IMPLEMENTATION OF TECHNIQUES FOR THE CONTROL OF A SYSTEM OF MAGNETIC LEVITATION

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Abstract— This paper discusses the implementation of the techniques for exact linearization by state feedback and pole placement by means of an algorithm that controls a magnetic levitation system, represented by a didactic kit. The algorithm was developed by using the programming language software Executive, employed here to communicate the computer with the kit. Initially, the exact linearization technique is applied with the aim of obtaining a linear system from the nonlinear state feedback formulation of the system. The linearization is made through of direct cancellation of nonlinear functions, which represent the phenomenological model of the system. With the linear system it becomes possible to apply classical techniques of the linear control such as the pole allocation technique, which is applied here. The obtained results from Matlab simulations and also from practical implementations shows that the control system of magnetic levitation is achievable.

Keywords— Non Linear Control; Exact Linearization; State Feedback; Magnetic Levitation.

1. INTRODUCTION

This article addresses the implementation of control techniques to a nonlinear Magnetic Levitation System (MLS) by using algorithms of control. The MLS was chosen because it has nonlinear dynamics, an area of great interest in studies, and also because of the physical system (a teaching kit) to be available for practical tests, allowing not only the experimental validation but also favouring the continuity work.

The MLS used is manufactured by ECP - Educational Control Product (www.ecpsystems.com) and will be described in more detail in section 2. The aim is to implement a control algorithm in the programming language of the kit, using the software Executive, to control the displacement of a hard magnetic over a glass guide. The movement is caused by the magnetic force produced by the magnetic field generated by applying an electric current in a coil.

The relationship of the electric current applied to the coil and the displacement of the magnetic disk is given by a nonlinear second order differential equation. Therefore, to control the position of the disk may be resorted to the use of nonlinear control techniques, such as fuzzy controllers, neural, neurofuzzy, exact linearization, among others. In the present work the exact linearization technique is used.

A linearization technique allows the accurate transformation of a nonlinear system into a linear one through the incorporation of compensating nonlinear in the meshes of the control system. The incorporation is done through feedback from the states of the nonlinear system. Thus, there is a need of accessing all the states. In the case of MLS studied states are able to be measured.

In order to apply state feedback, the system must have the dynamics written as $\dot{x} = f(x) + g(x)u$ where the functions *f(x)* and *g(x)* represent the nonlinearities of the state, *u* the control input, and *x* is the vector of state. Regarding the nonlinear functions, it is important that the nonlinearities of the physical system is well represented by the mathematical model, otherwise the control system will not act effectively on the basis of having been designed considering the cancellation of system nonlinearities. The mathematical model obtained for the MLS of the ECP was compared with actual data input and output of the plant and showed satisfactory results.

The exact linearization technique proposes to implement a control law u such that the system has an input/output linear relationship. There are several well known techniques for effective control of linear system, such as the pole placement technique (Ogata, 2006), which has been used here. In this case, the feedback gains are determined by allocating the poles of the transfer function of closed loop system in desired positions.

2. THE MODEL

2.1 Magnetic Levitation System

In the present work the MLS made by ECP was used and is shown in Fig. 1. It comprises two magnetic discs, a glass column, two laser sensors and two coils. The sensors are used to obtain the system response associated with the disc positions. The system input is given by the application of an electrical current to the coils. The physical system communicates with the computer via Digital Signal Processing (DSP) and a black box is responsible for the electrical current drivers and the energy supply.

This MLS can be classified according to two modes, SISO (Single Input Single Output) or MIMO (Multiple Input Multiple Output) and this depends on the desired system configuration. In the SISO mode only one disc is used whereas in the MIMO mode two discs are used. Here the MLS was configured to operate in the SISO mode.

Fig. 1. Magnetic Levitation System made by ECP.

The MLS manual (Parks, 1999) shows the mathematic model, based on the physical laws, which allows us to obtain its differential equation model. The development of the mathematic model is beyond the scope of this paper. Through the balance of forces, the equation is given by (Laithwaite, 1965).

$$\ddot{y} + \frac{c}{m}\dot{y} = \frac{F_m}{m} - g \qquad (1)$$

where:

y - magnetic disc position;

$\dot{y}$ - first derivative of the magnetic disc position;

$\ddot{y}$ - second derivative of the magnetic disc position;

c - air viscosity coefficient;

m - magnetic disc mass;

F_m - magnetic force applied to the magnetic disc.

The magnetic force can be written as (Parks, 1999)

$$F_m = \frac{i}{a(y+b)^4} \qquad (2)$$

where,

i - electrical current applied on the coil;

a and b - are constants related with the coil properties.

By substituting (2) in (1) gives a nonlinear relationship between the magnetic disc position and electrical current applied to the coil

$$\ddot{y} = -g - \frac{c}{m}\dot{y} + \frac{1}{ma(y-b)^4}i\,. \qquad (3)$$

2.2 Parameter Estimation

There are five parameters in (3): g, c, m, a and b. The first three are considered as constant and known (Parks, 1999) having the following values: g = 9.81 [m/s2], m = 0.12 [Kg] and c = 0.15 [Ns/m]. As for the parameters a and b are constants related with magnetic coil properties and must be estimated. In Silva (2009), the least square and Monte Carlo methods were used to estimate a and b. Accordingly with Silva (2009) and based on a cost function, the Monte Carlo method presented the best values for these parameters as being a = 0.95 and b = 6.28, which have been adopted through this paper.

3. THE FEEDBACK LINEARIZATION TECHNIQUE

Feedback linearization can be applied to a certain class of nonlinear systems, including the MLS studied, and enables to transform the original system models into equivalent models of a simpler form. The control scheme uses the exact linearization with state feedback based on the cancellation of nonlinear functions. However, to enable the application of the technique, the system dynamic must be represented by (Guadarbassi and Savaresi, 2001)

$$\dot{x} = f(x) + g(x)u \qquad (4)$$

where the functions *f(x)* and *g(x)* represent the nonlinearities of the states, u is the control system input and x is the state vector. Furthermore, three conditions must be satisfied.
The first condition is that the system is controllable, so that the matrix formed by vector fields *f(x)* and *g(x)* must have rank n, where n is the order of the system. The second condition is that the system is involutive. The third condition requires $g(x) \neq 0, \forall\ x$.

Once the conditions are satisfied it is possible to determine a diffeomorphism $Z = T(X)$. After this, the dynamic of the system given by (4) can be transformed into the form (Isidori, 1995)

$$\dot{Z} = AZ + B\beta^{-1}(Z)[u - \alpha(Z)] \qquad (5)$$

where Z is the new vector of states, A and B are constant matrices obtained from the model of the MLS, u the control input linearizante e $\alpha(Z)$ and $\beta(Z)$ are functions that represent the feedback of the states. A feedback signal for the nonlinear system is chosen in the form

$$u = \alpha(Z) + \beta(Z)v \tag{6}$$

Thus, the linear system can be written in the form

$$\dot{Z} = AZ + Bv \tag{7}$$

where v is the control signal (control law) for the system after linearization. The determination of v will be discussed in next section.

3.1 Linearization of the MLS Made by ECP

The model of the MLS made by ECP was presented in (3) and the two conditions for application of the exact linearization were presented in the last subsection. The variables of states and the feedback signal u can be set as follows:

$$u = i \qquad x_1 = y \qquad x_2 = \dot{y}\,. \tag{8}$$

The dynamic of the system given by (3) can be rewritten in the form given in (4)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ -g - \dfrac{c}{m}x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{1}{ma(x_1+b)^4} \end{bmatrix} u. \tag{9}$$

The functions *F(X)* and *G(X)* that contain the nonlinearities of the system can be set as follow

$$F(X) = \begin{bmatrix} x_2 \\ -g - \dfrac{c}{m}x_2 \end{bmatrix} \tag{10}$$

$$G(X) = \begin{bmatrix} 0 \\ \dfrac{1}{ma(x_1+b)^4} \end{bmatrix}. \tag{11}$$

The transformation *T(X)* can be set as (Khalil, 2002)

$$Z = \begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix} = T(X) = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}. \tag{12}$$

The functions $\alpha(Z)$ and $\beta(Z)$ can be calculated in the form given by:

$$\alpha(Z) = (mga + caZ_2)(Z_1 + b)^4\,. \tag{13}$$

$$\beta(Z) = ma(Z_1 + b)^4\,. \tag{14}$$

Finally, the feedback signal u can be rewritten by:

$$u = (mga + caZ_2)(Z_1 + b)^4 + ma(Z_1 + b)^4 u\,. \tag{15}$$

The application of the feedback signal u over the system given by (9) will cancel the nonlinearities and the system will be transformed into a linear system given by (7). In the same way but using (12) (Slotine, 1991)

$$\dot{Z} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ v \end{bmatrix}. \tag{16}$$

With the linear system one can design the linear controller. Here pole placement technique is used, an this will be described in the next section.

4. CONTROL USING POLE PLACEMENT

The pole assignment technique is actually the allocation of the poles of the closed-loop system at any desired position by means of a state feedback. Thus, one can combine the technique of state feedback linearization with this pole placement technique. The requirement is that the system is completely state controllable, that is the case of MLS studied.

Whereas the MLS can be written in the form of Equation (16), as shown above, one can propose a control law as:

$$v = -kZ \tag{17}$$

where $k = [k_1\ k_2]$ is called the feedback gain matrix and Z represents the state vector.

To determine k, three methods are widely circulated: Ackermann formula, the transformation matrix and the direct substitution (Franklin, et. Al., 1995). Since the MLS has order $n \leq 3$ the determination of using the method of replacing direct substitution becomes simpler.
In this method it is essential that the characteristic polynomial given by (18) to be known

$$p(s) = |sI - A + Bk| \tag{18}$$

where A and B are constant matrices and can be obtained by combining equations (7) and (16):

$$\dot{Z} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v \tag{19}$$

and therefore,

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Thus, the polynomial is given by:

$$p(s) = \begin{vmatrix} s & -1 \\ k_1 & s+k_2 \end{vmatrix} = s^2 + k_2 s + k_1 \tag{20}$$

Allocating the poles of the characteristic polynomial in the desired positions the values of k_1 and k_2 can determined. Thus, setting $s_1 = -1$ and $s_2 = -2$, we have that:

$$s^2 + k_2 s + k_1 = (s - s_1)(s - s_2) \tag{21}$$

or,

$$s^2 + k_2 s + k_1 = s^2 + 3s + 2 \tag{22}$$

and comparing both sides of (22), we obtain:

$$k_1 = 2 \text{ and } k_2 = 3$$

And therefore, the control action by pole placement for the MLS with feedback from the states will be:

$$v = -kZ = [-2 \quad -3]\begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix} = -2Z_1 - 3Z_2 \tag{23}$$

For the case of tracking a signal of reference:

$$v = r - kZ = r - [2 \quad 3]\begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix} = r - 2Z_1 - 3Z_2 \tag{24}$$

5. SIMULATIONS AND ANALISYS RESULTS

In order to apply exact linearization techniques for state feedback and pole placement in the real system's magnetic levitator ECP an algorithm has been developed in the programming language software Executive. The flowchart of operation for this algorithm can be seen in Figure 2. The model parameters used in the implementation were presented in subsection 2.1. The input signal applied to the magnetic levitation system was a step from 0 to 4 centimeters, so the disc should stabilize in the position of 4 cm.

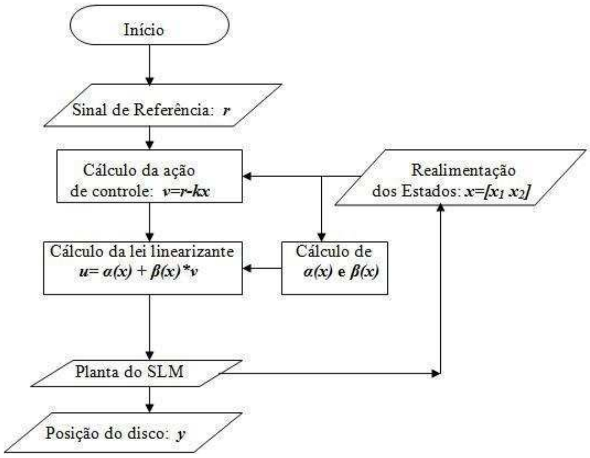

Figure 2. Flowchart of operation of the control algorithm.

Initially, the project control system by exact linearization and pole placement was simulated in Matlab/Simulink. The block diagram drawn can be seen in Figure 3.

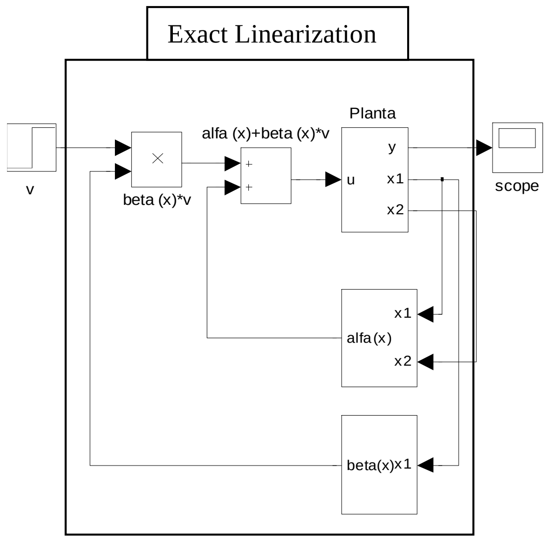

Figure 3. Block diagram in Matlab / Simulink for the system of exact linearization.

In the simulation, it was observed that the system showed an error in the scheme and eliminate it, it was necessary to obtain the transfer function of closed loop system to analyze the behaviour of the SLM. From the representation of the system by equations of state obtained previously, using Matlab, one can obtain the transfer function of closed loop system (Matsumura, 1974):

$$H(s) = \frac{8{,}882e^{-0{,}16}s + 1}{s^2 + 3s + 2} \tag{25}$$

By Final Value Theorem, when $t \to \infty$, $s \to 0$. Thus, by observing (25) one can see that the gain regime is given by:

$$H(0) = \frac{8{,}882 * e^{-0{,}16} * 0 + 1}{0^2 + 3 * 0 + 2} = \frac{1}{2} \qquad (262)$$

And to eliminate the error in the scheme, was inserted with a constant value opposite to that found in (26), k_3=2, at the input of the system by multiplying the reference signal *r*, as can be seen in the output of the system shown in Figure 4.

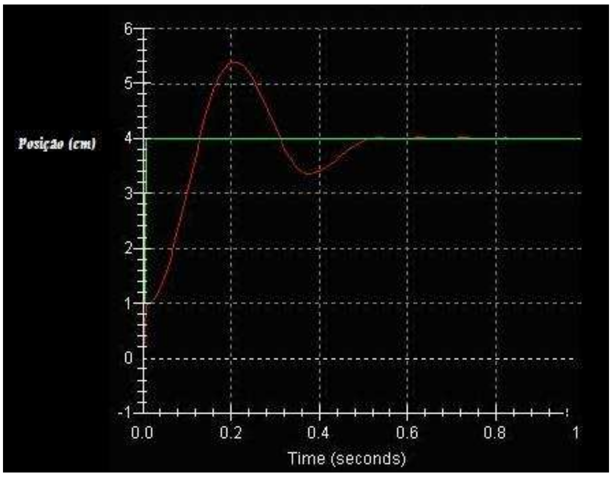

Figure 4. Comparison between the reference signal and the disc displacement without error in the scheme.

It is observed that the driver had a satisfactory answer since it was expected that the control signal would cause the system output to follow the reference signal input and this could be seen in the chart.

The current applied to the coil represents the control effort to track the position of the magnetic disk. Although the software does not provide the Executive Chart of the behaviour of the chain, we can analyze the voltage applied to the coil, as shown in Figure 5.

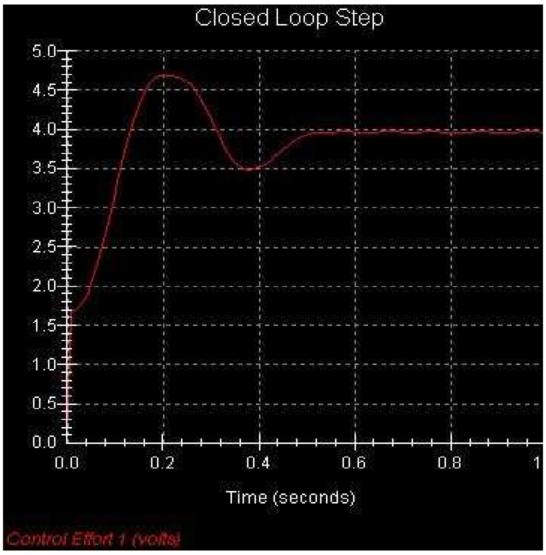

Figure 5. Behavior of the voltage applied to the coil.

Looking at the graph of Figure 5 one can see that the control effort is well associated with the behaviour of the position of the magnetic disk shown in Figure 4. In transitory control the effort varies until the system reached the steady state in about 0.6 seconds, and stabilizes when the disk reaches the desired height of 4 cm.

6. CONCLUSIONS

In the present work the combination of two techniques to control a magnetic levitation system, namely, exact linearization by state feedback and pole placement. It was found through the analysis of experimental results that the controller had a level of overshoot tolerate a satisfactory answer because, after the elimination of error in the scheme, the output signal tracked the input signal, which was expected. It was also observed that the effort to control showed a significant response when associated with the system.

For future work it will be analyzed the system output when applied to different reference signals and how the system had acted in the presence of disturbances. And yet, to implement adaptive systems techniques for correcting errors of nonlinear functions of the model.

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