Control Systems

2

“Systems Classification”

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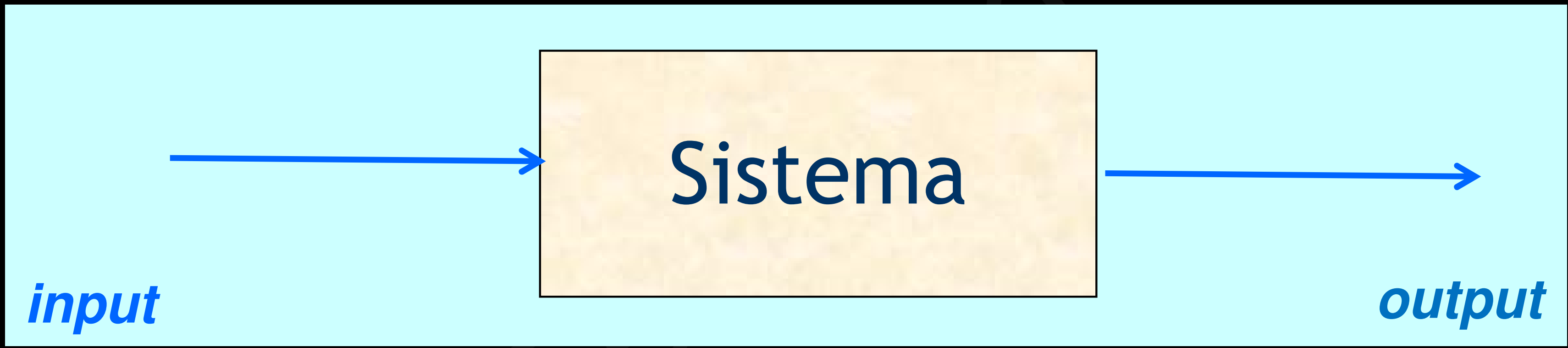
It is common to represent systems schematically using *black box*

Black box of a system with its ***input*** and the ***output***.

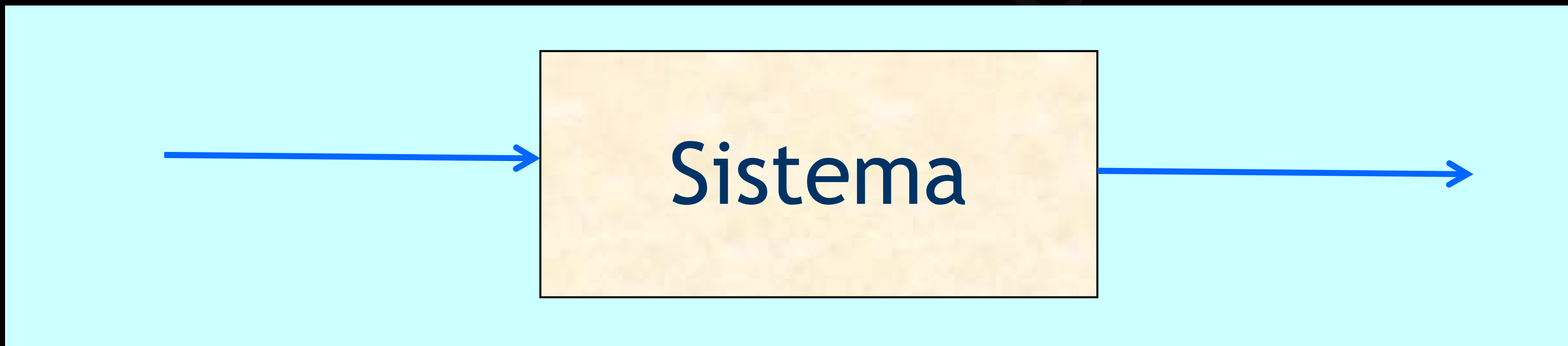


Other names for *input* and *output*?

Other names for input and output?

The ***input*** of a system sometimes is also called by '*control*' or even *the* '*excitation*' of the system.

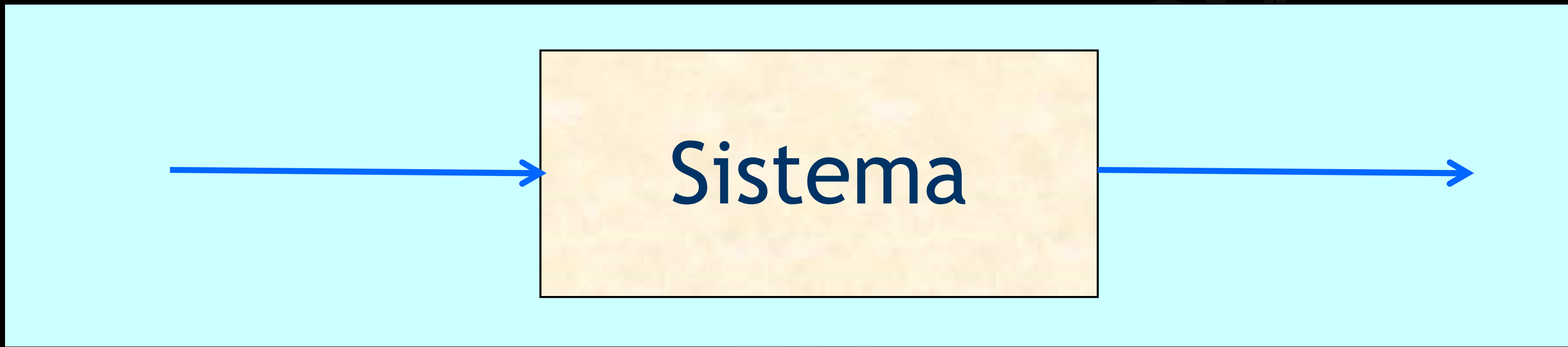
On the other hand, a ***output*** of a system sometimes is also called by '*answer*' or the '*observation*' of the system.



input
control
excitation

output
answer
observation

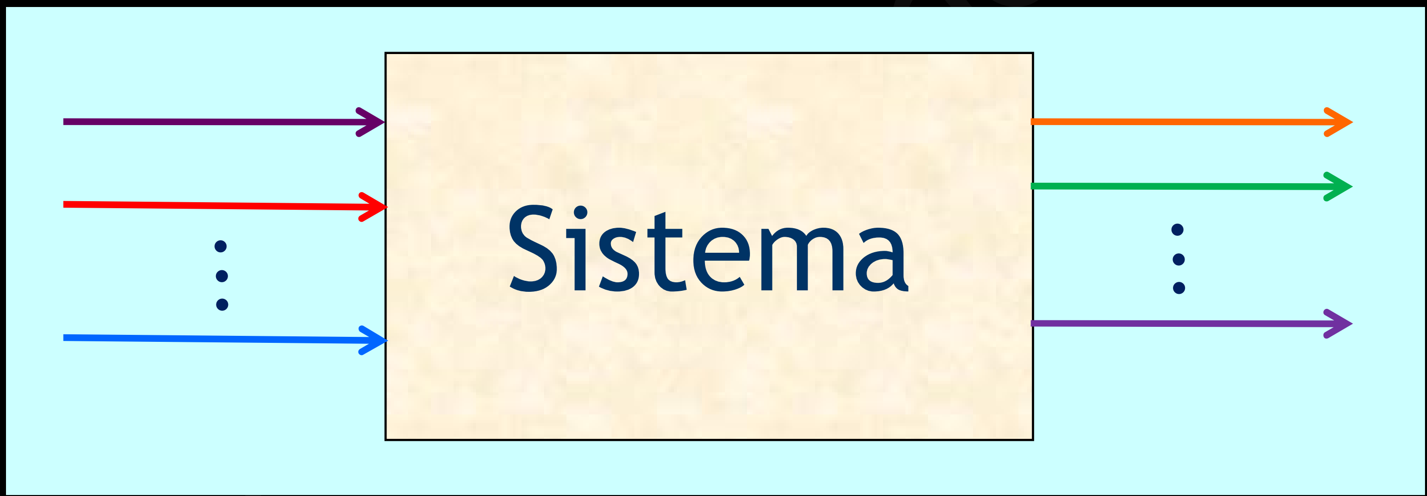
Systems



input	*output*
control	answer
excitation	*observation*

Actually, several systems have not only one input and one output but multiple ***inputs*** and/or multiple ***outputs***.

inputs



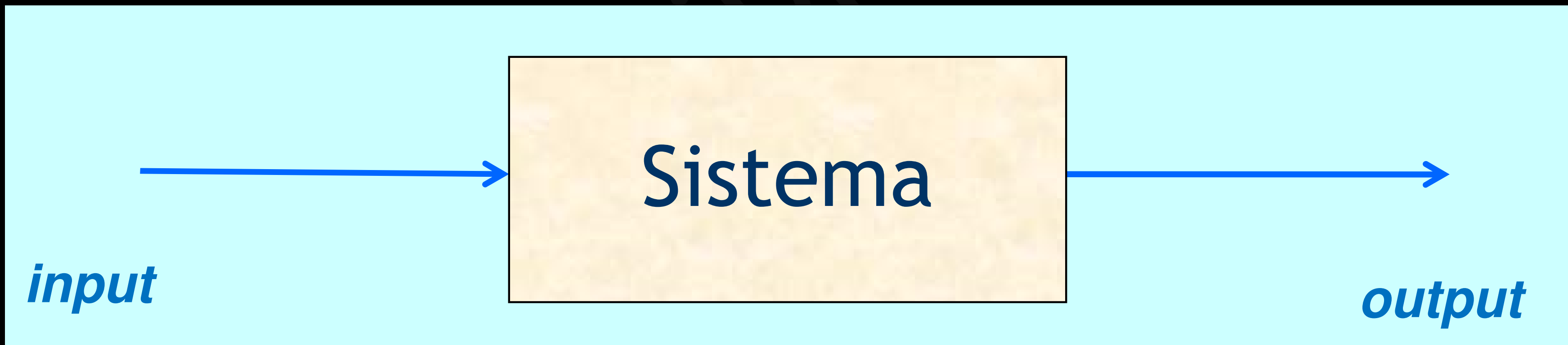
outputs

Black box of a system with multiples ***inputs*** and/or multiple ***outputs***

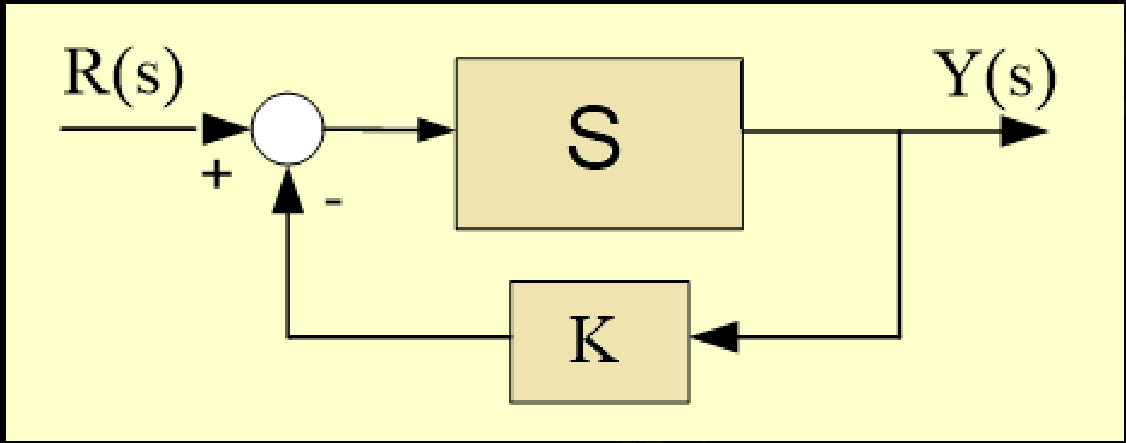
Systems

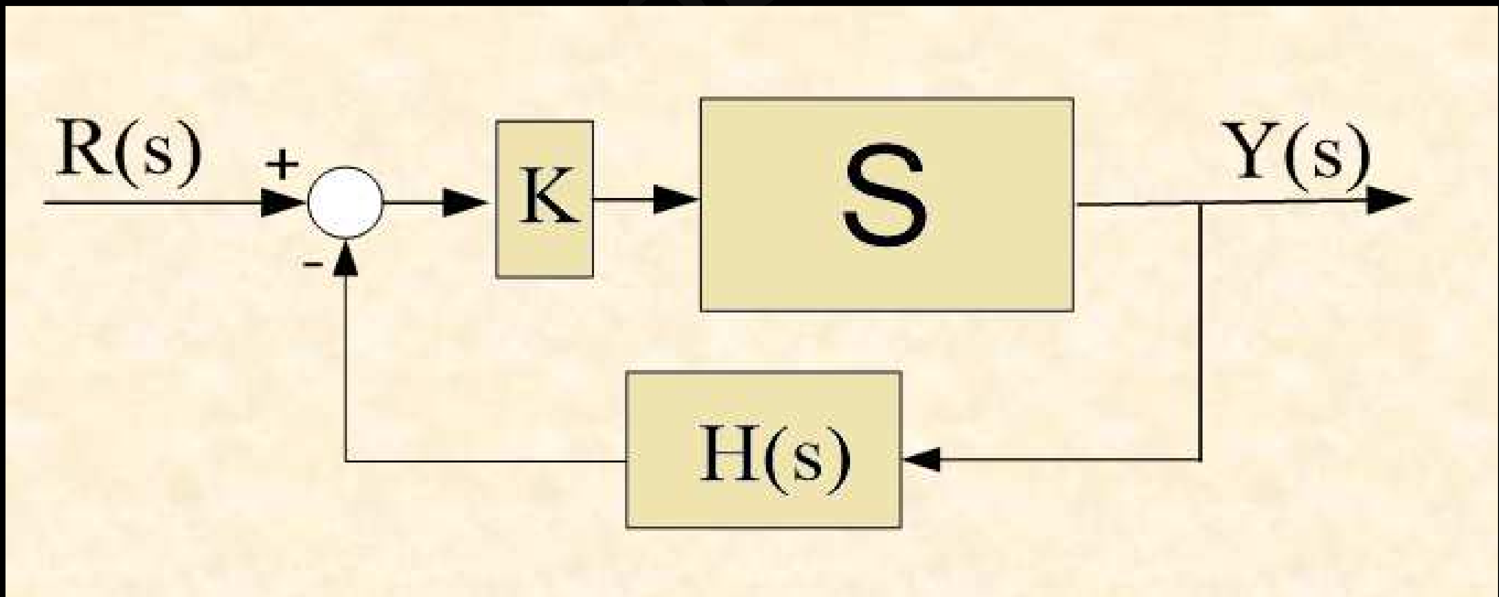
There is a way to represent systems using blocks and therefore it called by "*block diagram*", which will be seen in chapter 5.

Actually, the ***black box*** is a *block diagram* with only one block.



Other examples of "*block diagrams*".





Systems Classification

Systems Classification

Physical nature

- electrical;
- electronic;
- mechanical;
- electromechanical;
- thermic;
- hydraulic;
- optical;
- acoustic;
- chemical;
- informatics;
- aeronautical;
- aerospace;
- biological;
- biomedical;
- economic;
- sociological;
- socioeconomic;
- etc.

the majority of complex systems are combinations of the various sub-systems of different natures.

Systems Classification

Physical nature

Medical equipment in a surgery room, examples of ***systems*** developed in biomedical engineering.

Systems Classification

Physical nature

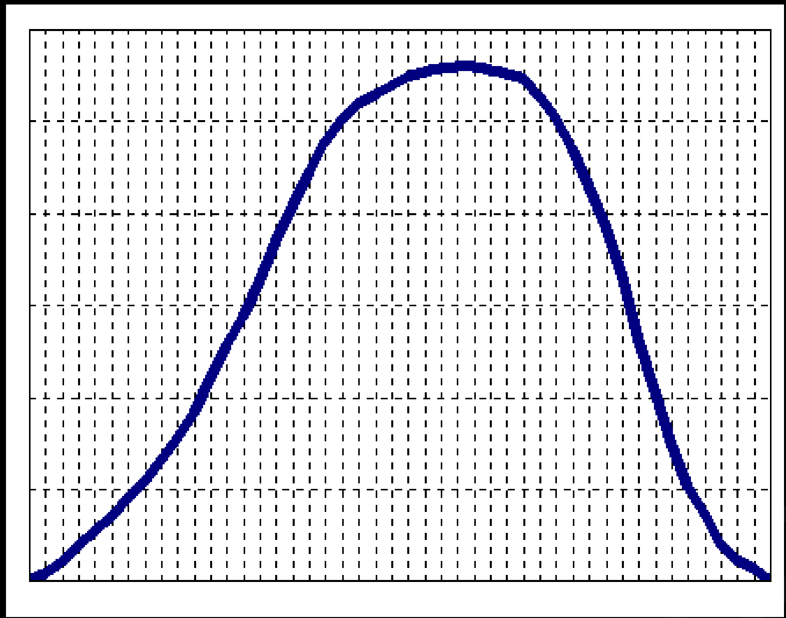
Therefore, in that area of '*health*' also can be found many systems of bioengineering, that is, biologic e biomedical systems simultaneously with mechanical, electrical ou electronic systems.

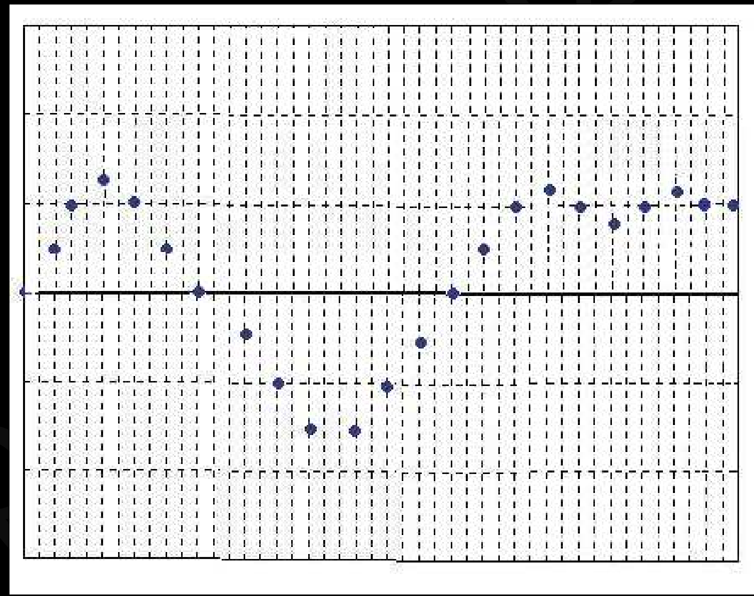
An *artificial limb*, or each device utilized in surgeries are some examples of biomedical systems.

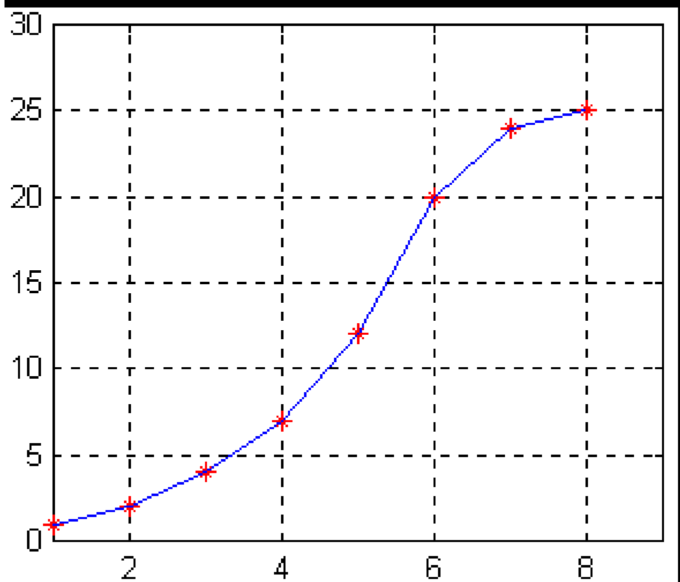
Systems Classification

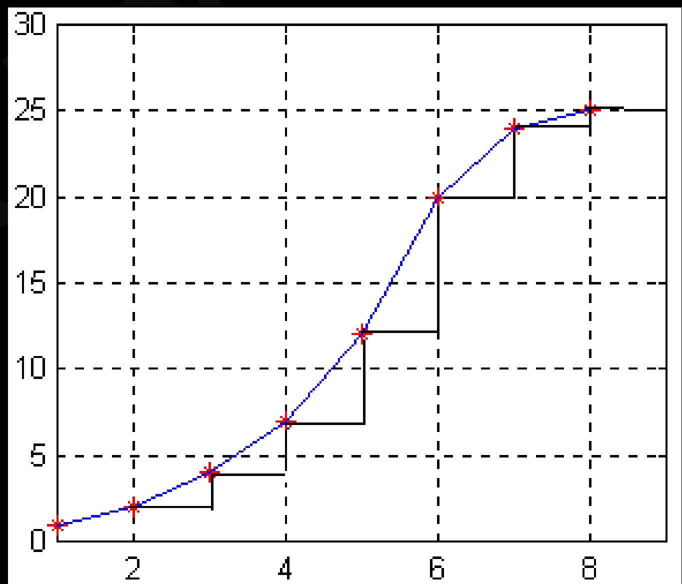
Continuity in time

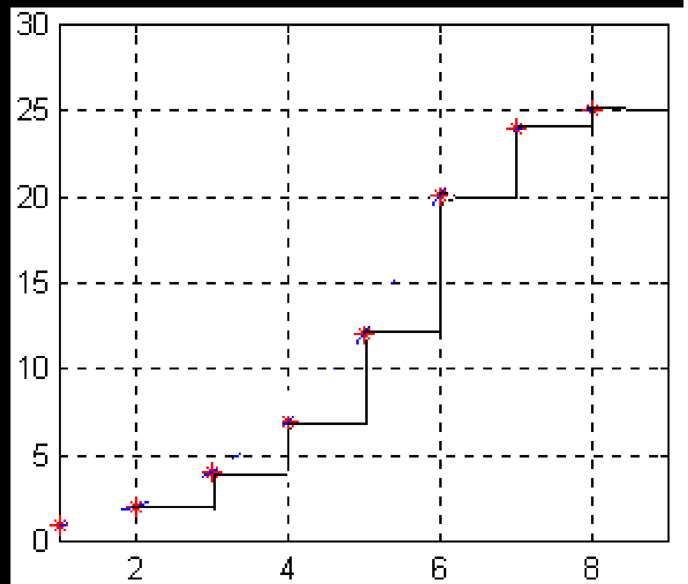
- continuous
- discrete
- discretized











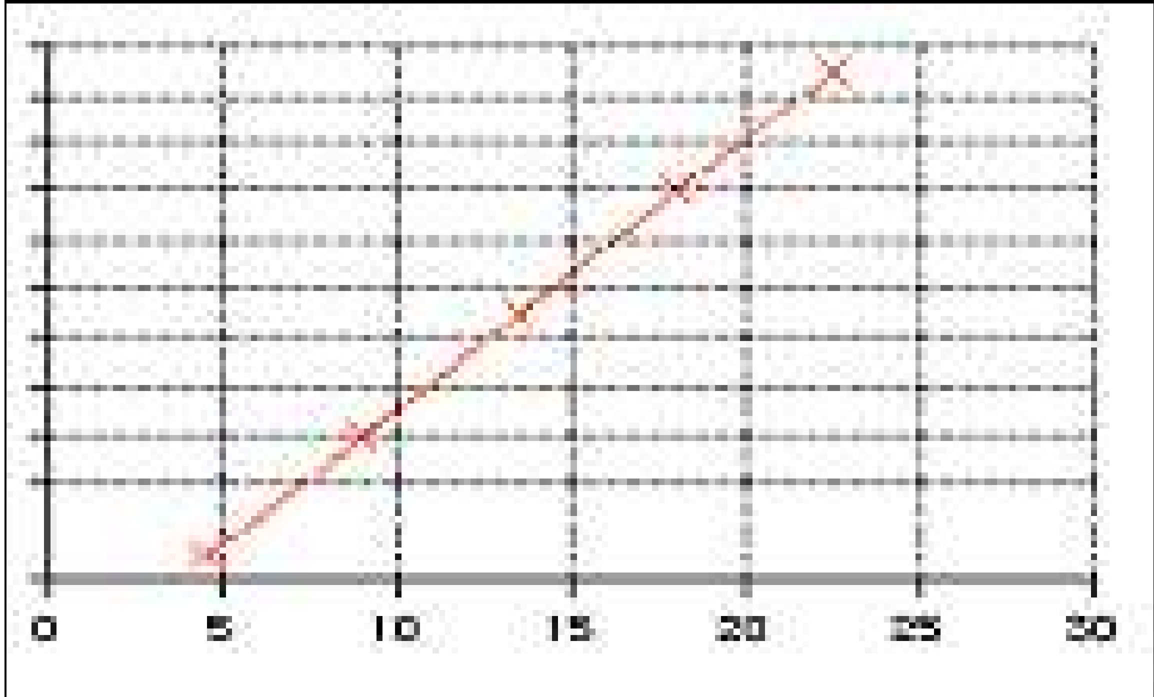
Continuity in time

When a system which has been discretized is stored in "*bits*" (sequence of "*zeros*" and "*ones*") then it said that the system has been digitalized.

This is the case of *audio digital systems (music in mp3) or in video*, and even the *electrocardiograms and electroencephalograms exams* that are digitally stored no computer.

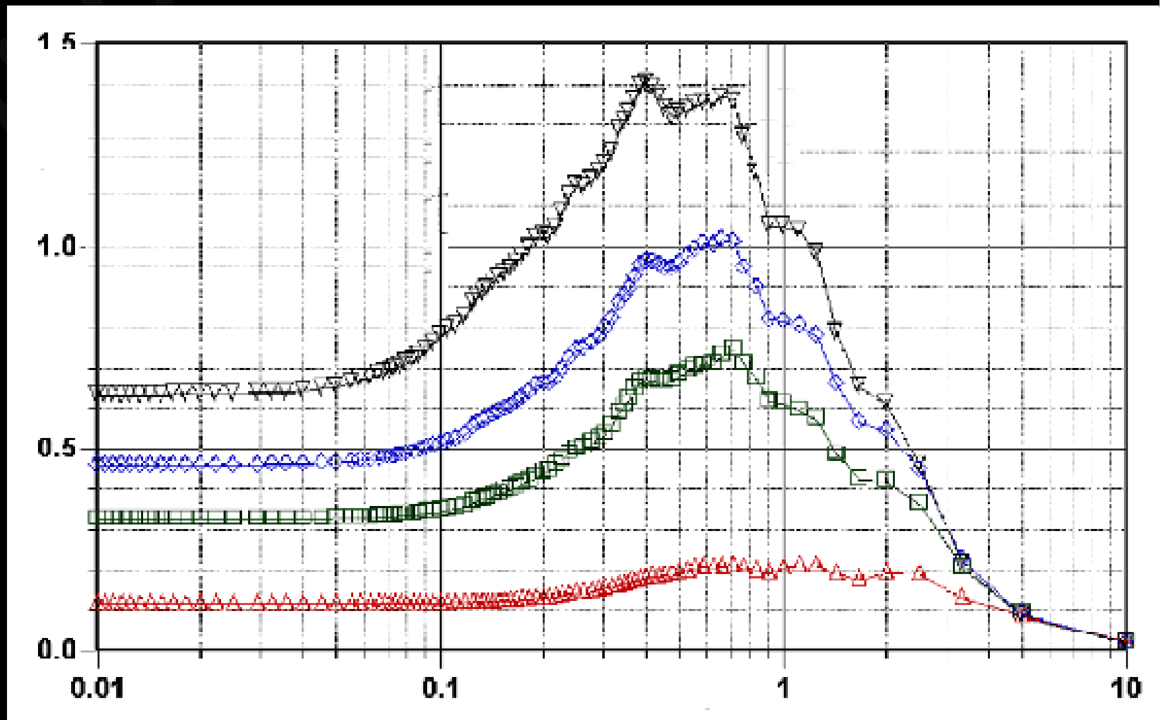
The *digitalization* it is not the same that the *discretization*. Normally are used several "*bits*" to store each discrete position.

Systems Classification



Linearity

- linear
- nonlinear



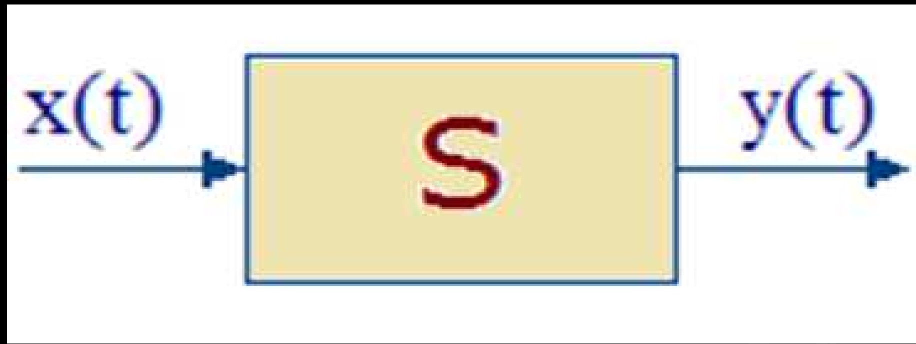
Linearity

homogeneity: when the input x is multiplied by a value k; then the output y is also multiplied by this same value k;

additivity: when the input is the sum of x_1 and x_2, that produce individually the outputs y_1 and y_2 respectively; then the output is the sum of the outputs y_1 e y_2.

Linearity

continuous case

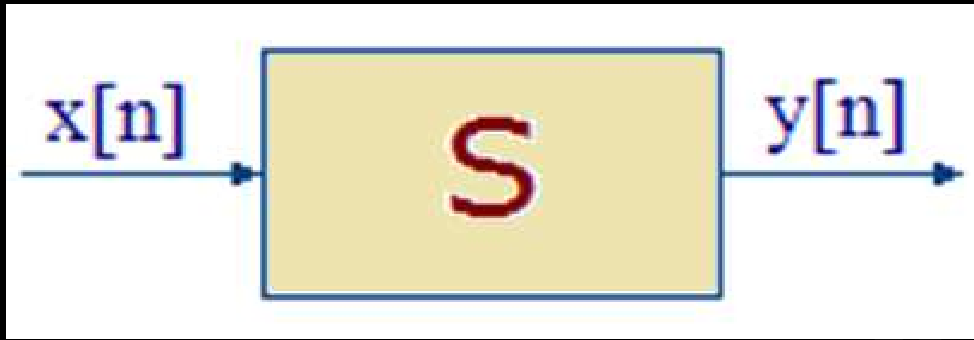


homogeneity

additivity

Linearity

discrete case

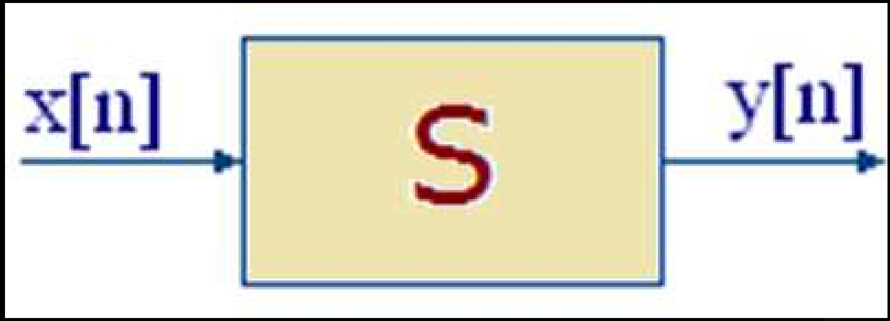


homogeneity

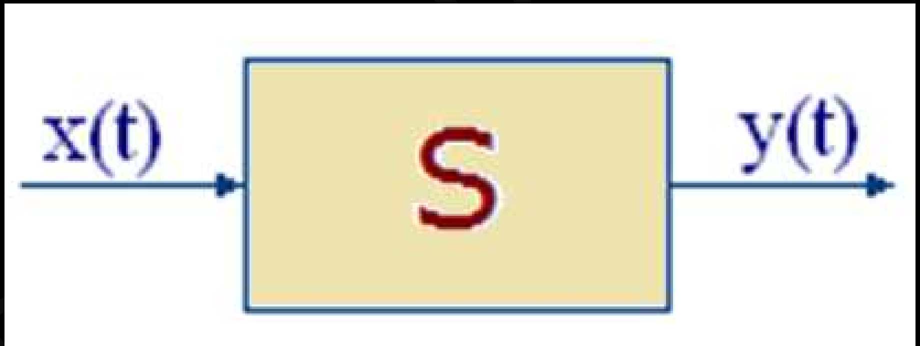
additivity

Systems Modelling

The creation a model or a scheme
to represent the system



Black box diagram schematic of a system
discrete case



Black box diagram schematic of a system
continuous case

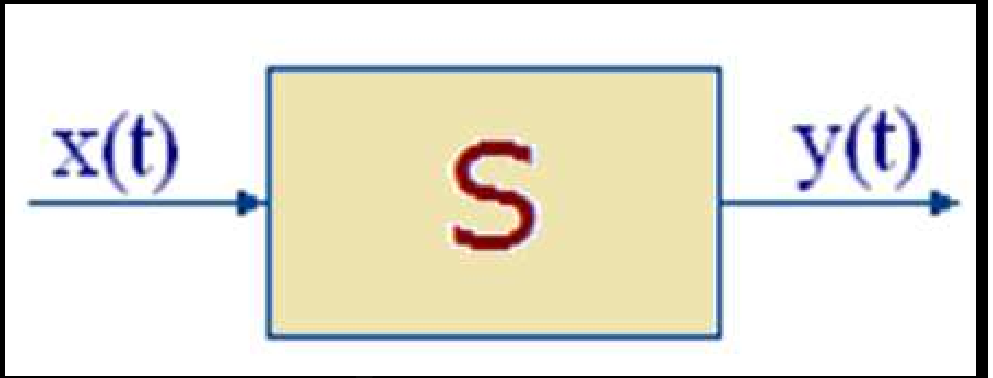
Systems Modelling

The representation of systems can be of several different forms:

- *with Difference equations* [discrete case]
- *with Ordinary differential equations* (ODE) [continuous case]
- *with Partial differential equations* (PDE) [continuous case]
- *with Retarded equations* [continuous case]
- *with Tables* [discrete case]
- *with Flowcharts or Data flow diagram* [either discrete or continuous case]
- *with Integral equations* [continuous case]
- *with Integro-differential equations* [continuous case]

Ordinary differential equations (ODE)

Examples



$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} - y = \frac{dx}{dt} + 3x$$

$$\frac{d^2y}{dt^2} + 6t\frac{dy}{dt} + y = \frac{dx}{dt} - (t-4)x$$

$$\frac{d^2y}{dt^2} + 5\,\frac{dy}{dt} + 2y = \frac{dx}{dt} - x(t+3)$$

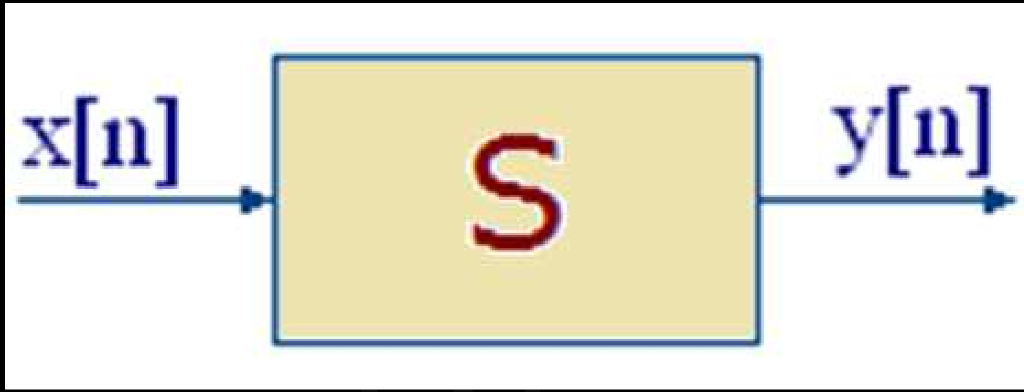
$$3y'' - 2y' + y = x'x$$

$$10\,y'' + 2y' - y = e^{x}$$

$$\frac{d^2y}{dt^2} + x\frac{dy}{dt} + y = 0$$

Difference equations

Examples



$$y[n] + 7\,y[n-1] + 2\,y[n-2] = x[n] - 4\,x[n-1]$$

$$y[n] = -4\,x[n-1]$$

$$y[n] - 5\,n\,y[n-1] = x[n+1] - 2\,x[n]$$

$$y[n] = 2\,(x[n])^2 - 4\,x[n]$$

$$y[n] = \sqrt[3]{|x[n]|^2}$$

Partial differential equations (PDE)

Examples

$$\frac{\partial^2 u}{\partial t^2} = k\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) =$$

$$= k\,(u_{xx} + u_{yy} + u_{zz})$$

'equação de onda'
("*wave equation*")

$$\frac{\partial u}{\partial t} = k\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) =$$

$$= k\,(u_{xx} + u_{yy} + u_{zz})$$

'equação de calor'
("*heat equation*")

Retarded equations

Examples

$$y(t) = x(t - \delta)$$

$$y'(t) + y(t) - 3y(t - \tau) = x(t)$$

$$y[n] = 5x[n - n_\delta]$$

Algebraic equations

Examples

$$y(t) = 2x(t) - 5$$

$$y(t) = -x^2(t) + 2x(t)$$

$$y[n] = x^3[n] + 5x^2[n] + 6x[n]$$

$$y[n] = \cos(x[n]) + 0{,}5$$

Other Examples (ODE)

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} - y = \frac{dx}{dt} + 3x$$

continuous and linear system

$$\frac{d^2y}{dt^2} + 6t\frac{dy}{dt} + y = \frac{dx}{dt} - (t-4)x$$

continuous and linear system

$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 2y = \frac{dx}{dt} - x(t+3)$$

continuous and linear system

$$3y'' - 2y' + y = x'x$$

continuous and nonlinear system

$$10y'' + 2y' - y = e^x$$

continuous and nonlinear system

$$\frac{d^2y}{dt^2} + x\frac{dy}{dt} + y = 0$$

continuous and nonlinear system

Other Examples (*Difference equations*)

$$y[n] + 7\,y[n-1] + 2\,y[n-2] = x[n] - 4\,x[n-1]$$

discrete and linear system

$$y[n] = -4\,x[n-1]$$

discrete and linear system

$$y[n] - 5\,n\,y[n-1] = x[n+1] - 2\,x[n]$$

discrete and linear system

$$y[n] = 2\left(x[n]\right)^2 - 4\,x[n]$$

discrete and nonlinear system

$$y[n] = \sqrt[3]{\left|x[n]\right|^2}$$

discrete and nonlinear system

Other Examples (PDE)

$$\frac{\partial^2 u}{\partial t^2} = k\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) =$$

$$= k\,(u_{xx} + u_{yy} + u_{zz})$$

"wave equation"

continuous and linear system

$$\frac{\partial u}{\partial t} = k\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) =$$

$$= k\,(u_{xx} + u_{yy} + u_{zz})$$

"heat equation"

continuous and linear system

Other Examples (*Retarded equations*)

$$y(t) = x(t - \delta)$$ continuous and linear system

$$y'(t) + y(t) - 3y(t - \tau) = x(t)$$ continuous and linear system

$$y[n] = 5x[n - n_\delta]$$ discrete and linear system

Other Examples (*Algebraic equations*)

$$y(t) = 2x(t) - 7$$ continuous and nonlinear system

$$y(t) = -x^2(t) - 2x(t) + 3$$ continuous and nonlinear system

$$y[n] = -2x[n]$$ discrete and linear system

$$y[n] = 1 - \cos(2\pi \cdot x[n])$$ discrete and nonlinear system

Systems Classification
(*continued*)

Systems Classification

Time variance

- time-variant
- time-invariant

A *time-invariant* system is the one that doesn't matter when the input is applied.

Any input signal $x(t)$ will always produce the same an output signal $y(t)$, either now or later.

Actually, no system is *time-invariant*.

However in practical terms, we consider *time-invariant* several systems which variation in time is very slow.

Examples

Time variance

$$y[n] - 5\,n\,y[n-1] = x[n+1] - 2\,x[n]$$

Time-variant system

$$\frac{d^2y}{dt^2} + 6t\frac{dy}{dt} + y = \frac{dx}{dt} - (t-4)\,x$$

Time-variant system

LTI systems - Linear time-invariant systems

Systems Classification

Examples (*Time variant/invariant*)

$$y[n] + 7y[n-1] + 2y[n-2] = x[n] - 4x[n-1]$$

Time-invariant system

$$y[n] = -4x[n-1]$$ Time-invariant system

$$y[n] = 2(x[n])^2 - 3n\,x[n]$$ Time-variant system

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} - y = \frac{dx}{dt} + 3x$$ Time-invariant system

$$\frac{d^2y}{dt^2} + x\frac{dy}{dt} + ty = 0$$ Time-variant system

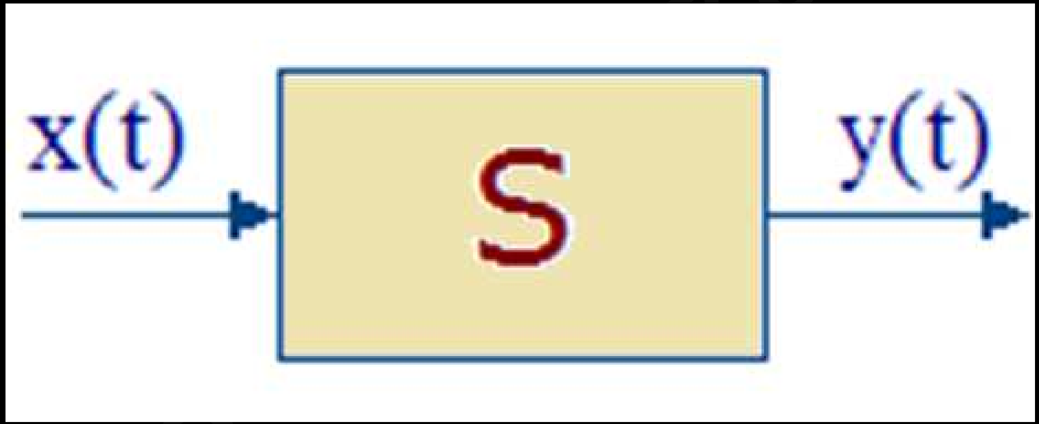
$$3y'' - 2y' + y = x'x$$ Time-invariant system

$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 2y = \frac{dx}{dt} - x(t+3)$$ Time-invariant system

Systems Classification

Random nature

- deterministic
- stochastic



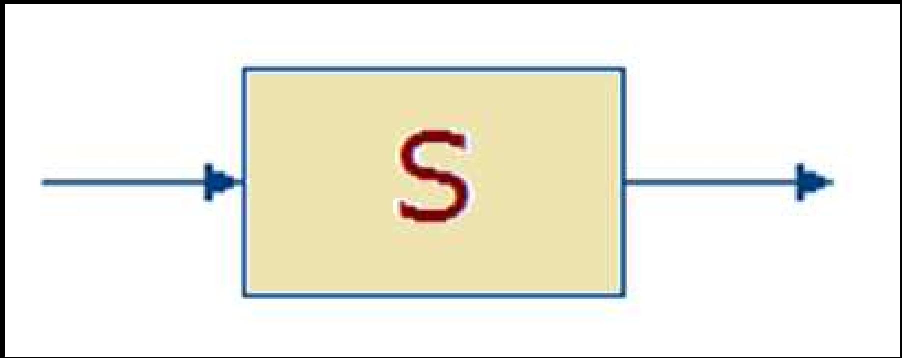
A ***deterministic*** system is the one that is not under the influence of any random perturbation, that is, does not have uncertainties.

The output signal $y(t)$ for an input signal $x(t)$ can be calculated (or "*determined*") with precision when the system model is known.

As matter of fact, no system is *deterministic*. Every system has some type of 'uncertainty' or *random character* and therefore they are called ***stochastic***.

Systems Classification

Memory



- memory systems
- memoryless systems

A *memoryless* system is such that: an output signal at the time instant t_1 depends only on the input signal of that instant t_1.

$$y[n] = 2(x[n])^2 - 4x[n]$$

memoryless systems

$$y(t) = -x^2(t) - 2x(t) + 3$$

memoryless systems

since the output $y[n]$, or $y(t)$, depends on input $x[n]$, or $x(t)$, only at that time instants ('t' ou 'n').

Memory

Examples

$$y[n] = \sqrt[3]{|x[n]|^2}$$

memoryless systems

$$y(t) = 2\,x(t) - 7$$

memoryless systems

$$y[n] = -2x[n]$$

memoryless systems

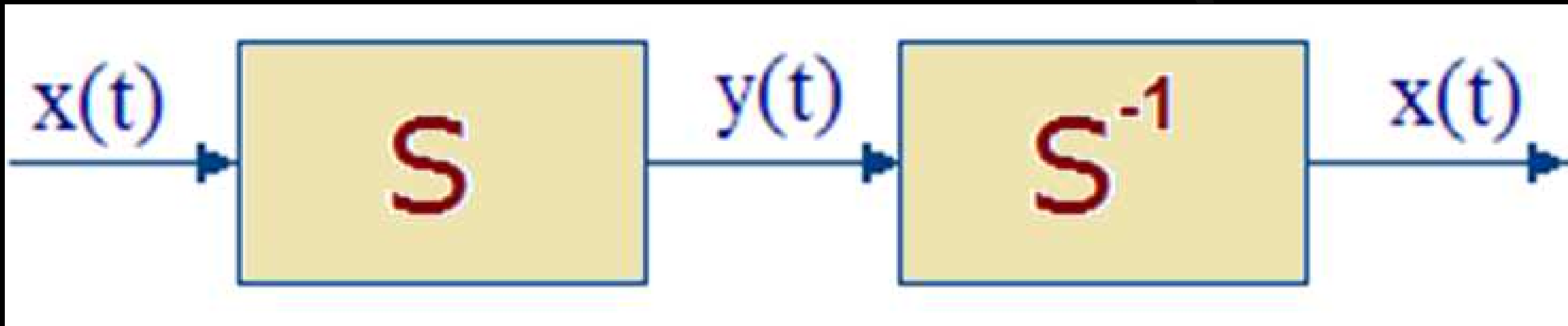
$$y[n] = 1 - \cos(2\pi \cdot x[n])$$

memoryless systems

Invertibility

- invertible
- non invertible

A system is invertible if different *inputs* will drive to different *outputs*.



It is possible to find an inverse system S^{-1} for which the *input* $y[n]$, or $y(t)$, will produce the *output* $x[n]$, or $x(t)$, respectively.

$$y(t) = 2\,x(t) - 7$$

invertible system

$$y[n] = -2x[n]$$

invertible system

Invertibility

$$y(t) = 2\,x(t) - 7$$

$$y[n] = -2x[n]$$

For the above systems, each *input* signal x produces an exclusive *output* signal y, different from the *output* for any other *input*.

Because of that, any *input* signal x can be expressed in terms of the *output* signal y as:

$$x(t) = \tfrac{1}{2}\,(y(t) + 7)$$

$$x[n] = -\,y[n]/2$$

Invertibility

continuous case

retarded system (“*time delay equation*”)

$$y(t) = x(t - \delta)$$

The *output* $y(t)$ reproduces the *input* $x(t)$ with a delay of δ units of time.

advanced system (“*time advance equation*”)

$$x(t) = y(t + \delta)$$

The *output* signal $x(t)$ reproduces what is going to be the *input* signal $y(t)$ in δ units of time later.

retarded system and advanced system are invertible systems and one is the inverse of the other

Invertibility

discrete case

retarded system (“*time delay equation*”)

$$y[n] = x[n - n_\delta\ ]$$

The *output* y[n] reproduces the *input* x[n] with a delay of δ units of time.

advanced system (“*time advance equation*”)

$$x[n] = y[n + n_\delta]$$

The *output* signal x[n] reproduces what is going to be the *input* signal y[n] in δ units of time later.

retarded system and advanced system are invertible systems and one is the inverse of the other

Systems Classification

Causality

- causal (or non-anticipative)
- non-causal (or anticipative)

A system is called ***causal*** (ou non-anticipative) if the *output* at the instant t_1 depends only on the *input* at time instants $t < t_1$.

If the *output* at the instant t_1 depends on the *input* at instants $t > t_1$ then this system anticipates what is going to happen and therefore it is called "*anticipative*" or *non-causal*.

In our real physical world, if a variable 't' (or 'n' in the discrete case) represents the time, then there is a dynamics that evolves with time and therefore it is not possible to have a '*non-causal*' system since it is not possible to predict the future.

Nevertheless there are cases in which this variable 't' (or 'n' in the discrete case) may represent other parameter or another physical measure (that is not the time) and in that way causal system may be possible to occur.

Summarizing:

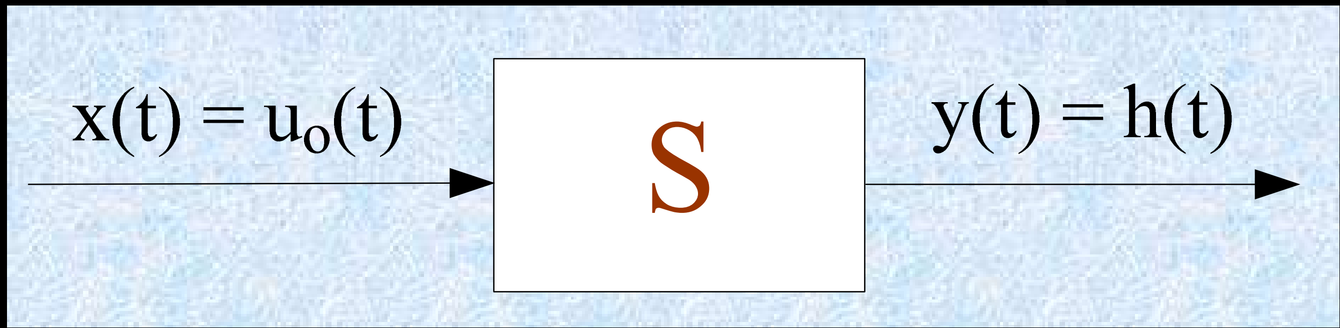
Systems Classification

- Physical nature
- Continuity in time
- Linearity
- Time variance
- Random nature
- Memory
- Invertibility
- Causality

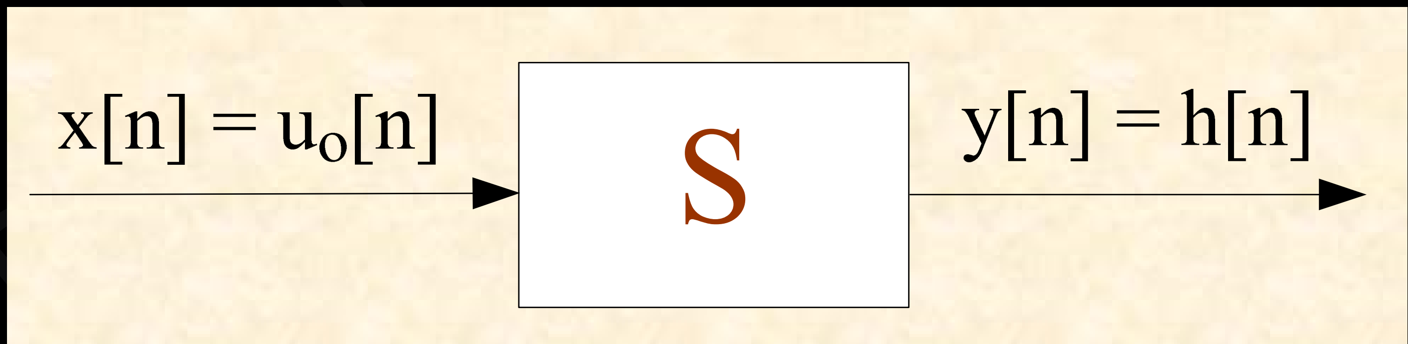
Linear Time Invariant systems (LTI)

Linear time invariant systems (LTI systems)

$h(t)$ “*impulse response*”



$h[n]$ “*impulse response*”



Linear Time invariant systems (LTI)

$$y(t) = h(t) * x(t) = \int_{-\infty}^{+\infty} h(t-\tau) \cdot x(\tau) \cdot d\tau$$

convolution integral

continuous case

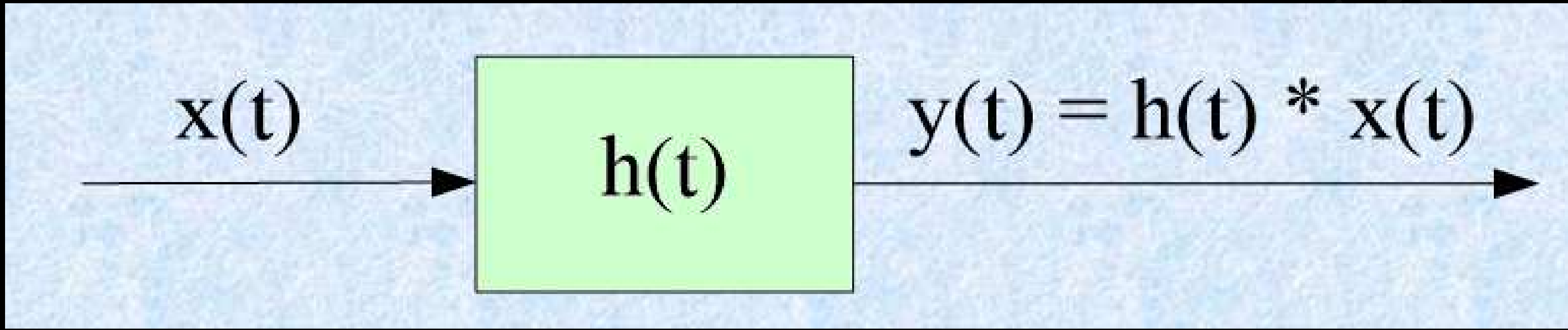
$$y[n] = h[n] * x[n] = \sum_{k=-\infty}^{+\infty} h[n-k] \cdot x[k]$$

convolution sum

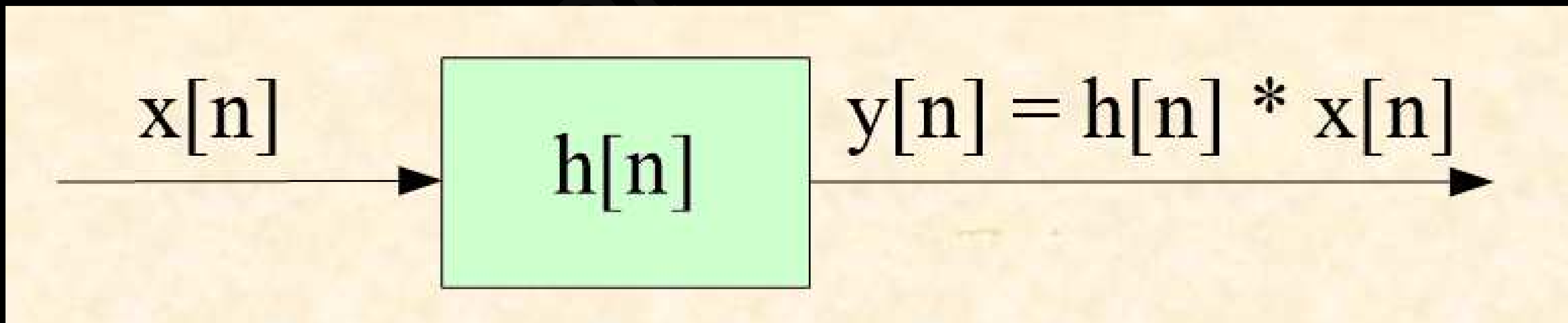
discrete case

Linear Time invariant systems (LTI)

continuous case



discrete case



Linear Time invariant systems (LTI)

Properties of the Convolutions

- Commutative
- Distributive
- Associative

Commutative:

$$h[n] * x[n] = x[n] * h[n] \qquad h(t) * x(t) = x(t) * h(t)$$

Distributive:

$$\left(h_1[n] + h_2[n]\right) * x[n] = h_1[n] * x[n] + h_2[n] * x[n]$$

$$\left(h_1(t) + h_2(t)\right) * x(t) = h_1(t) * x(t) + h_2(t) * x(t)$$

Associative:

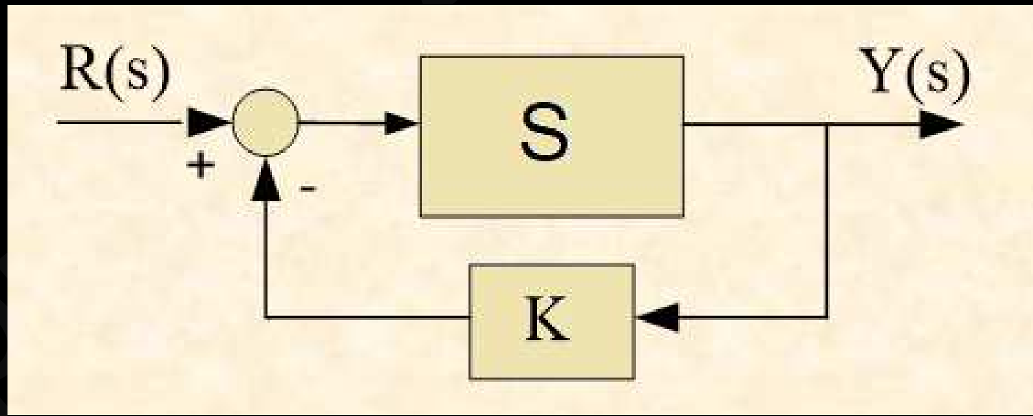
$$\left(h_1[n] * h_2[n]\right) * x[n] = h_1[n] * \left(h_2[n] * x[n]\right)$$

$$\left(h_1(t) * h_2(t)\right) * x(t) = h_1(t) * \left(h_2(t) * x(t)\right)$$

Systems Theory studies many other *topics* as well as other *types of systems* that will only be studied in the Master's course

Some further *topics* that are studied in System theory

- *Modelling*
- *Parameter Identification*
- *Control systems*
- *Optimization*
- *Simulation*
- *Feedback*
- *State estimation*
- *Stability*
- *Robust systems*
- *Fault tolerant systems*
- *Parallel or distributed processing*
- *Fuzzy systems*
- *Intelligent systems*



(*not all of these topics will be seen in the present discipline*)

Here below we mention some subjects that are normally treated in another area of systems, the "Intelligent Systems":

- the recognition of things and objects
- face recognition
- our everyday tasks such as: *walk*, *speak*, *read*, *write*, *climb and go down stairs*, *remember names*, *facts or things*, *drive (a vehicle)*, *identify a traffic sign*, *cooking*, *sewing*, etc. etc.
- tasks such as: *sing*, *dançing*, *play an instrument*, *compose*, *write a text*, *painting*, or *others activities that involve* ***art***.

(*these topics above will not be seen here in this discipline*)

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Thank you!

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