

## Optimal PID Tuning Method Applied to a Sucker Rod Pump System of Petroleum Wells

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### Abstract

Studies have shown that the satisfactory operation of oil wells with sucker rod pumps is due to the techniques and methods able to control the performance of the well. The Polytechnic School of Federal University of Bahia, through the Artificial Lift Lab (LEA), has nowadays a reduced model of a plant of sucker rod pump system with an artificial well of 32m of height fully instrumented, with full access and visible of the downhole well. A kind of resource of the LEA is a laboratorial support to validate existing models and to base experimentally new studies. Among some studies already performed with this sucker rod pump, there is the use of a dynamic model to control the fluid level in the annular well with PID controller (that is directly associated with oil productivity of the pump unit). However, there are in literature, several techniques for tuning PID controllers (e.g.: Ziegler-Nichols, Cohen and Coon, IMC, Integral Criteria, Pole placement). The objective of this paper is to use an optimal tuning method to PID controllers presented in [1] and to apply in the control of the fluid level in the annular well of the pump unit. This optimal tuning method is based on solution of an optimization problem through the minimization of a global objective function, that is composed by local objective functions. Thus, this method may incorporate some model uncertainties, PID control algorithms, process perturbations, manipulated variable restrictions (in this application the manipulated variable is the downhole pump outflow) and overshoot. Finally, the results with a PID controller using this optimal method were compared with other PID tuning methods available in the literature.

**Keywords:** Sucker rod pump; Optimal tuning; Process control; Artificial lift; Petroleum lift.

### 1. Introduction

The rod sucker pump system is the artificial lift method most used in the current on-shore petroleum industry due to the simplicity of its equipments and installations [2]. This method is also considered as the first technique used to lift oil up from wells. Studies show that his popularity is related to low costs of investments and maintenance, deep and outflow flexibility, good energy efficiency and the possibility for operating in different fluid compositions and viscosities in a wide range of temperature [3].

Although this lift method is already well-known and widely used, there are some opportunities of improving the operational conditions, especially, to deal with production control strategies of the pump unit for increasing the system productivity. The development of low cost downhole fiber optic sensors turned possible the measurement of downhole variables that assists the production monitoring and the application of new control strategies [4-5-6]. In this context, The Polytechnic School of Federal University of Bahia, through the Artificial Lift Lab (LEA), has nowadays a reduced plant of a sucker rod pump with an artificial well of 32m of height fully instrumented, with full access and visible. All components of this sucker rod pump system are industrial and the plant also has a supervisory system to data acquisition and control. However, in terms of production control of the pump unit, the presence of some uncertainties in parameters in the dynamic system model may jeopardize the good performance of a conventional controller (e.g.: PID). These uncertainties, in the case of the sucker rod pump, normally are related to or fluid characteristics in the well either associated with the electrical and mechanical assembly.

Among some studies already performed with this sucker rod pump, there is the use of a dynamic model to control the fluid level in the annular well with PID controller (that is directly associated with oil productivity of the pump unit). However, there are in literature, several techniques for tuning PID controllers (e.g.: Ziegler-Nichols, Cohen and Coon, IMC, Integral Criteria, Pole placement) [7-8-9-10-11]. The objective of this paper is to use an optimal tuning method to PID controllers presented in [1] and to apply in the control of the fluid level in the annular well of the pump unit. This optimal tuning method is based on solution of an optimization problem through the minimization of a global objective function, which is composed by local objective functions. In this way, the tuning technique developed here could add in the optimization problem the model uncertainties, PID control algorithms, process perturbations, manipulated variable restrictions (in this application the manipulated variable is the downhole pump outflow) and overshoot.

In section 2 the sucker rod pump system is briefly explained. The necessary mathematics to the development of the optimal tuning method is presented in section 3. The simulation and analysis results are discussed in section 4.

Finally, some remarks about the controller performance are presented in section 5.

**2. The Sucker Rod Pump System**

**2.1. System Description**

In this artificial lift method a rotatory movement of or an electric motor either combustion motor localized on surface of the pump unit is converted in alternative movement of the rods column. This same column transmits the alternative movement to the pump components that are located in downhole well, that are responsible to elevate the fluid from reservoir to the surface. The sucker rod pump system could be divided in downhole and surface elements, (see Fig. 1)

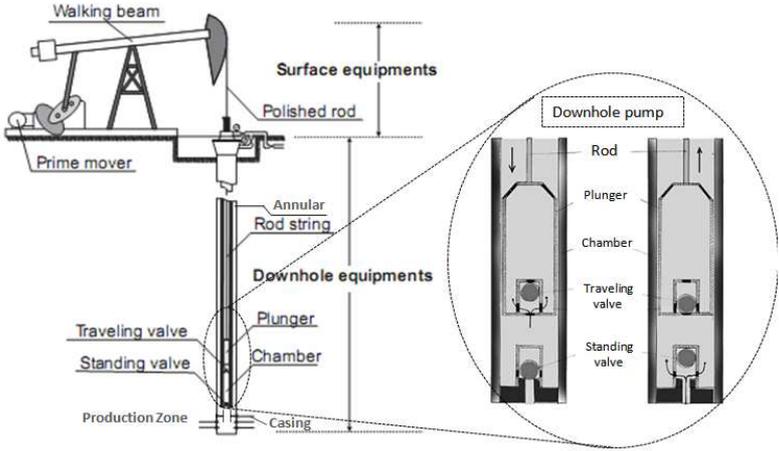


Figure 1: Components of a rod sucker pump system

The rods column is the link between the pump unit localized on the surface and the downhole pump. The downhole pump is a kind of alternative pump of positive displacement and simple effect, in other words, the fluid is displaced in a one way direction of the alternative movement. The function of the downhole pump is to provide energy (increasing the pressure) to the fluid from reservoir [12-13].

**2.2. The Fluid Dynamic Model in the Annular Well**

The production performance of a rod sucker pump system is directly associated with fluid level in annular well [14]. Thus, it is necessary to obtain the fluid-level-dynamic-model in annular well and his relationship with some of variables of entire system. As follow in Fig. 2

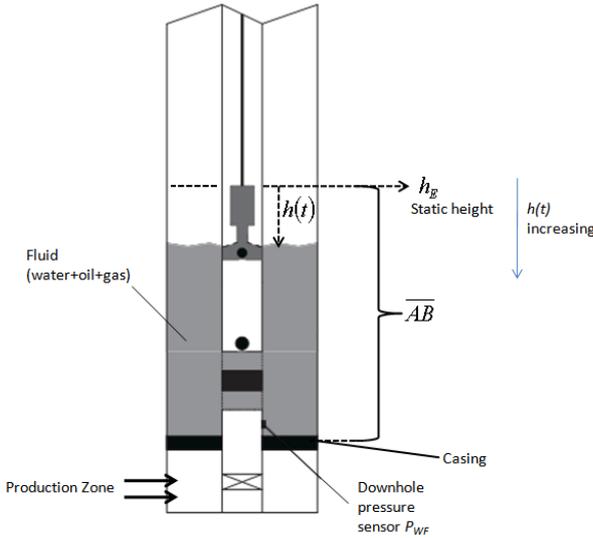


Figure 2: Downhole well scheme with sucker rod pump system

It is possible to obtain the volumetric balance given by an ordinary differential equation of the drainage of the annular well as follow in Eq. (1)

$$Q_{AN}(t) + Q_R(t) = Q_B(t) \quad (1)$$

where  $Q_R(t)$  is the outflow from the reservoir to the annular well,  $Q_{AN}(t)$  is the outflow from the annular well to the production well (where is localized the downhole pump) and  $Q_B(t)$  is the downhole pump outflow. The outflow from the annular well to the production well  $Q_{AN}(t)$  is given by

$$Q_{AN}(t) = A_{AN} \dot{h}(t) \quad (2)$$

where  $\dot{h}(t)$  is the level ratio of the  $h(t)$  in the annular well and  $A_{AN}$  is the transversal section area of the annular. The annular area is calculated as follow:  $A_{AN} = \frac{\pi}{4} \left( (D_{INT}^{CAS})^2 - (D_{EXT}^{PROD})^2 \right)$ , where  $D_{INT}^{CAS}$  is the internal diameter of the casing pipe,  $D_{EXT}^{PROD}$  is the external diameter of the production well. The outflow from the reservoir to the annular well  $Q_R(t)$  is given by

$$Q_R(t) = PI(P_S - P_{WF}(t)) \quad (3)$$

Where  $PI$  is called Productivity Index,  $P_S$  is the static pressure and is the static pressure and  $P_{WF}$  is the well flowing pressure (also called downhole pressure). The static pressure  $P_S$  is given by

$$P_S = P_{CAS}^S + \gamma_F \overline{AB} \quad (4)$$

where  $P_{CAS}^S$  is the casing pressure in statics conditions (the pump system is down and there is no production),  $\gamma_F$  is the specific weight of the fluid (that may be a composition of water, oil and gas),  $\overline{AB}$  is length between the static level  $h_S$  and the casing. In this work the pressure of the gas column on the fluid level in annular well will not take in count. The reference point is the point where  $h_S$  occur. The well flowing pressure  $P_{WF}$  is given by

$$P_{WF}(t) = P_{CAS}^D + \gamma_F [\overline{AB} - h(t)] \quad (5)$$

where  $P_{CAS}^D$  is the casing pressure in dynamics conditions (the pump system is on and there is production) and  $h(t)$  is the level as indicated in Fig. 2 in the time  $t$ . It will be assumed here  $P_{CAS}^S \cong P_{CAS}^D$

$$A_{AN} \dot{h}(t) + IP\gamma_F h(t) = Q_B(t) \quad (6)$$

It could rewrite Eq. (6) as follow in Eq. (7)

$$\dot{h}(t) = -\frac{PI\gamma_F}{A_{AN}} h(t) + \frac{1}{A_{AN}} Q_B(t) \quad (7)$$

It could be observed that the dynamic model in Eq. (7) is a linear relationship given by a first order ordinary differential equation.

### 3. The Tuning Method

#### 3.1. SISO Systems

In this work will be presented the development of the tuning method formulated in [1] only to SISO systems, since that is the case of the sucker rod pump system. The method was formulated by using as a start point a typical feedback control loop. Consider the diagram indicated in Fig. 3

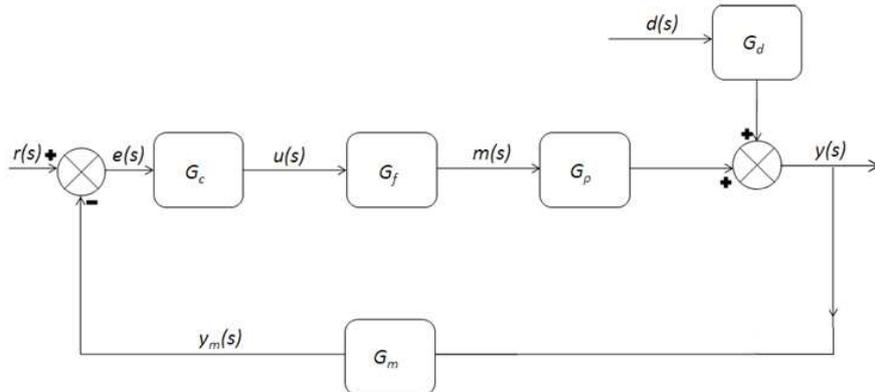


Figure 3: Feedback control loop diagram

where  $G_c$  and  $G_p$  are, respectively, the transfer function of the controller and the transfer function of the process. In the same way  $G_f$  and  $G_m$  are, respectively, the transfer function of the final controller element and the transfer function of the measurement element. The block  $G_d$  refers to the transfer function associated to the disturbance. From a generic point of view, a tuning method has the objective to determine the parameters values of the controller that optimize a determined criterion. Thus, this criterion (denoted here by  $J$ ) is a function of the parameters ( $P$ ) and it could be expressed, for example, by

$$\min_P J(P) \quad (8)$$

In terms of tuning methods of process controllers, there are criteria to the optimization, such as IAE, ISE, ITAE and ITSE. In this case, it is possible to use a generic criterion that is comprised by the linear combination of these well-known criteria [15-16]

$$\min_P J(P) = \min_P (\alpha_1.IAE(P) + \alpha_2.ISE(P) + \alpha_3.ITAE(P) + \alpha_4.ITSE(P)) \quad (9)$$

where  $\alpha_1$ ,  $\alpha_2$ ,  $\alpha_3$ , and  $\alpha_4$  are weights for each integral criteria.

There are some processes that the overshoot ( $OS$ ) is not desired. It could be minimized by adding an extra term in the function  $J$

$$\min_P (\alpha_1.IAE(P) + \alpha_2.ISE(P) + \alpha_3.ITAE(P) + \alpha_4.ITSE(P) + \alpha_5.OS(P)) \quad (10)$$

In most of processes, it could be desired that the manipulated variable ( $u$ ) behave smoothly. In other words, it is desired that the movements of the manipulated variable are restricted. In this case, it could be achieved from the minimization of these movements

$$\min_P (\alpha_1.IAE(P) + \alpha_2.ISE(P) + \alpha_3.ITAE(P) + \alpha_4.ITSE(P) + \alpha_5.OS(P) + \alpha_6.\Delta u(P)) \quad (11)$$

Equation (11) is a function not only of the controller parameters. The processes present different responses according to the changes in set points and load values and, moreover, different results are obtained from different types of input signals (pulse, step, ramp, etc.). In this case, it could be defined each combination of signal input and the type of problem (servo or regulator) as a control problem (CP), it is possible to write

$$\min_P (\alpha_1.IAE(P, CP) + \alpha_2.ISE(P, CP) + \alpha_3.ITAE(P, CP) + \alpha_4.ITSE(P, CP) + \alpha_5.OS(P, CP) + \alpha_6.\Delta u(P, CP)) \quad (12)$$

In the same way the objective function could be defined as a combination of criteria, this concept could be generalized to the combination of control problems. In this case

$$\min_P \sum_{j=1}^N (\alpha_{1,j}.IAE(P, CP_j) + \alpha_{2,j}.ISE(P, CP_j) + \alpha_{3,j}.ITAE(P, CP_j) + \alpha_{4,j}.ITSE(P, CP_j) + \alpha_{5,j}.OS(P, CP_j) + \alpha_{6,j}.\Delta u(P, CP_j)) \quad (13)$$

In order to reduce the notation, it could be defined a generic criterion  $C$ , composed by several control problems, linearly weighted by using weights

$$\min_P \sum_{j=1}^N \gamma_j \sum_{i=1}^L \alpha_{i,j}.C_i(P, CP_j) \quad (14)$$

where  $C_i$  are the criteria of evaluation adopted (integral criteria, overshoot, manipulated variable),  $N$  is the number of control problems,  $L$  is the number of criteria used. The constants  $\gamma_j$  are weights for each control problem. The presence of the weights  $\gamma_j$  is to facilitate the tuning design. Because it could be attributed clearly a weight for each control problem, and still turning possible to use the weights  $\alpha_{i,j}$  in normalized form. In others words, for each control problem one has

$$\sum_{i=1}^L \alpha_{i,j} = 1 \quad (15)$$

It could be observed that the objective function in Eq.(14) may incorporate the model of the process ( $M$ ).

$$\min_P \sum_{j=1}^N \gamma_j \sum_{i=1}^L \alpha_{i,j}.C_i(P, CP_j, M_j) \quad (16)$$

An important question in tuning controllers is the guarantee that the parameters obtained will provide a stable system output. Other question is the presence of model uncertainties. Furthermore, these uncertainties in the phenomenological model can commit better result. It could be observed that the solution of the function in Eq.(16)

will guarantee a stable response only if the model uncertainties are within the range of the chosen models of the process. In case of instability, the value of the objective function will tend to infinite and, therefore, it will not be a solution.

### 3.2. Numerical Aspects

The tuning problem, as it was presented in [1], is characterized as a non-linear programming problem with restrictions. In order to the solution of this problem was used a SQP (Sequential Quadratic Programming) algorithm. To the solution of the differential equations to the simulation of the transfer functions in Laplace domain was used the fourth order Runge-Kutta. The initialization of the tuning method depends on the initial estimates of the controller parameters. These parameters are automatically normalized by the program of the tuning method to facilitate the convergence of the method. One must provide the maximum and minimum values for these controller parameters, or to use the default values of the program. One could face some numerical problems of instability by the use of non-normalized values. However, with the normalized parameters one could obtain good results even with not so refined initial estimates. The default values of the program are as follow

- Weights  $\alpha_{i,j}$ :  $\alpha_{1,j} = \alpha_{2,j} = \alpha_{3,j} = \alpha_{4,j} = 1$  e  $\alpha_{5,j} = \alpha_{6,j} = 0$ ;
- Initial Estimates  $K_c = 0$ ,  $\tau_i = 1s$ , e  $\tau_D = 1s$ ;
- Minimum parameters  $K_c = 0$ ,  $\tau_i = 0.01s$ , e  $\tau_D = 0.01s$ ;
- Maximum parameters  $K_c = 1000$ ,  $\tau_i = 9999s$ , e  $\tau_D = 9999s$ ;

As will be presented in the next section, the tuning method described here shows good results to the sucker rod pump system even if the default values of the tuning program are used.

### 4. Simulations and Analysis Results

The Matlab Simulink was used to simulate the tuning method presented for the level control of the fluid in the annular well of the sucker rod pump in the present work. The transfer function of the nominal model of the plant is

$$M(s) = b_m \frac{1}{s + a_m} \Rightarrow b_m = 62,1; a_m = 3,1 \quad (17)$$

The data used here were obtained from real tests with the sucker rod pump system (physical system) in the LEA. The values to the model in Eq.(7) are:  $IP = 4,96527 \times 10^{-9} m^3 s^{-1} Pa$ ,  $A_{AN} = 0,0161 m^2$ , and  $\gamma_F = 9800 Nm^{-3}$ . By considering an unfavorable situation for the process, from the standpoint of stability, one uses 10% (plus) of uncertainties in the parameters of the model

$$M_e(s) = b_e \frac{1}{s + a_e} \Rightarrow b_e = 68,31; a_e = 3,41 \quad (18)$$

Thus, it could be used two control problems. In other words  $N = 2$  in Eq.(16). One of the problems will use the nominal model of the plant in Eq.(17), and the other one will use the model with uncertainties of 10 % in Eq.(18). Since there is no restriction in the manipulated variable and in the overshoot (here the manipulated variable is the downhole pump outflow) the weights associated with these two terms are set as  $\alpha_{5,1} = \alpha_{5,2} = \alpha_{6,1} = \alpha_{6,2} = 0$ . The other weights are set as  $\alpha_{1,1} = \alpha_{1,2} = \alpha_{2,1} = \alpha_{2,2} = \alpha_{3,1} = \alpha_{3,2} = \alpha_{4,1} = \alpha_{4,2} = 1$ . The weights  $\gamma_i$  could be chosen to give more importance to the nominal transfer function in Eq.(17) and less to transfer function in Eq.(18) with uncertainties in the parameters. The weights were chosen as  $\gamma_1 = 0.8$  and  $\gamma_2 = 0.2$ . The PID controller equation was used as follow

$$u(s) = Kc \left[ (r - y) + \frac{(r - y)}{\tau_I s} + \frac{\tau_D s(-y)}{\tau_D s + 1} \right] \quad (19)$$

By following the same methodology used in [17] to evaluate the performance of the tuning techniques of PID controllers available in the literature, was used the Ziegler-Nichols method (Z-N) as reference, and the objective function ( $F_O$ ) below as a measurement of this performance

$$F_O = \frac{(F_{IAE} + F_{ISE} + F_{ITAE} + F_{ITSE})}{4} \quad (20)$$

where

$$F_{IAE} = \frac{IAE}{IAE_{Z-N}} \quad (21)$$

$$F_{ISE} = \frac{ISE}{ISE_{Z-N}} \quad (22)$$

$$F_{ITAE} = \frac{ITAE}{ITAE_{Z-N}} \quad (23)$$

$$F_{ITSE} = \frac{ITSE}{ITSE_{Z-N}} \quad (24)$$

This objective function was chosen to allow the comparison between the methods used in this paper. It is clear that the objective function is only a criterion to evaluate the tuning problems and it could assume other configuration. Figure 3 shows the comparison between the four methods tested: IAE, IMC, Z-N, and the method proposed by [1] (here will be used only the word Method for it). The simulations were performed regarding the input  $r(s)$  as a unit step signal with amplitude equal to 1.

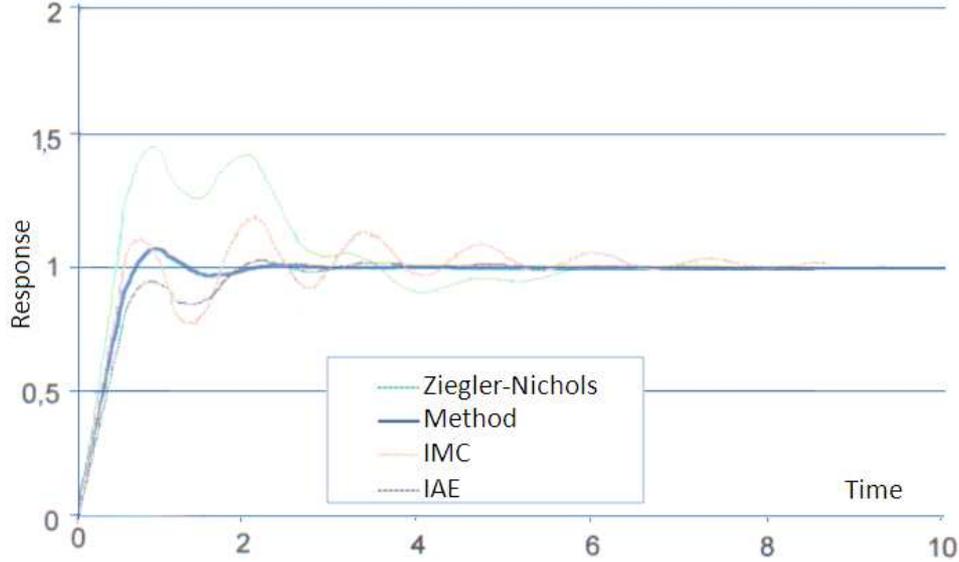


Figure 3: Comparison between the methods tested.

Table 1 shows the controller parameters for each tuning method used in the test and the correspondent value of the objective function  $F_O$ .

Table 1: tuning parameters of the PID controller and the value of the objective function  $F_O$ .

<i>Method used</i>	Z-N	Method	IMC	IAE
$K_c$	9.677	8.513	9.933	7.648
$\tau_I$	0.970	6.847	5.060	6.612
$\tau_D$	0.243	0.010	0.247	0.219
$F_O$	1.0	0.545	0.763	0.642

According to Eq.(7) the reference signal to be tracked is the downhole pump outflow (that is the manipulated variable). It could be observed in Fig.(3) that the reference was tracked by all tuning methods. The process variable that is the fluid level in annular well could be observed in Fig.(3) with oscillations in transitory, but stable in steady state for all methods tested.

It could be seen from the values of the objective function  $F_O$  in table 1 that the method proposed in [1] presents the best performance in tuning the controller parameters for the PID controller of the sucker rod pump system. It also shows the fast response with little overshoot among the tuning methods tested.

## 5. Conclusions

In this paper the optimal PID tuning method was applied in order to control the fluid level in annular well of a sucker rod system. It could be observed through the simulations and analysis results that the reference was tracked by the tuning method presented in this paper with few oscillations (little overshoot) in transitory, but stable in steady. For future work this adaptive controller should be implemented in the real physical system.

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