Feedback Linearization and Model Reference Adaptive Control of a Magnetic Levitation System

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Topics

- Introduction
- Magnetic Levitation System Model
- Exact Linearization with State Feedback
- Model Reference – Direct Approach
- MRAC Applied to the MSL after the Linearization
- Adaptive Control Scheme
- Simulation and Results
- Conclusions

Introduction

- Magnetic Levitation System;
- Non-linear system in the form:

$$\dot{X} = F(X) + G(X)u$$

- Control Technique – Exact Linearization with State Feedback;
- Representation of the dynamics of real plant and the uncertainties present in the phenomenological model;
- Controller using Model Reference Adaptive Control - Direct Approach (MRAC)
- Convergence of estimates to their correct values and stability of the system.

Magnetic Levitation System made by ECP

The Model

- Problem statement;
- Magnetic Levitation System – balance of forces:

$$\ddot{y} + \frac{c}{m}\dot{y} = \frac{F_m}{m} - g \qquad (1)$$

- The magnetic force F_m could be written in the form:

$$F_m = \frac{i}{a(y+b)^4} \qquad (2)$$

- By substituting (2) in (1):

$$\ddot{y} = -g - \frac{c}{m}\dot{y} + \frac{1}{ma(y-b)^4}i \qquad (3)$$

Non-linear system!

Magnetic Levitation System

Where:

y - magnetic disc position;
$\dot{y}$ - first derivative magnetic disc position;
$\ddot{y}$ - second derivative magnetic disc position;
c - air viscosity coefficient;
m - magnetic disc mass;
i - electrical current applied on the coil;
g - is the acceleration of gravity;
a and b - are constants related with the coil properties.

The Model

- There are five system parameters: *g, c, m, a e b*;
- In this work: *g* = 9,81 [*m/s2*], *m* = 0,12 [*Kg*] e *c* = 0,15 [*Ns/m*] - are provided by the manual (Parks, 1999);
- The parameters a and b are constants related with magnetic coil properties ;
- In this work were used a = 0,95 e b = 6,28. These values were estimated in previous works (Silva, 2009).

Magnetic Levitation System

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Exact Linearization with State Feedback

- The system dynamic must be represented by:

$$\frac{dX}{dt} = F(X) + G(X)u \qquad (4)$$

- $F(X)$ and $G(X)$ represent the nonlinearities of the states, u is the control system input and X is the state vector.
- Two conditions must be satisfied:

1) The first one is that the system must be controllable. For this first condition the matrix formed by vectorial fields in (5) must contain order n, where n is the system order

$$[ad_F^0 G,\ ad_F^1 G, \dots, ad_F^{n-1} G] \qquad (5)$$

2) The second one is that the system be involutive. It means that the distribution expressed in (6) also be involutive.

$$D = span\left\{ad_F^0 G\ ad_F^1 G \dots ad_F^{n-1} G\right\} \qquad (6)$$

Exact Linearization with State Feedback

- Once the conditions are satisfied it is possible to determine a diffeomorphism $Z = T(X)$:

$$\dot{Z} = EZ + F\beta^{-1}(Z)[u - \alpha(Z)] \qquad (7)$$

- A feedback control signal u_f for the nonlinear system is chosen in the form :

$$u_f = \alpha(Z) + \beta(Z)u \qquad (8)$$

  where $\alpha(Z)$ and $\beta(Z)$ represent the states feedbacks
- Thus, the linear system can be written in the form in:

$$\dot{Z} = EZ + Fv \qquad (9)$$

  where v is the input signal for the system after linearization;
- The dynamic of the system given by (3) can be rewritten in :

$$u_f = i \qquad x_1 = y \qquad x_2 = \dot{y}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ -g - \frac{c}{m} x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{ma(x_1 + b)^4} \end{bmatrix} u_f \qquad (10)$$

Exact Linearization with State Feedback

- The transformation $Z=T(X)$ can be set in the form:

$$Z = \begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix} = T(X) = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{11}$$

- The functions $\alpha(Z)$ and $\beta(Z)$, can be calculated in the form given by:

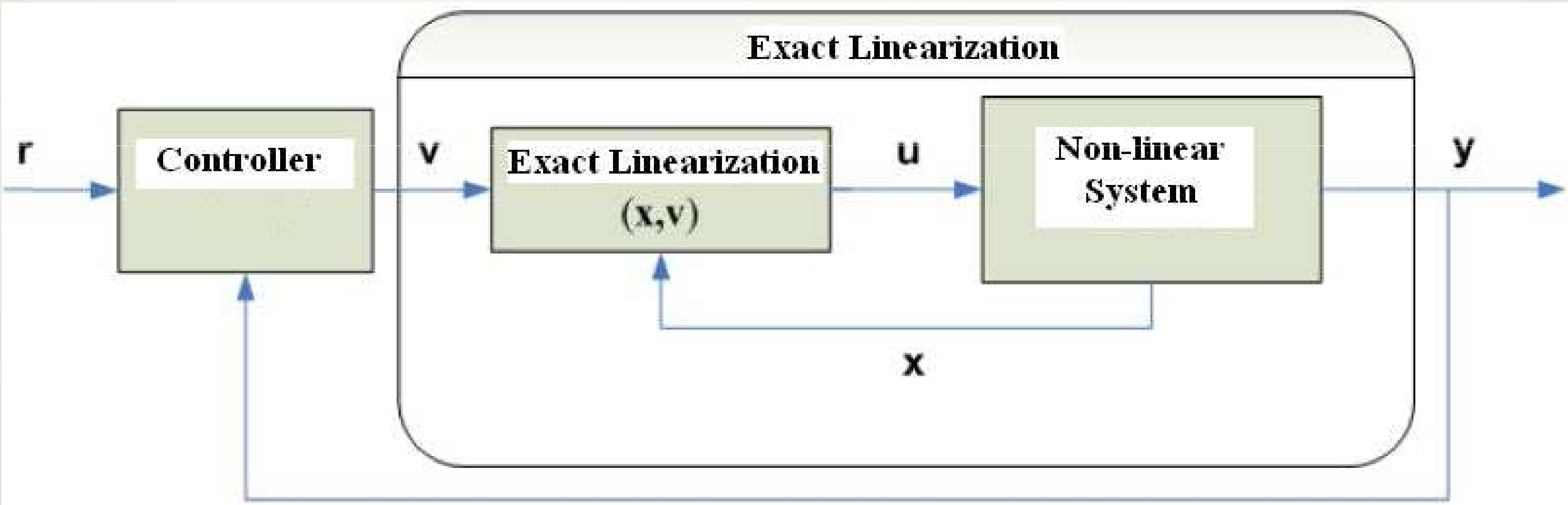
$$\alpha(Z) = (mga + caZ_2)(Z_1 + b)^4 \qquad \beta(Z) = ma(Z_1 + b)^4 \tag{12}$$

- Finally, the feedback control signal u could be rewritten

$$u_f = (mga + caZ_2)(Z_1 + b)^4 + ma(Z_1 + b)^4 v \tag{13}$$

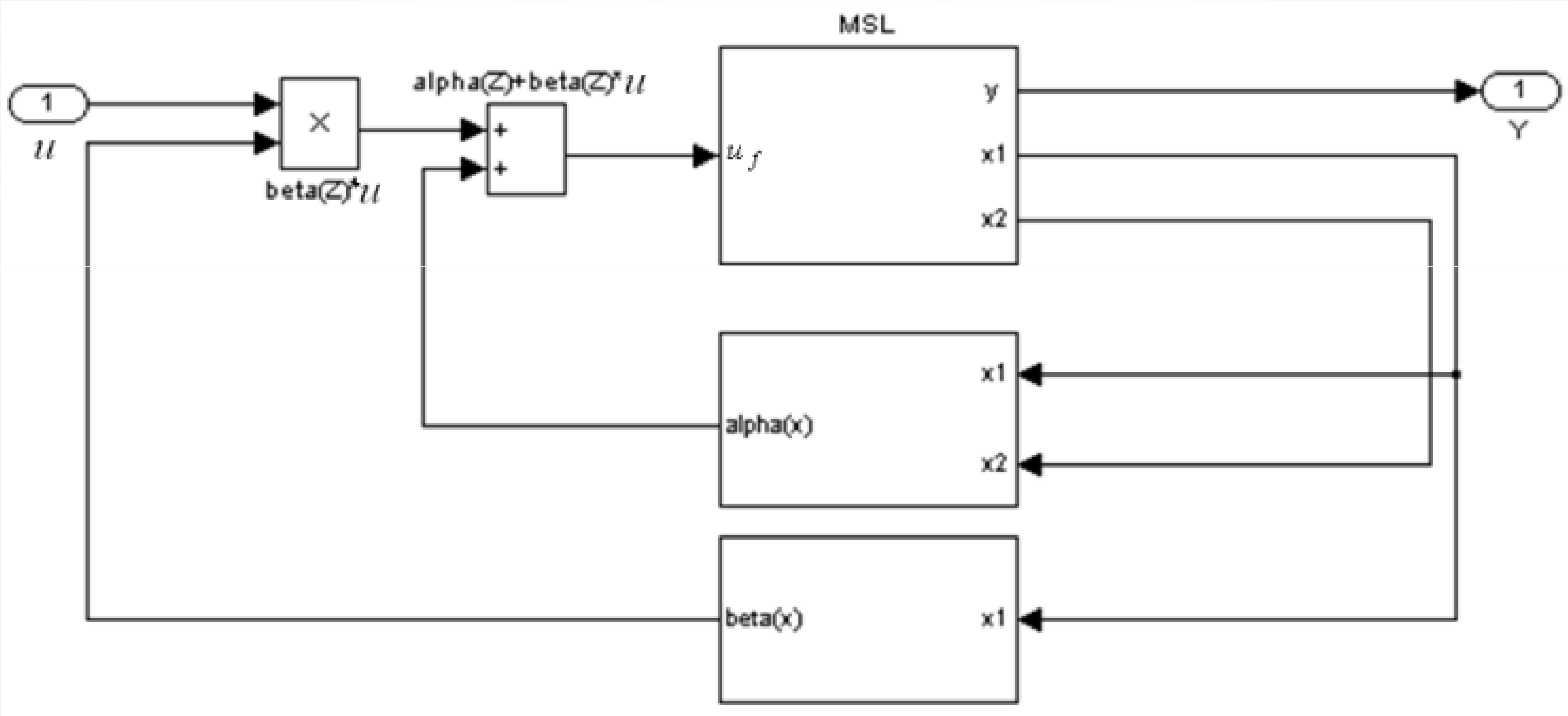
Exact Linearization with State Feedback

Block diagram using the exact linearization technique



Exact Linearization with State Feedback

Block diagram implemented in Matlab/Simulink for the exact linearization over the MLS

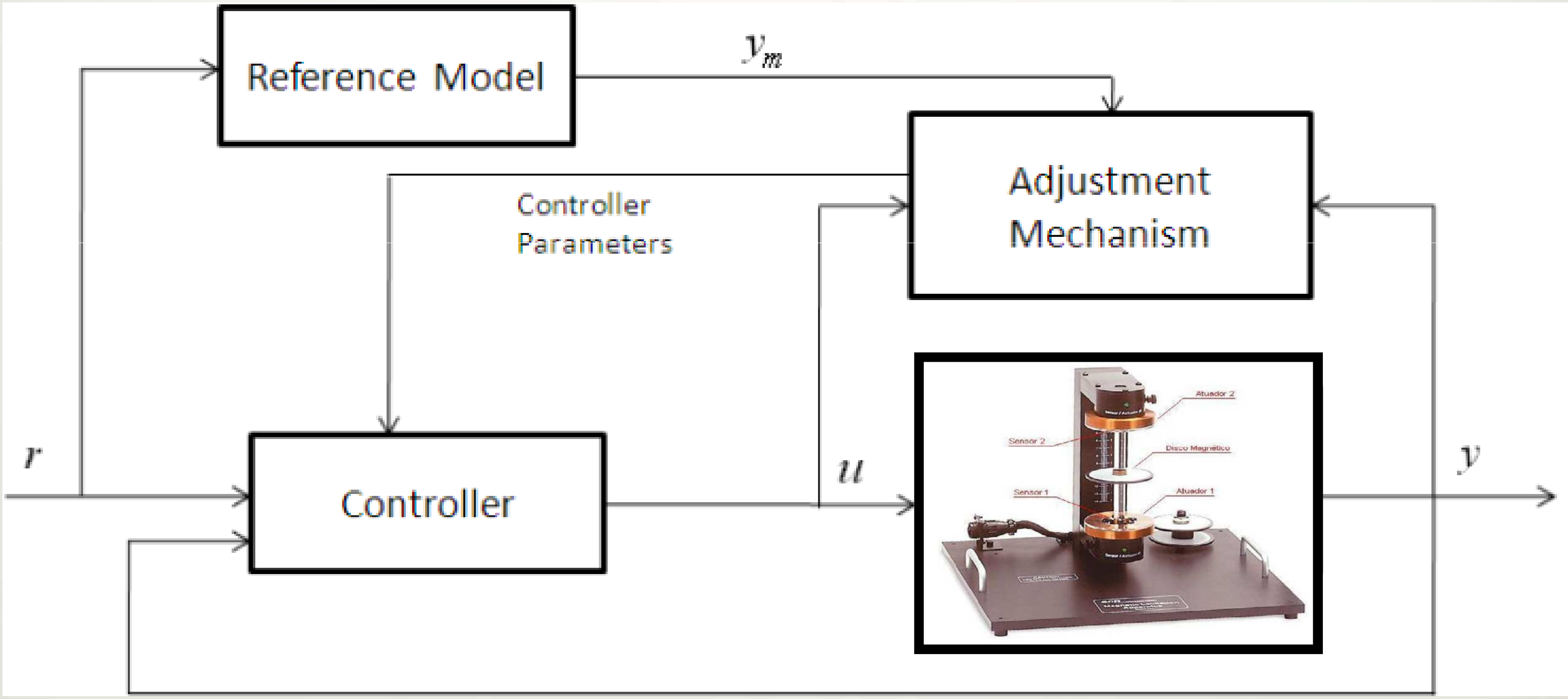


Model Reference – Direct Approach

- Model Reference Adaptive Control (MRAC) is one of main techniques in adaptive control
- The changes in the controller parameters are provided by the adjustment mechanism with the objective to minimize the error between the system under control and a model reference output (that is the desired response).

Model Reference – Direct Approach

Block diagram of the general idea of MRAC



MRAC Applied to the MSL after the Linearization

- Stability theory from the input-output view is applied to the MSL after the exact linearization. Once the dynamics are now linear, the control problem will be formulated as model-following
- The derivation of the MRAC will follow the 3 steps below (Aström and Wittenmark, 2008):
  – *Step 1:* Find a controller structure that admits perfect output tracking;
  – *Step 2*: Derive an error model of the form

$$\varepsilon = G_1(p)\{\phi^T(t)(\theta^0 - \theta)\} \quad (14)$$

where $G_1(p)$ is a Strictly Positive Real (SPR) transfer function in p, θ^0 is the process parameters (or the true controller parameters), and θ is the controller parameters (or the adjustable controller parameter).

  – *Step 3*: Use the parameter adjustment law

$$\dot{\theta}(t) = \gamma\phi\varepsilon \quad (15)$$

where γ is the adaptation gain, φ an auxiliary vector of filtered signals and ε the error signal.

Adaptive Control Scheme

- The set of equations needed to implement the MRAC system can be summarized as follow

$$ym = \frac{B_m}{A_m} r = G_m(s) \qquad \eta = -\left(\frac{1}{P_1} u + \phi^T \theta \right) \qquad \dot{\theta}(t) = \gamma \phi \varepsilon$$

$$e_f = \frac{Q}{P} e = \frac{Q}{P} \left(y - y_m \right) \qquad \varepsilon = e_f + \frac{b_0 Q}{A_0 A_m} \eta \qquad u(t) = -\theta^T (P_1 \phi) \tag{16}$$

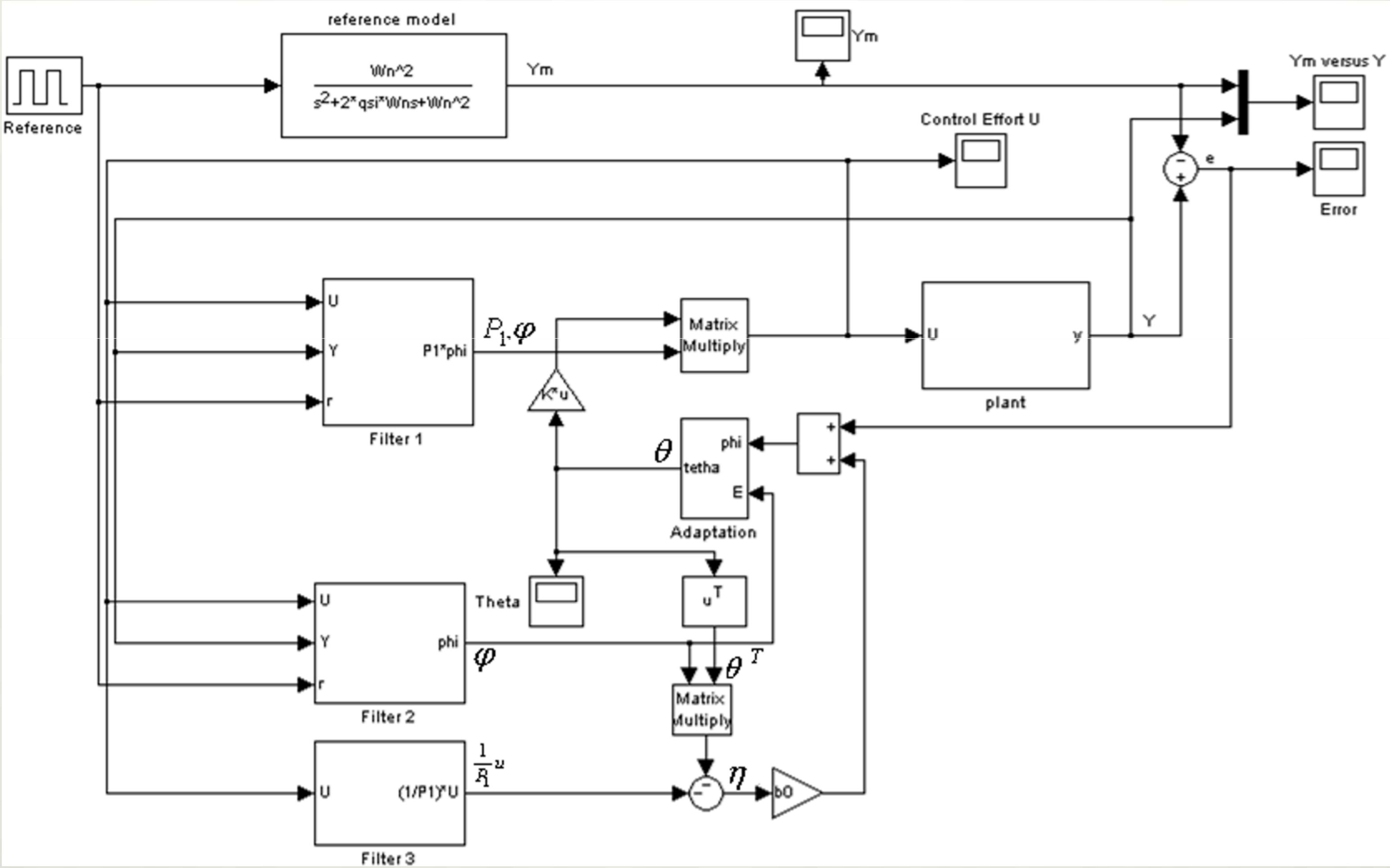
Where:

$G_m(s)$ is the model reference transfer function	η is called the error augmentation	$\dot{\theta}(t)$ parameter adjustment law
e_f is called the filtered error	ε is called the augmented error	$u(t)$ is the control law

- A_0 , A_m, B_m, Q, P, and P_1 are polynomials. The parameter b_0 is the high-frequency gain.
- The error model in (14) is the same defined in (16). It is also linear in the parameters and satisfies the requirements of the *step 2*, and the parameters will be updated by $\dot{\theta}(t)$
- The stability of the closed-loop system is obtained by considering that $b_0.Q/(A_0.A_m)$ is SPR and that signals in φ are bounded

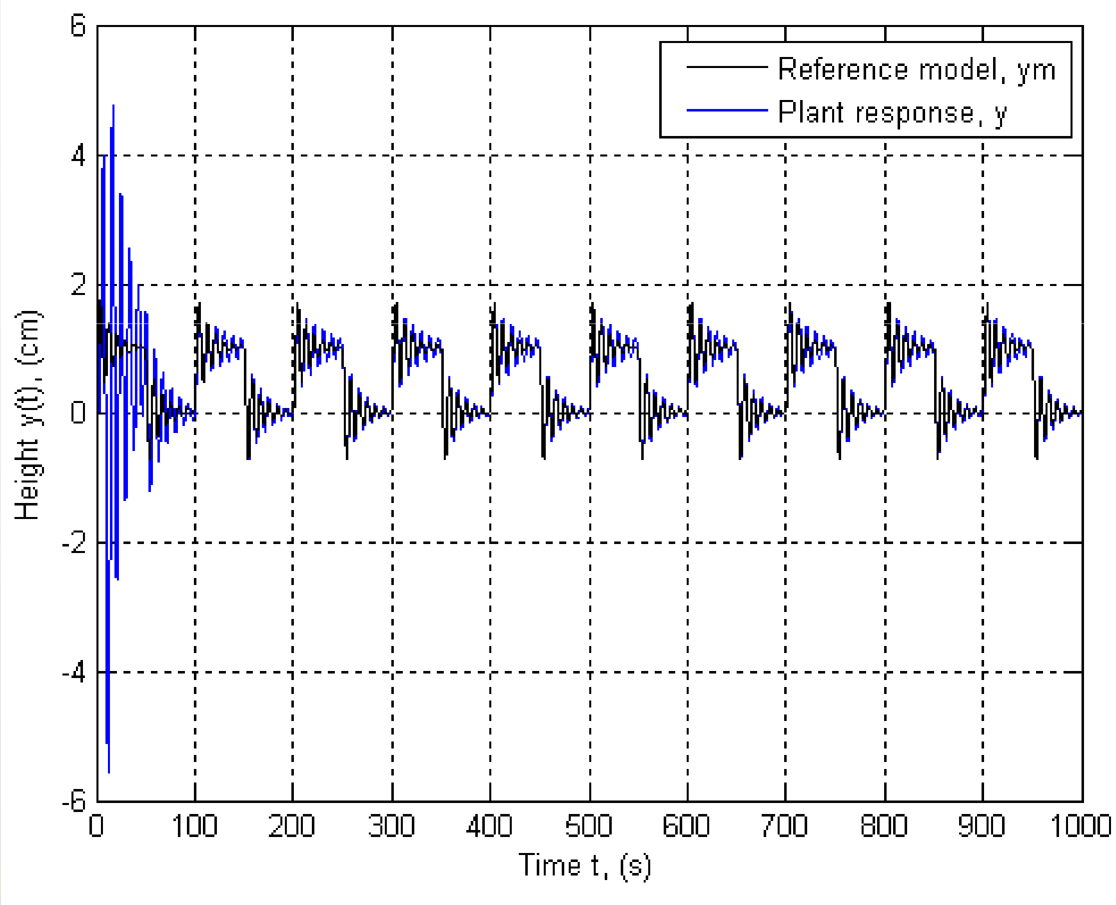
Simulation and Results

Block diagram in Simulink with the adaptive controller structure



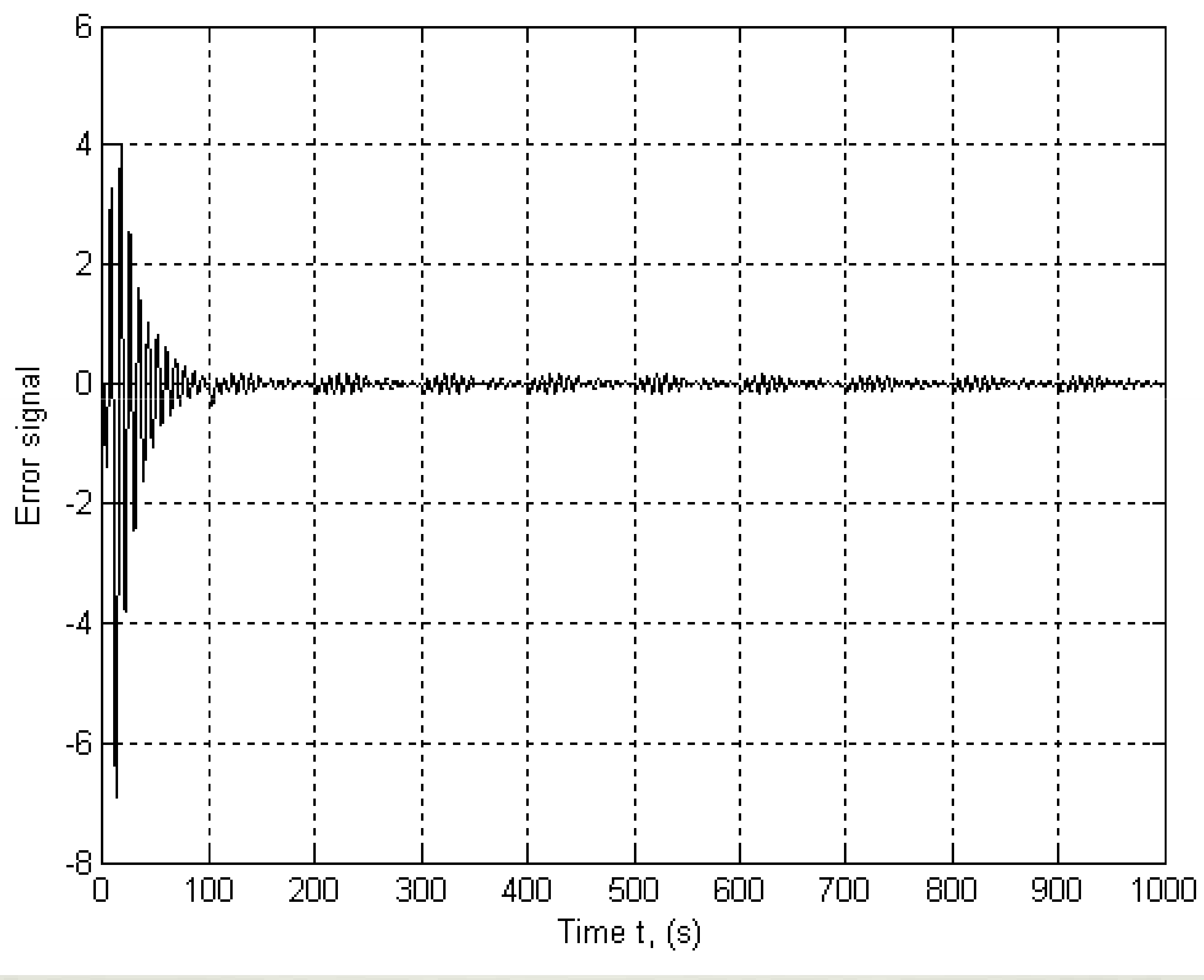
Simulation and Results

System response with the adaptive proposed controller and square wave reference signal



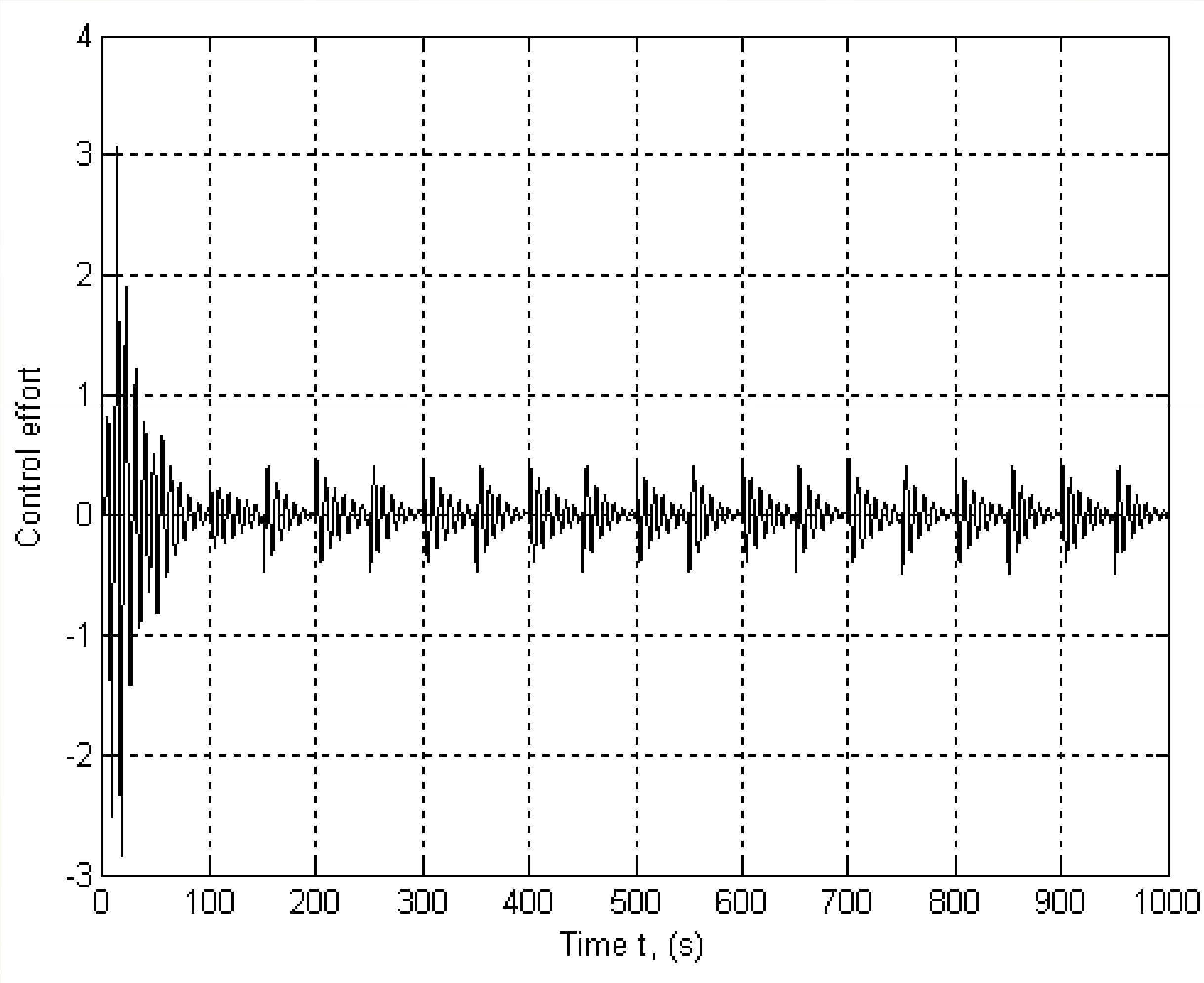
Simulation and Results

Error signal



Simulation and Results

Control Effort



Conclusions

- It was presented the combination of two techniques to control a magnetic levitation system:
  - Exact linearization with state feedback
    - Advantage: linear system - linear controller;
    - Disadvantages: model uncertainty; estimation of nonlinear functions;
  - Model Reference Adaptive Control
    - It can be used to deal the presence of the model uncertainties;

- It could be observed that the desired response (output signal of the model reference) was tracked by the plant response.
- The error signal could be seen bounded and near to zero and the control effort could be seen also bounded.

Future works

- For future work this adaptive controller should be implemented in the real physical system.

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Thank you!

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