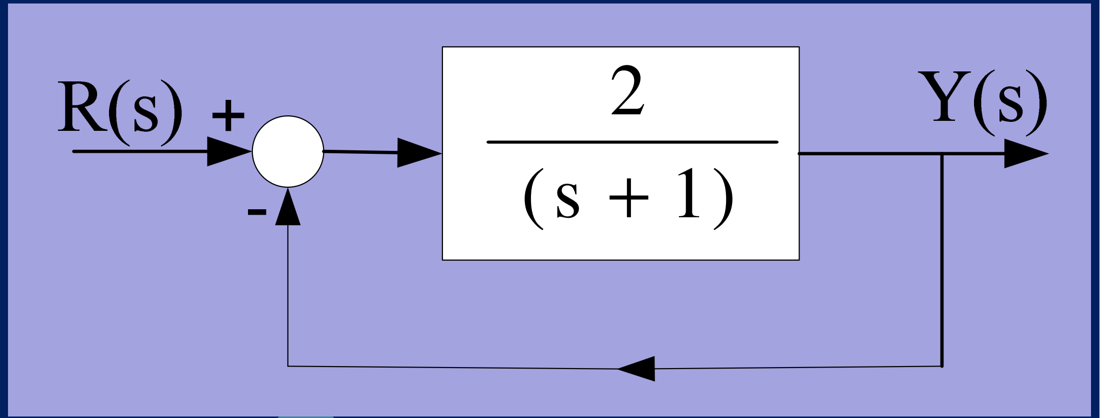
$$= 2/3$$

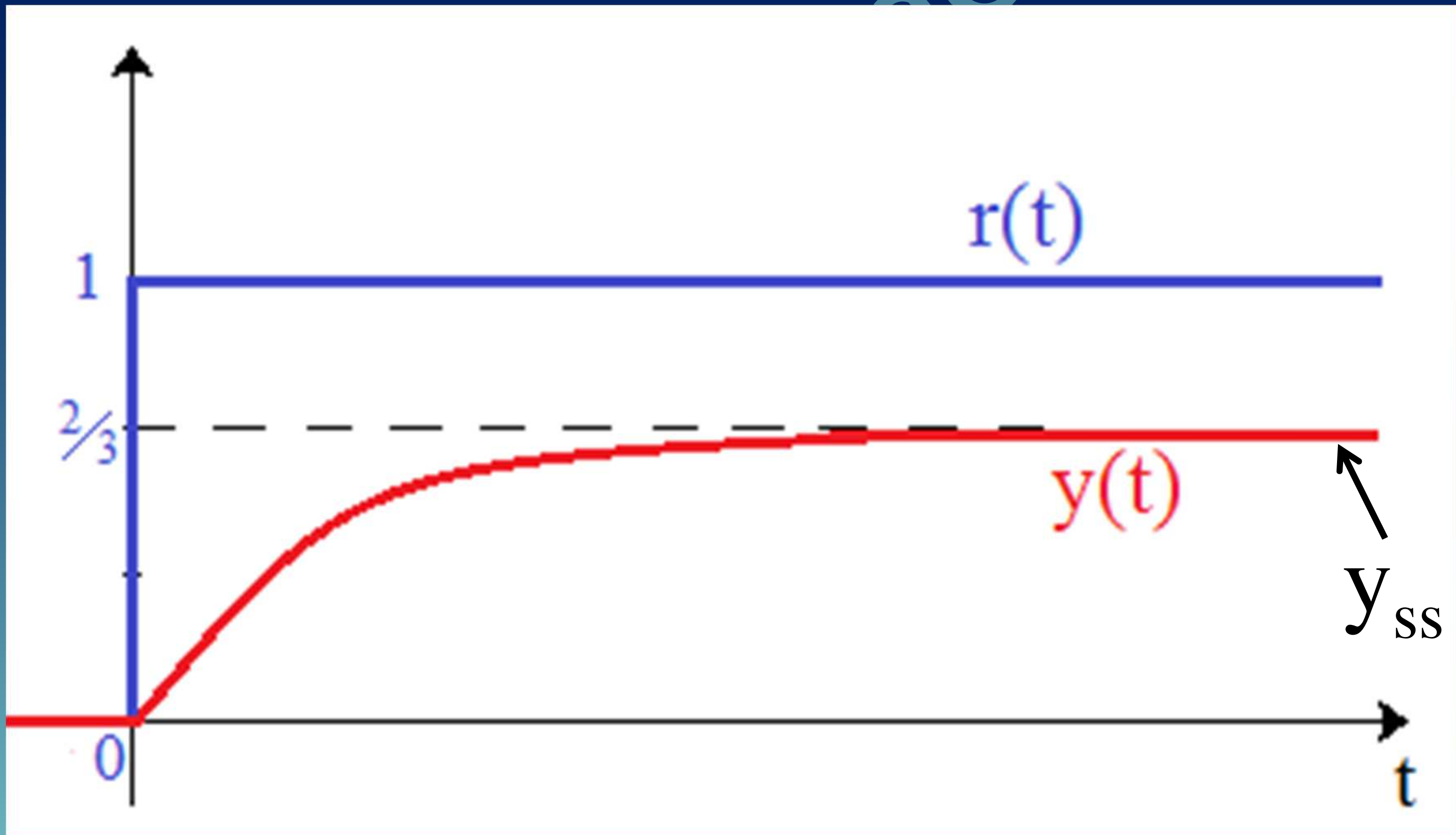
Example 5:
(continued)





Example 5:
(continued)



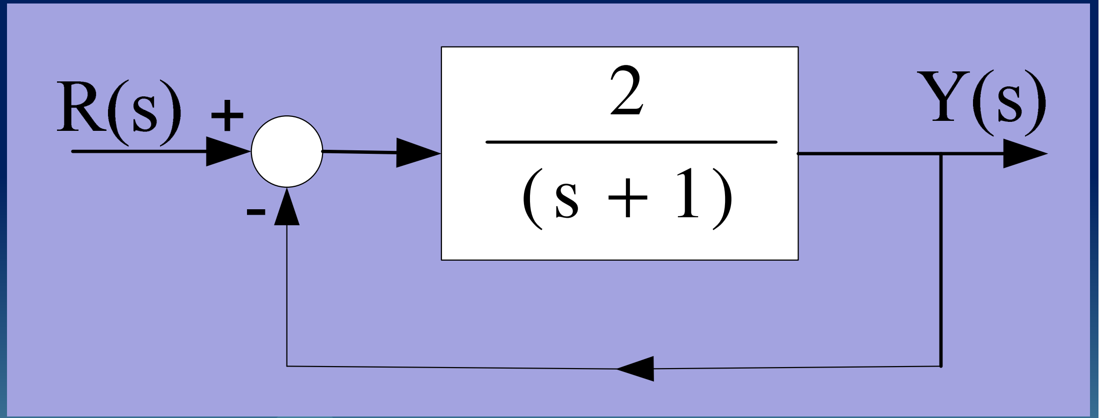


= 2/3

Example 5:

(continued)

$$R(s) = \frac{1}{s}$$



error $e(t)$

$$E(s) = \frac{R(s)}{[1+G(s)H(s)]}$$

$$= \frac{1/s}{\left[1+\dfrac{2}{(s+1)}\right]}$$

$$= \frac{(s+1)}{s(s+3)}$$

error in

steady state e_{ss}

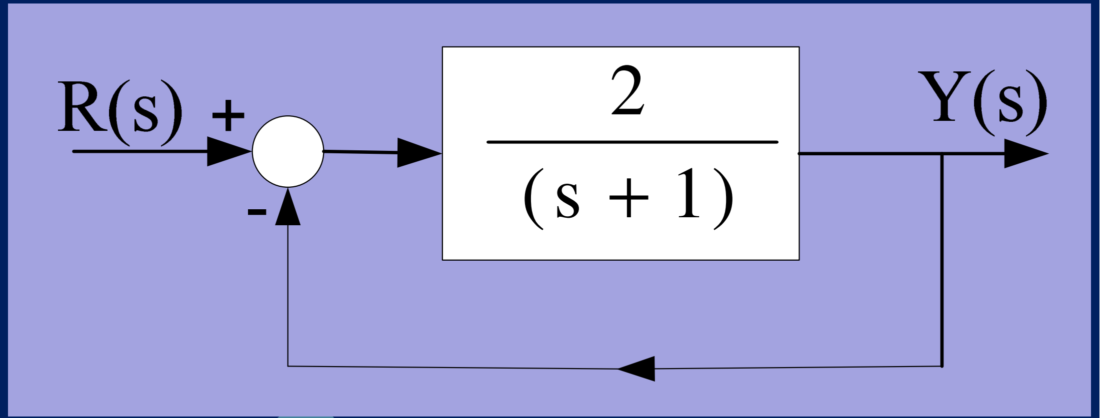
$$e_{ss} = \lim_{s\to 0} s\cdot E(s) =$$

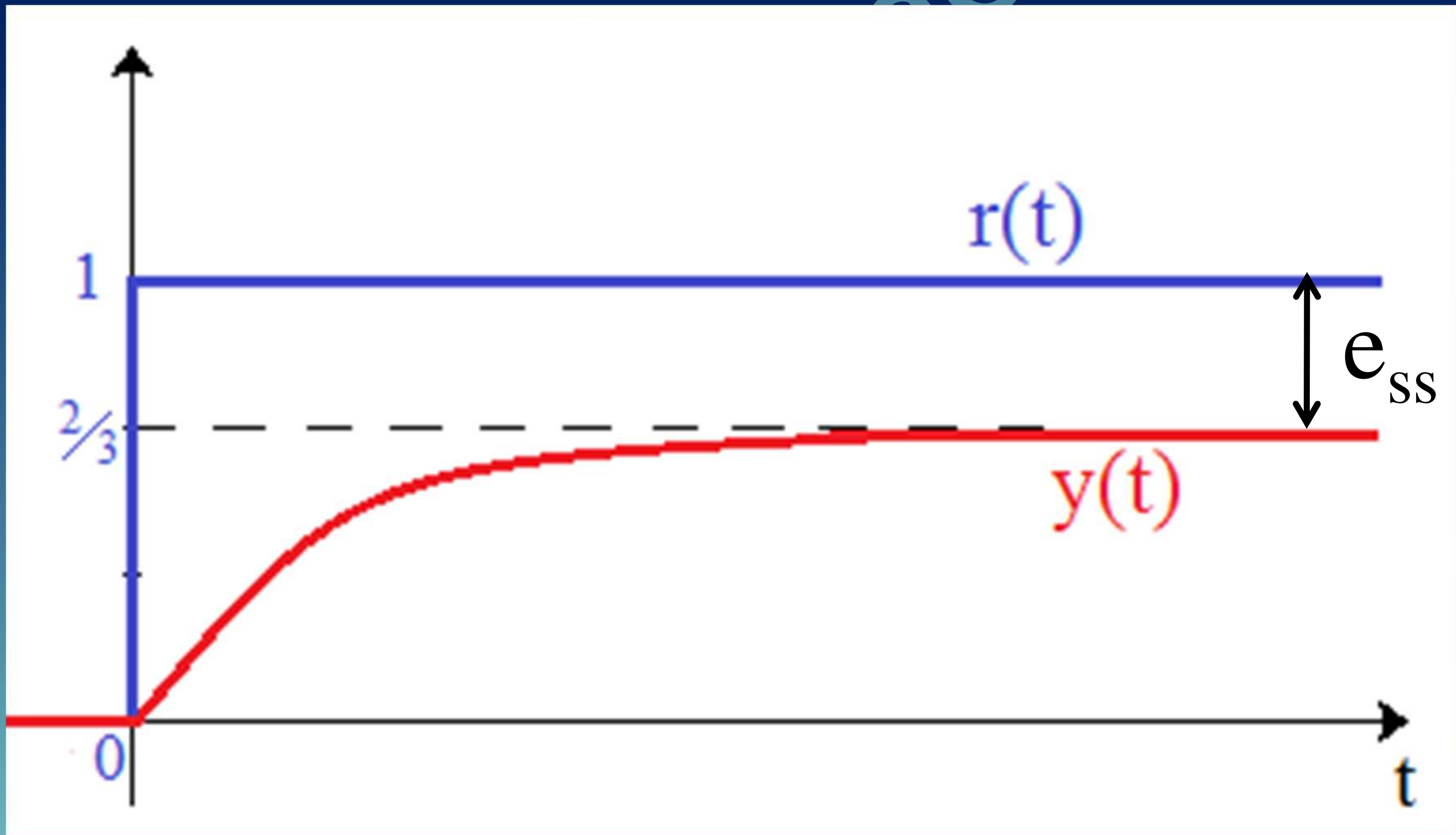
$$= \lim_{s\to 0} \cancel{s}\cdot\frac{(s+1)}{\cancel{s}(s+3)}$$

$$= 1/3$$

Example 5:

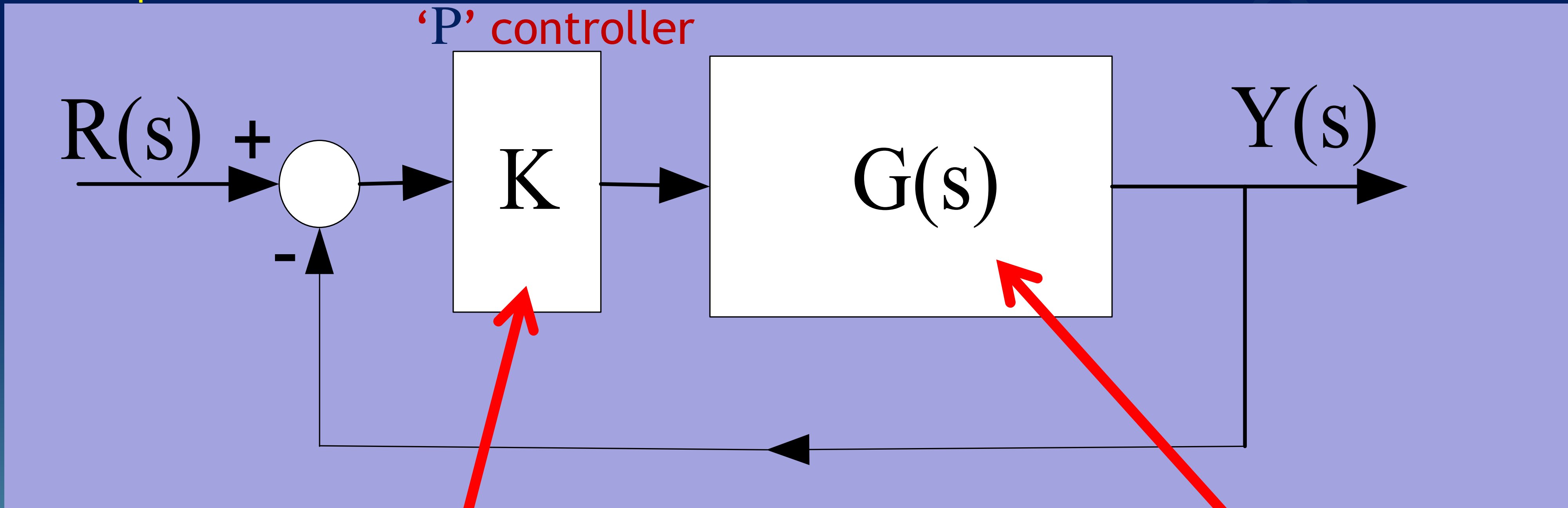
(continued)





= 1/3

Example 6:

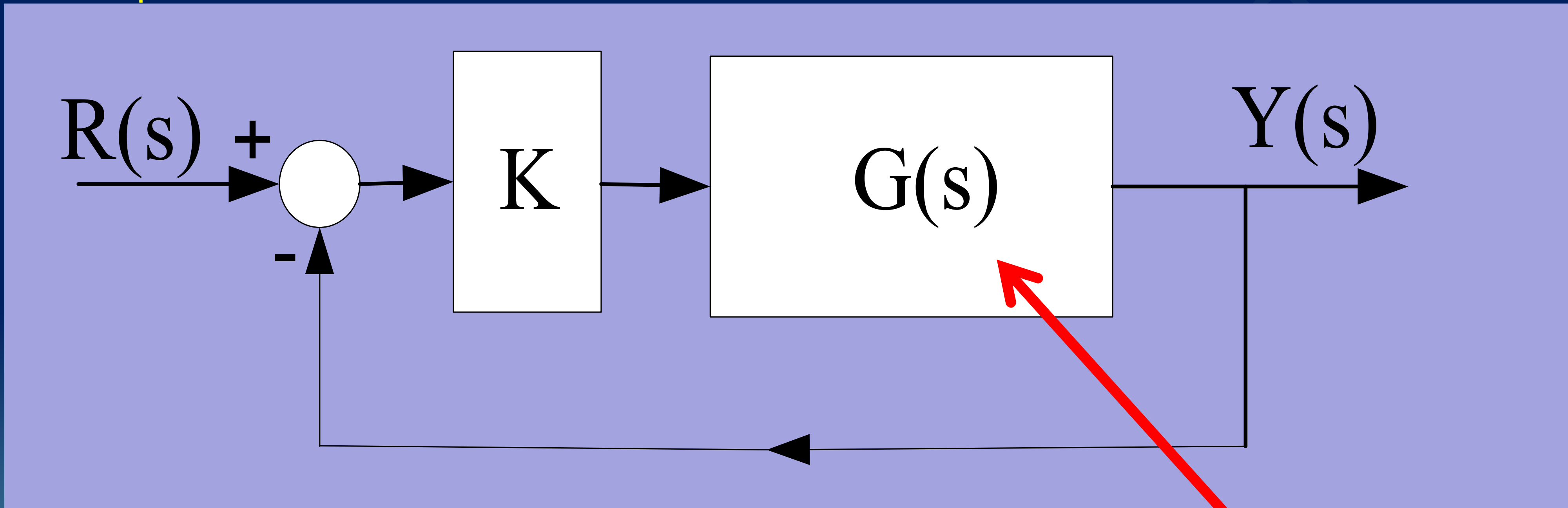


But now we have K, namely,
a "*proportional controller*"
or the "*type* P *controller*"

$$G(s) = \frac{2}{(s+1)}$$

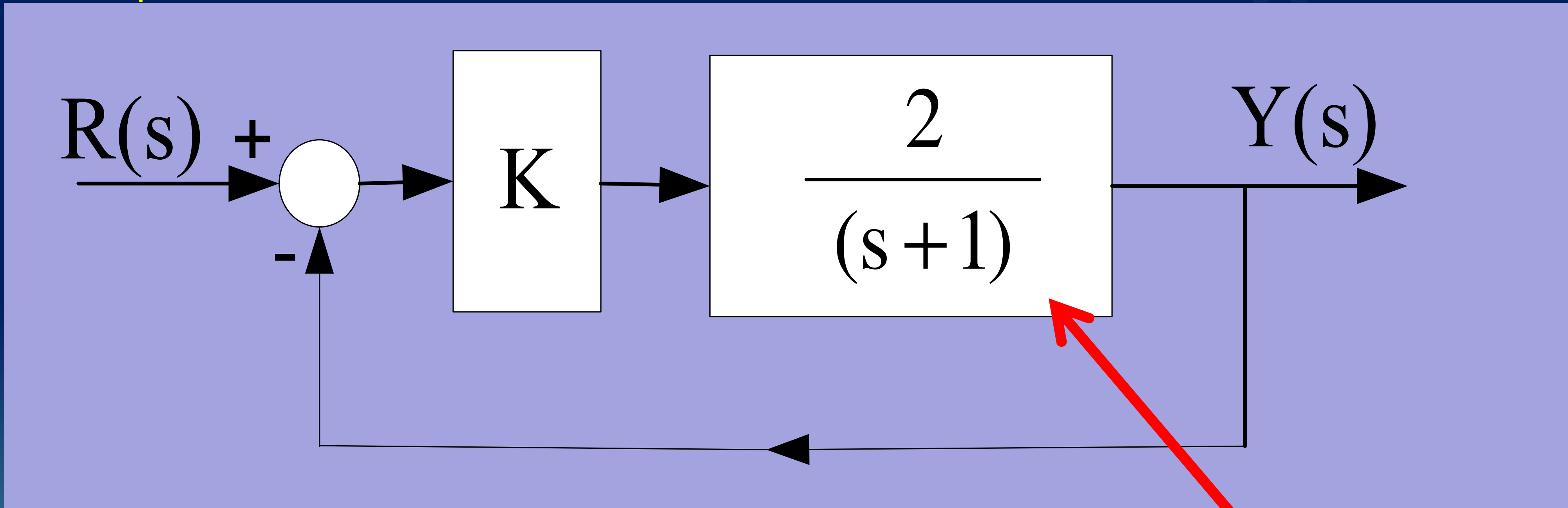
the same as in the previous example

Example 6: (continued)



$$G(s) = \frac{2}{(s+1)}$$

Example 6: (continued)



input r(t):

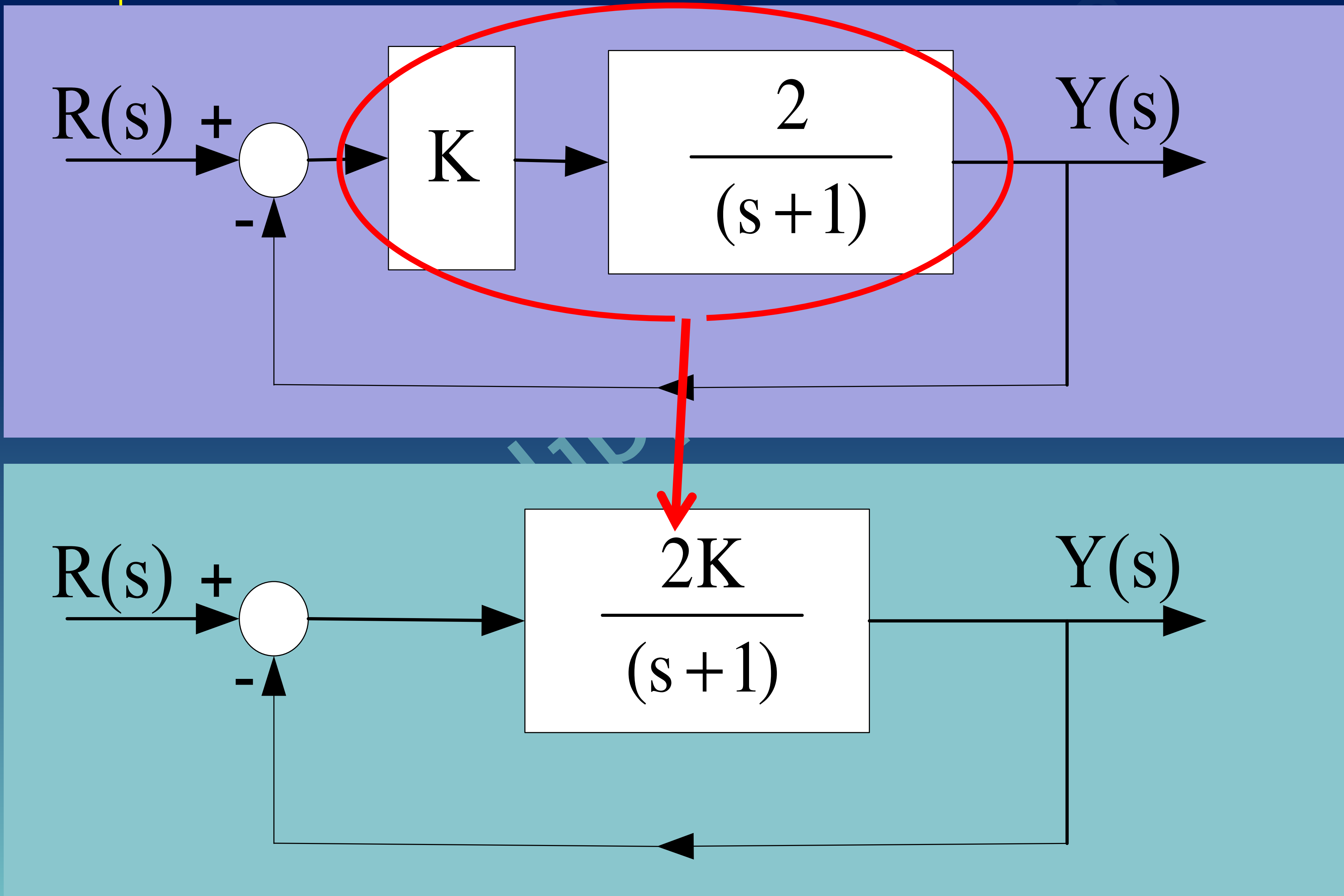
$$r(t) = u_1(t)$$

r(t) = unit step

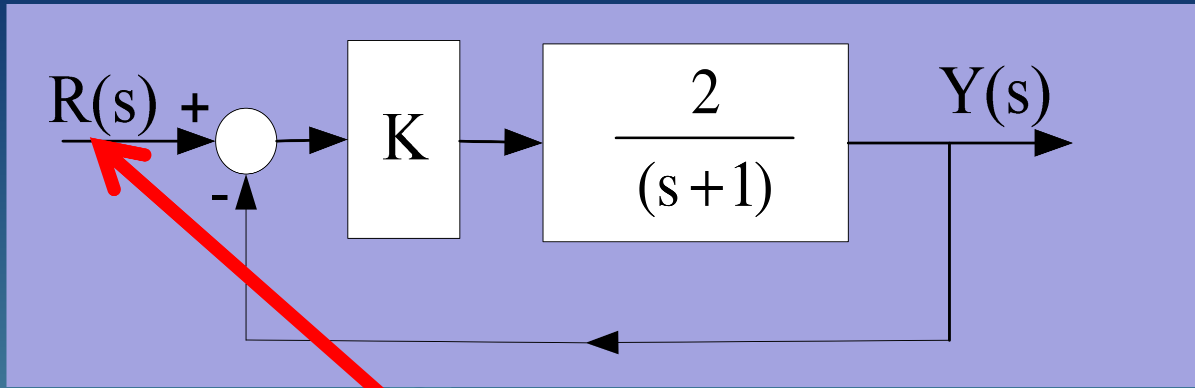
(*again*)

$$G(s) = \frac{2}{(s+1)}$$

Example 6: (continued)



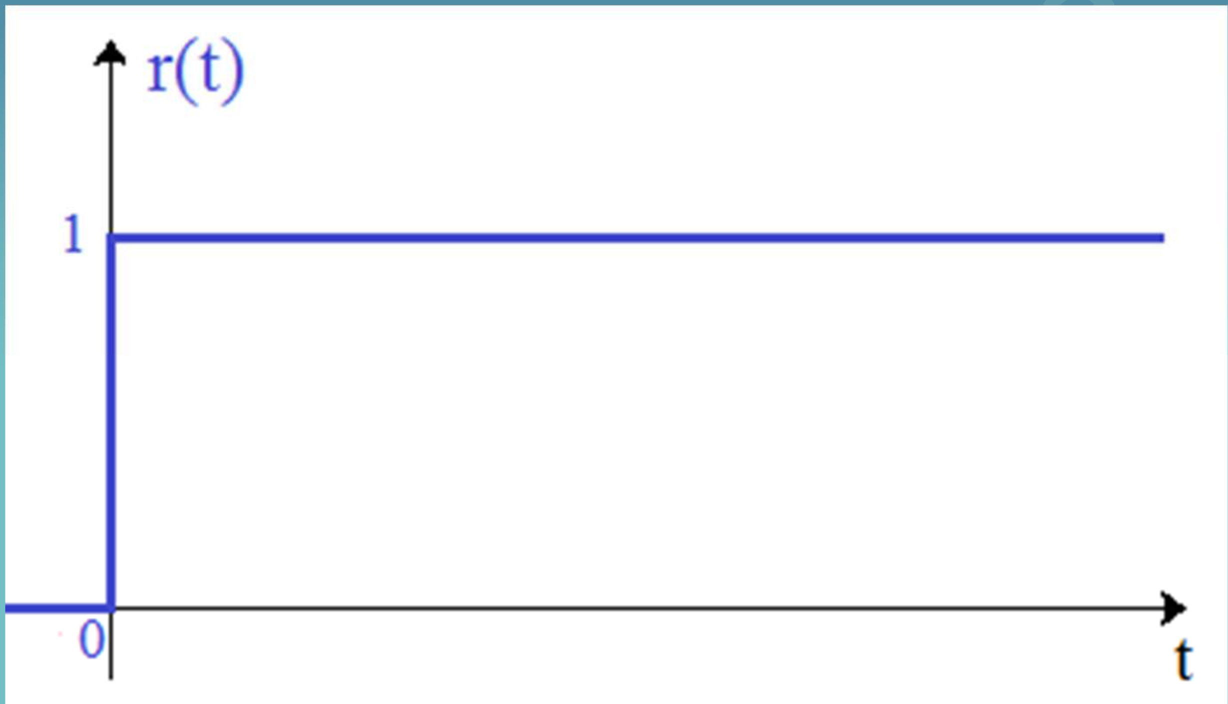
Example 6: (continued)



input r(t):

$$r(t) = u_1(t)$$

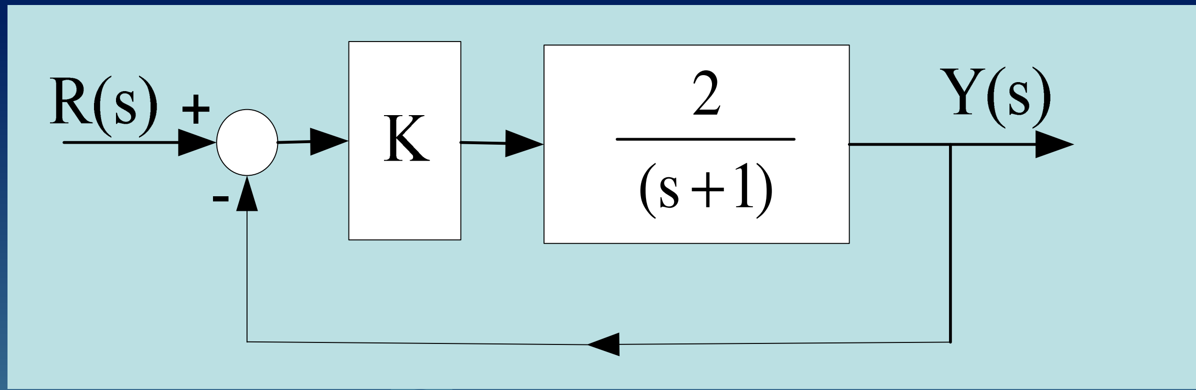
r(t) = unit step



$$R(s) = \frac{1}{s}$$

Example 6: (continued)

$$R(s) = \frac{1}{s}$$



output y(t)

$$Y(s) = \frac{G(s)}{[1+G(s)H(s)]} \cdot R(s)$$

$$= \frac{\dfrac{2K}{(s+1)}}{\left[1+\dfrac{2K}{(s+1)}\right]} \cdot \frac{1}{s}$$

$$= \frac{2K}{s(s+2K+1)}$$

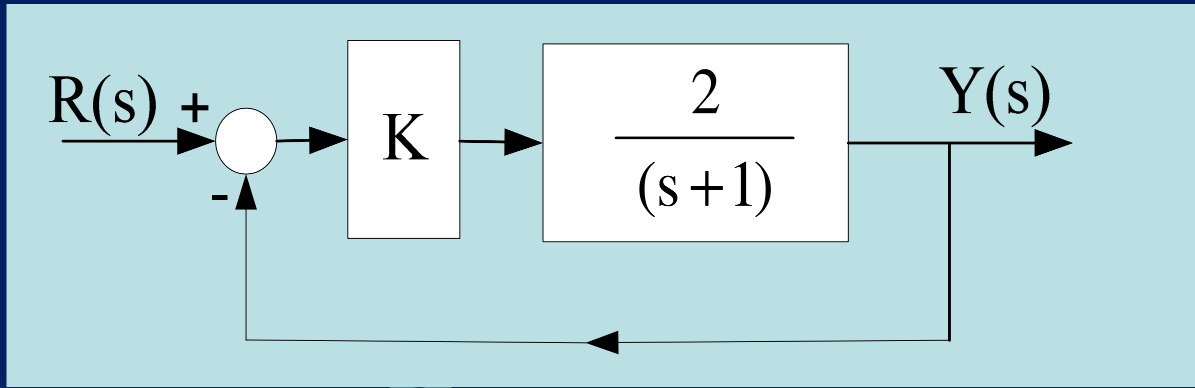
output in steady state y_{ss}

$$y_{ss} = \lim_{s \to 0} s \cdot Y(s) =$$

$$= \lim_{s \to 0} \cancel{s} \cdot \frac{2K}{\cancel{s}(s+2K+1)}$$

$$= \frac{2K}{(2K+1)}$$

Example 6: (continued)



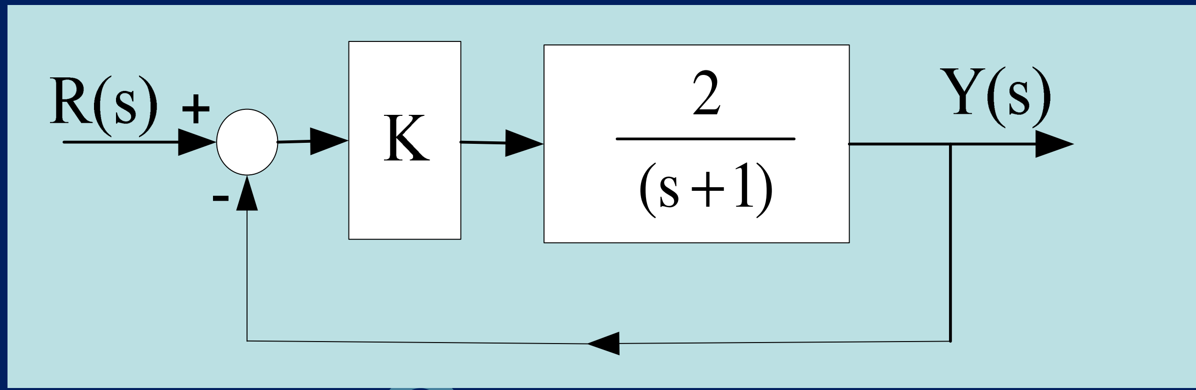
output in steady state y_{ss}

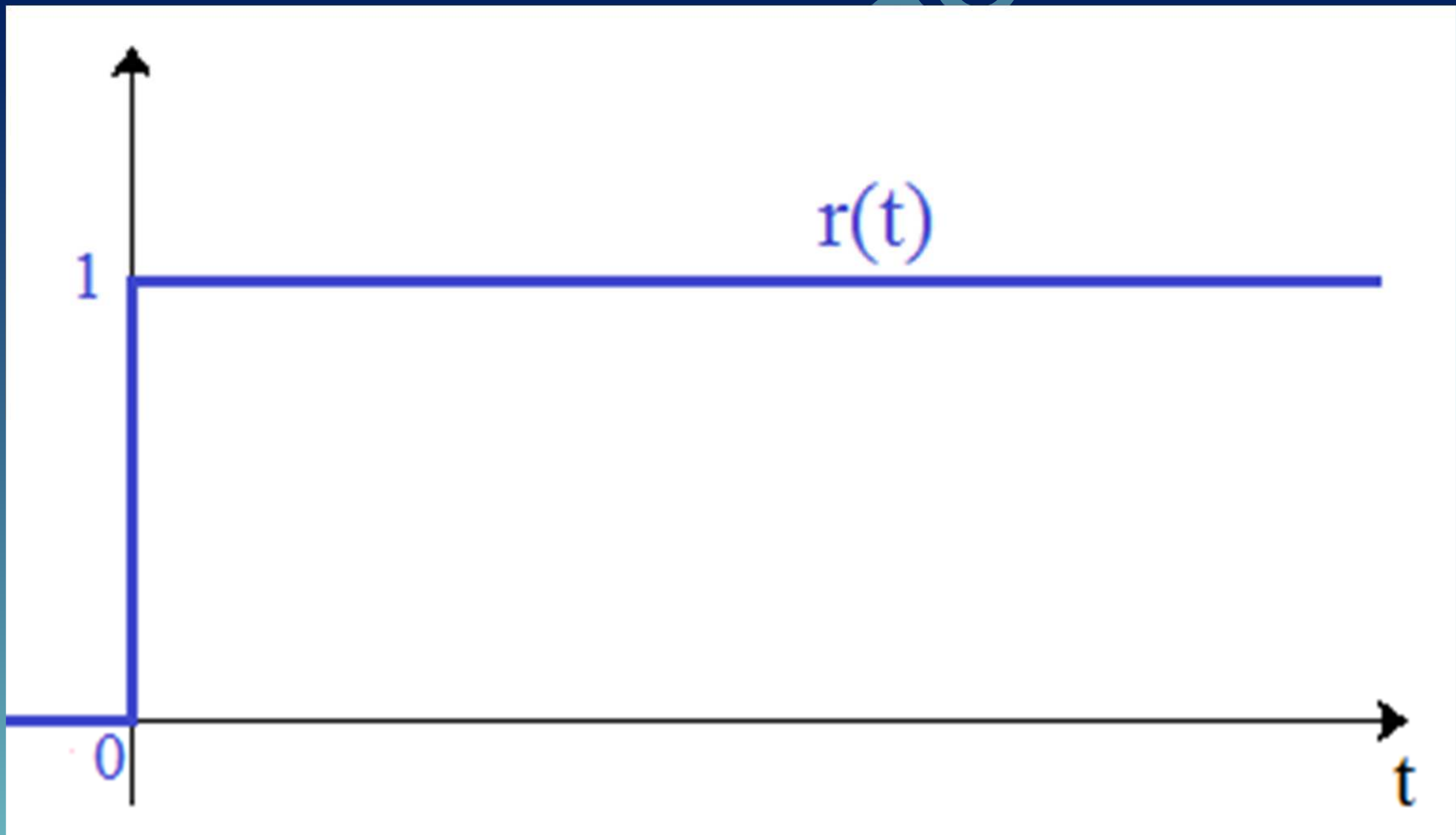
if K is large enough $\Rightarrow$ $y_{ss} = \frac{2K}{(2K+1)} \cong 1$

or

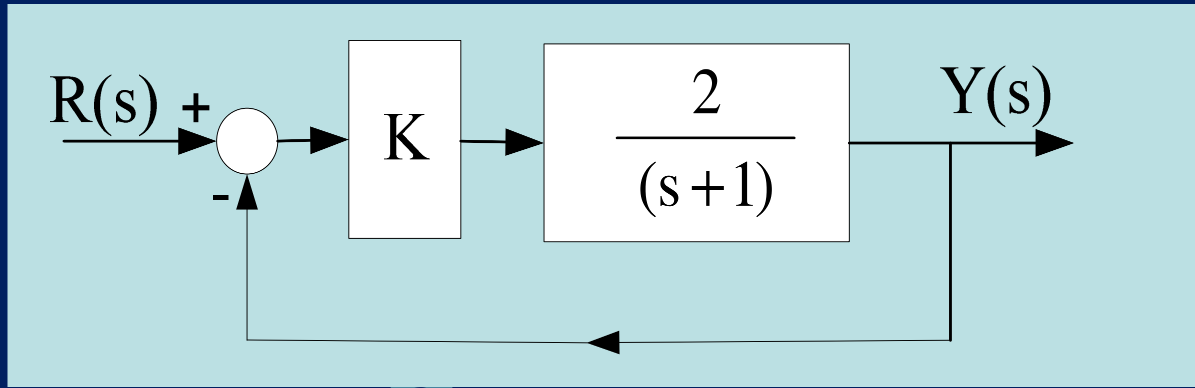
$$y_{ss} \to 1 \quad \text{when} \quad K \to \infty$$

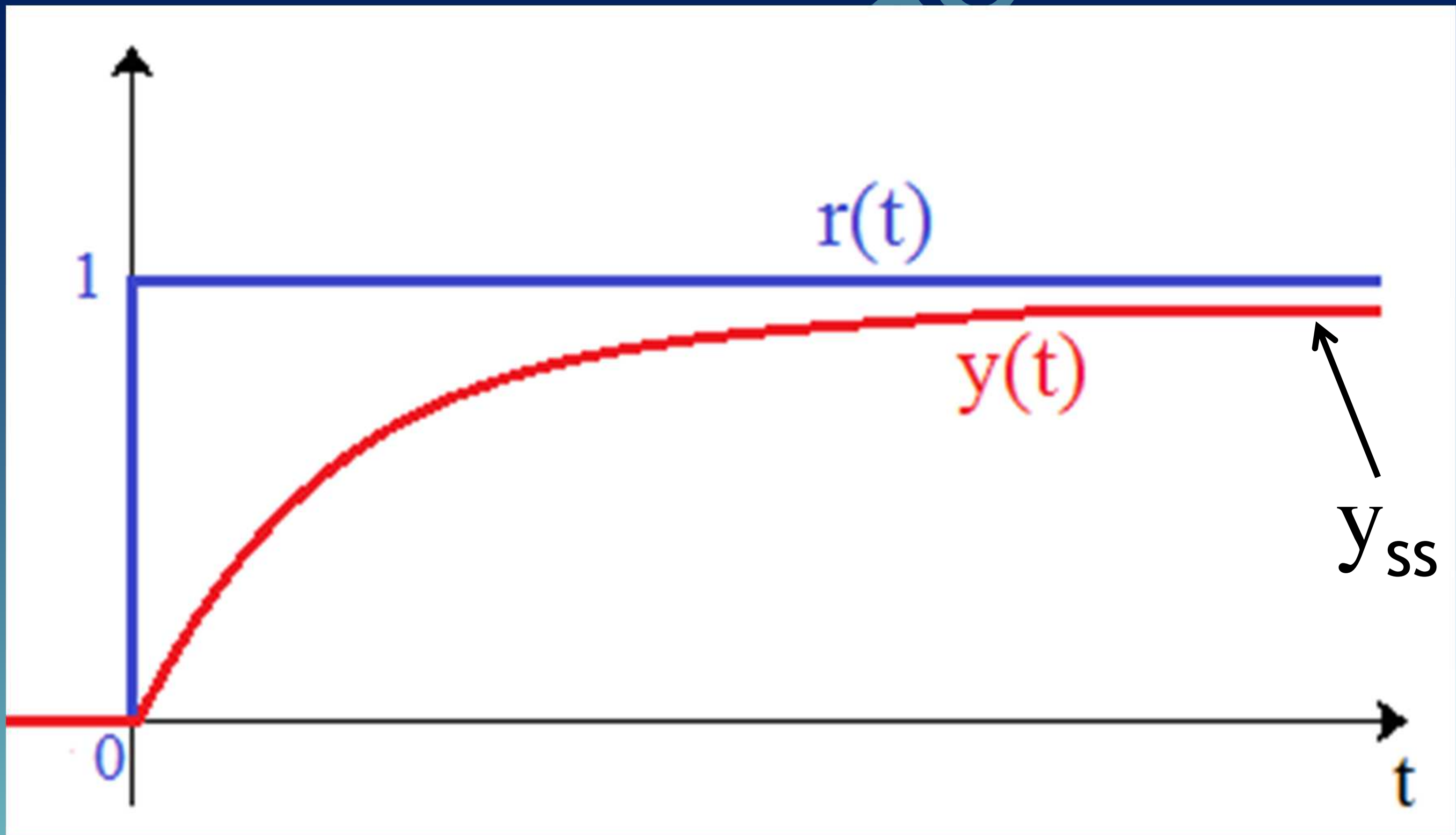
Example 6: (continued)





Example 6: (continued)



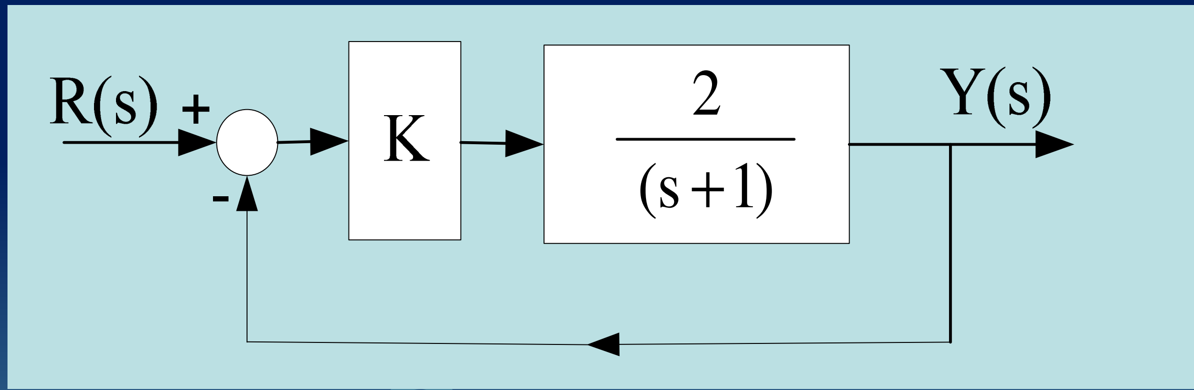


≈ 1

Block Diagrams & error

Example 6: (continued)

$$R(s) = \frac{1}{s}$$



error $e(t)$

$$E(s) = \frac{R(s)}{[1+G(s)H(s)]}$$

$$= \frac{1/s}{\left[1+\frac{2K}{(s+1)}\right]}$$

$$= \frac{(s+1)}{s\,(s+2K+1)}$$

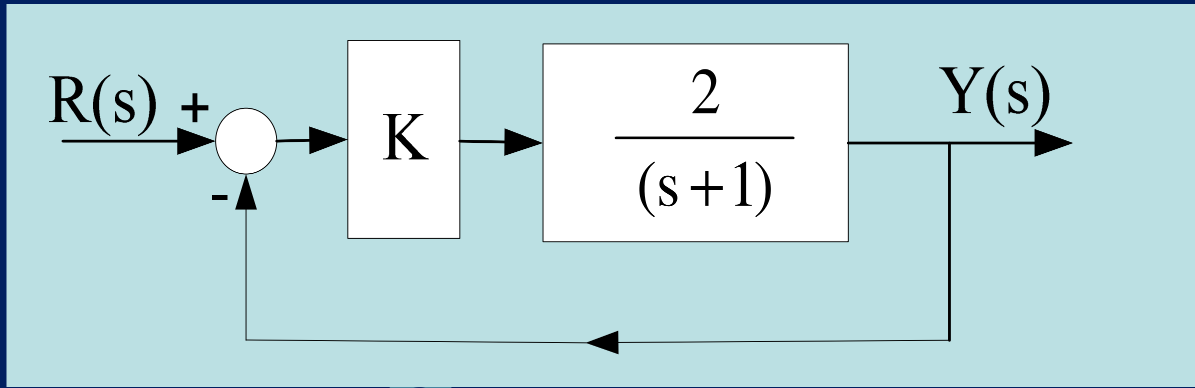
error in steady state e_{ss}

$$e_{ss} = \lim_{s\to 0} s\cdot E(s) =$$

$$= \lim_{s\to 0} \cancel{s}\cdot\frac{(s+1)}{\cancel{s}(s+2K+1)}$$

$$= \frac{1}{(2K+1)}$$

Example 6: (continued)



error in *steady state* e_{ss}

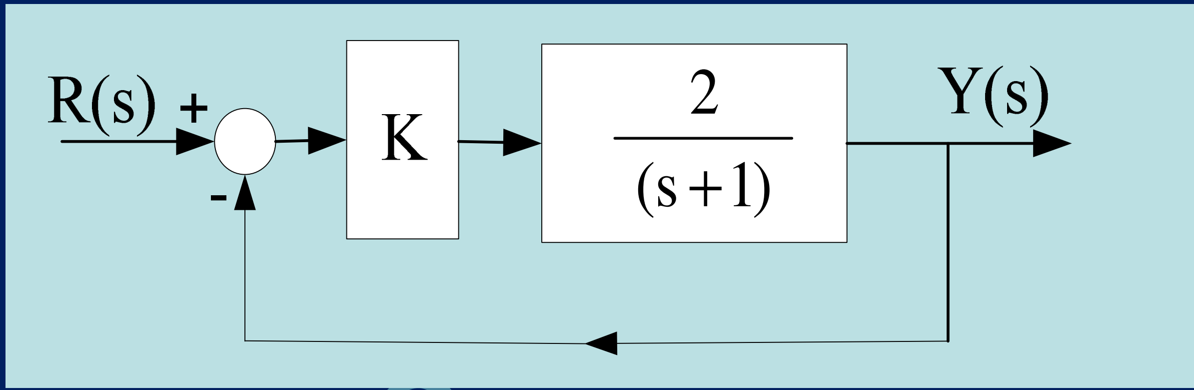
if K is large enough

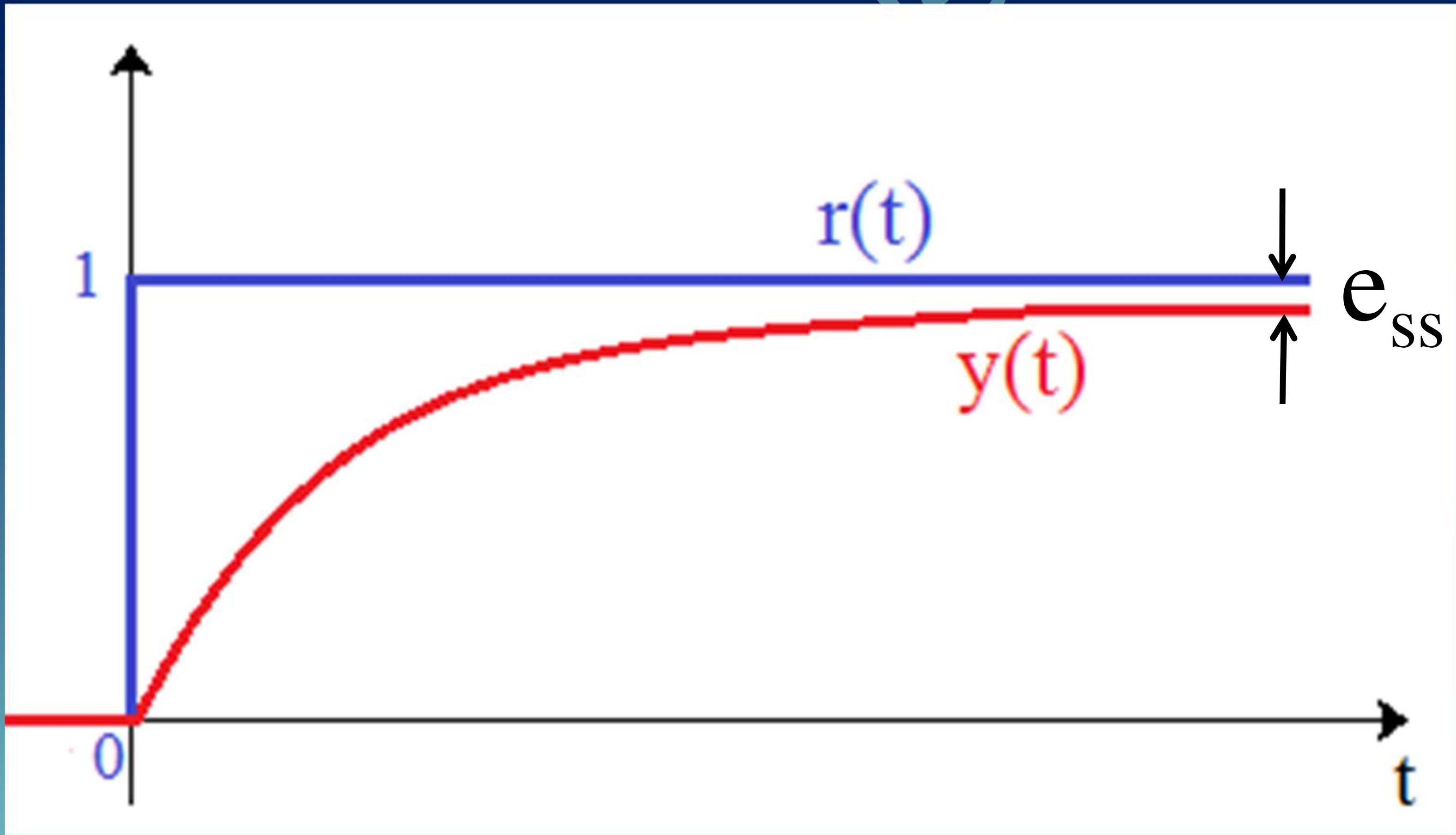
$$e_{ss} = \frac{1}{(2K+1)} \cong 0$$

or

$$e_{ss} \rightarrow 0 \quad \text{when} \quad K \rightarrow \infty$$

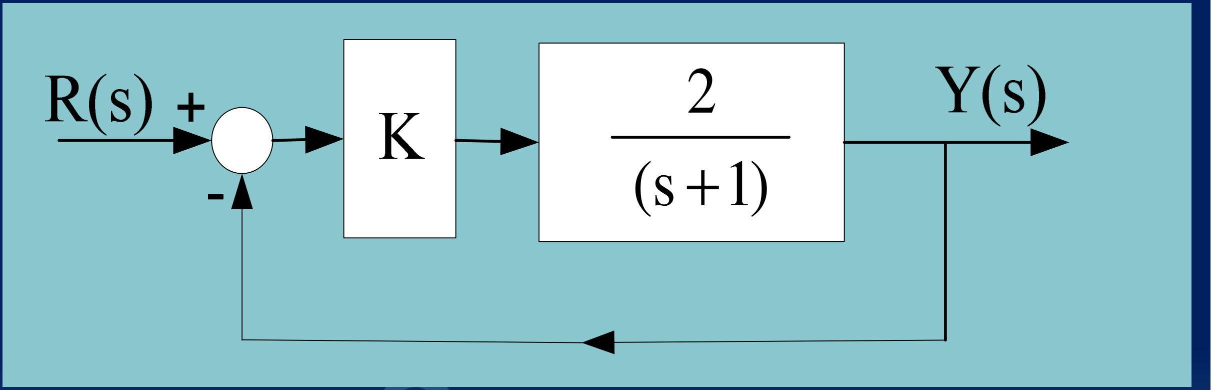
Example 6: (continued)





$e_{ss} \to 0$

Example 6: (continued)



which is the value that we should adjust K such that the *steady state error*

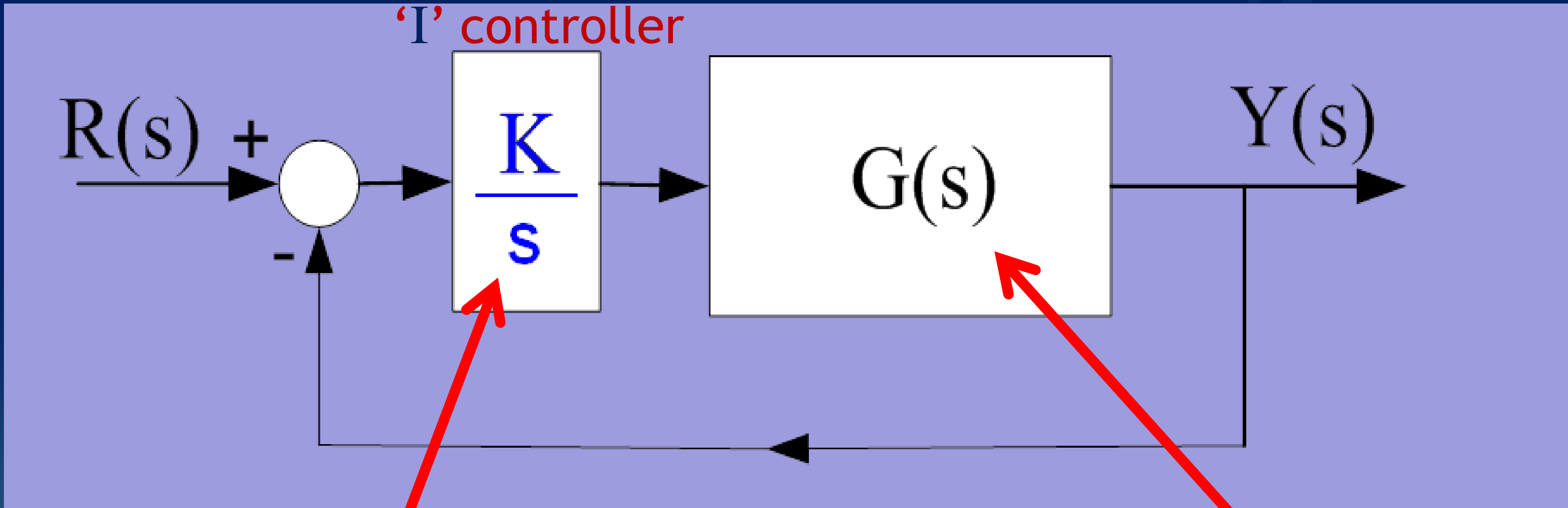
$$e_{ss} < 0{,}01$$

$$e_{ss} = \frac{1}{(2K+1)} < 0{,}01 = \frac{1}{100} \Rightarrow 100 < (2K+1)$$

$$\Rightarrow K > 99/2 \Rightarrow K > 49{,}5$$

we should choose $K > 49{,}5$

Example 7:



Using K/s, we get a "*integral controller*" or a "*type* I *controller*".

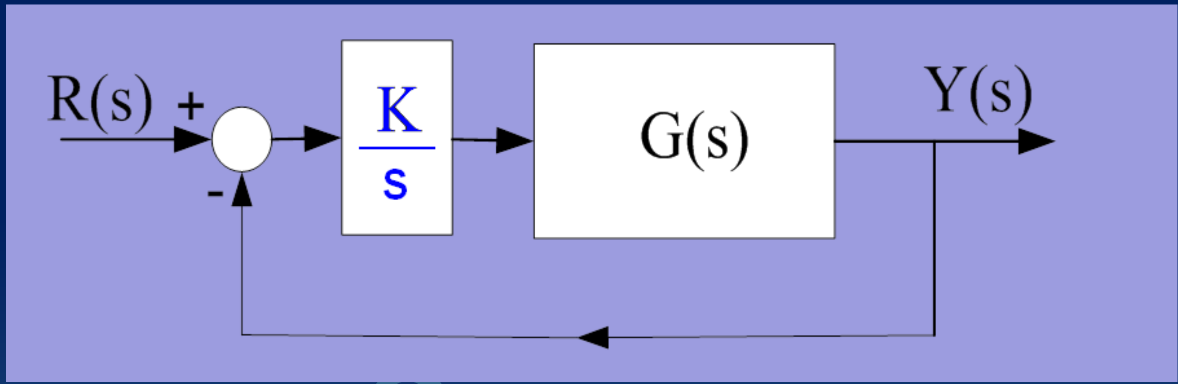
$$G(s) = \frac{2}{(s+1)}$$

the same as in the previous example

Example 7:

(continued)

$$R(s) = \frac{1}{s}$$



error $e(t)$

$$E(s) = \frac{R(s)}{[1+G(s)H(s)]}$$

$$= \frac{1/s}{\left[1+\dfrac{2K}{s(s+1)}\right]}$$

$$= \frac{(s+1)}{(s^2+s+2K)}$$

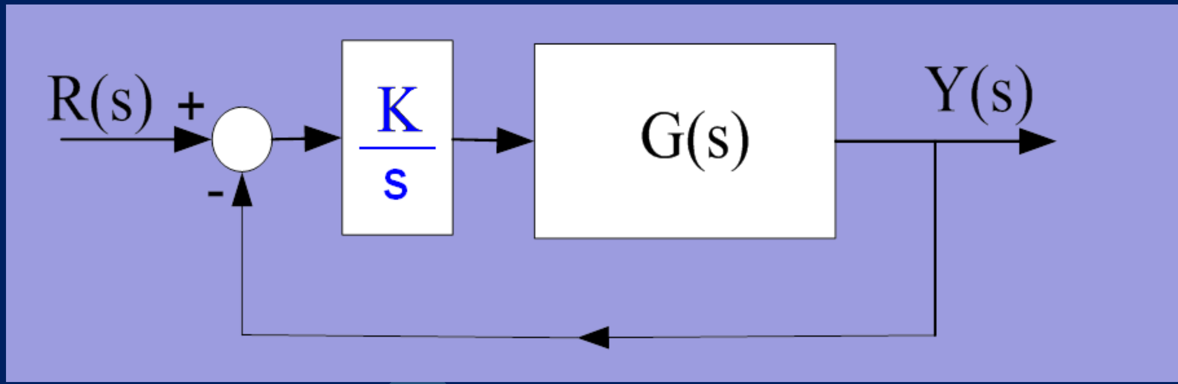
error in steady state e_{ss}

$$e_{ss} = \lim_{s\to 0} s\cdot E(s) =$$

$$= \lim_{s\to 0} s\cdot\frac{(s+1)}{(s^2+s+2K)}$$

$$= 0$$

Example 7:
(continued)

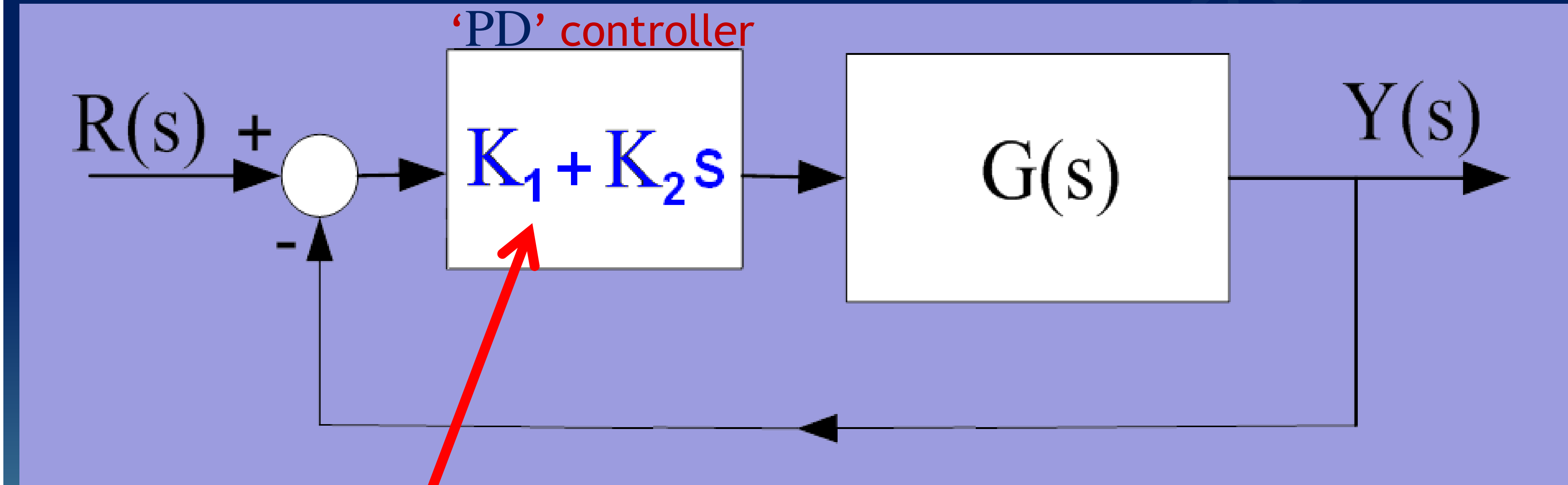


thus, now it is possible, for example, to adjust K such that the *steady state error* is

$$e_{ss} = 0$$

we should choose $K > 0$

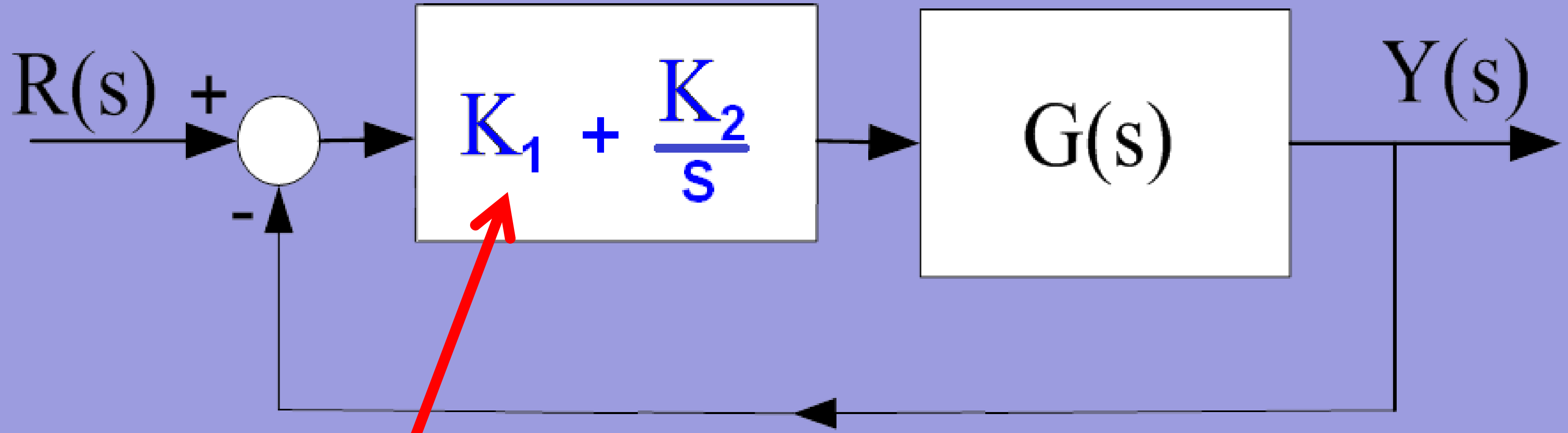
There are several types of controllers



Using $K_1 + K_2 s$, we get a "*proportional derivative controller*" or a "*type* PD *controller*"

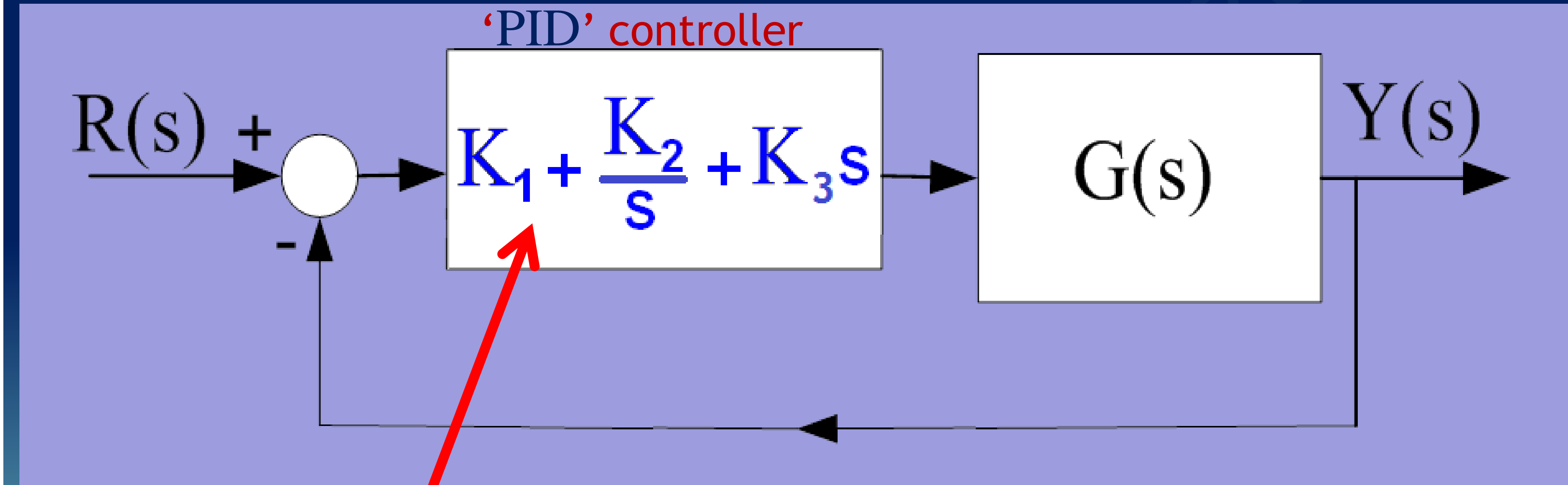
There are several types of controllers

'PI' controller



Using $K_1 + K_2/s$, we get a "*proportional integral controller*" or a "*type* PI *controller*"

The most general case is a 'PID controller'



Using $K_1 + K_2/s + K_3s$, we get a "*proportional integral derivative controller*" or a "*type PID controller*"

FACULDADE ENGENHARIA

Departamento de
Engenharia Eletromecânica

Thank you!

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