Exact Linearization and Fuzzy Logic Applied to the Control of a Magnetic Levitation System

Luiz H. S. Torres, Carlos A. V. Vasconcelos, Jr., Leizer Schnitman, and J. A. M Felippe de Souza

***Abstract*— In recent years the area of nonlinear control systems has been the subject of many studies. Computational developments have enabled more complex applications to provide solutions to nonlinear problems. The purpose of this paper is to use a combination of two techniques to control a nonlinear system: the Magnetic Levitation System. First, the exact linearization technique with state feedback is applied to obtain a linear system. Second, the linearization is made via direct cancellation of nonlinear functions, which represent the phenomenological model of the system. Finally, to deal with the presence of uncertainty in the system model, an adaptive controller is used. The controller is based on fuzzy logic to estimate the functions that contain the nonlinearities of the system. The fuzzy system is a zero-order Takagi-Sugeno-Kang structure and the adaptive controller is implemented in a simulated environment (Matlab Simulink ©). The methodology guarantees the convergence of the estimates to their optimal values, and in turn the overall stability of the system. The results show the controller output signal tracks a reference input signal. For future work this adaptive controller should be implemented in a real physical system.**

I. Introduction

In recent years the area of nonlinear control systems has been the subject of many studies. Computational developments have enabled more complex applications to provide solutions to nonlinear problems. This paper shows a combination of a linearization technique and Artificial Intelligence (AI) to control a Magnetic Levitation System (MLS). This system was chosen since it has nonlinear dynamics and a didactic kit of the physical system is available to perform tests and to continue with future work.

The MLS used is manufactured by ECP – Educational Control Products (www.ecp.com) and will be described in more detail in section II. The aim of this work is to control a magnetic disc movement over a glass column as a result of the application of an electrical current on a coil [1],[2].

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The relationship between the electrical flow and the magnetic disc movement is given by a second order nonlinear ordinary differential equation of the type $\dot{x} = \phi(t,x) = Ax + B[F(x) + G(x)u]$. Several nonlinear control strategies can be used to control the disc position, for example, fuzzy, neural network, adaptive control, exact linearization [3],[4]. In this paper both the exact linearization technique and fuzzy will be used.

Exact linearization enables a transformation from a nonlinear system to a linear system through the addition of nonlinear compensators in the system control loops [5],[6]. However, the exact linearization technique with state feedback requires a mathematic model that represents the dynamics of the real plant well. Furthermore, the uncertainties in the phenomenological model cannot guarantee good results can commit better results. To deal with some uncertainties in the system model, an adaptive controller is used [7]. The controller is based on fuzzy logic to estimate the functions $F(x)$ and $G(x)$ that contain the nonlinearities of the system. The fuzzy system is a zero-order Takagi-Sugeno-Kang (TSK) structure and the adaptive controller is implemented in a simulated environment [8],[9]. The methodology adopted guarantees the convergence of the estimates to their optimal values, and in turn the overall stability of the system [10].

II. The Model

A. Magnetic Levitation System

In this paper the MLS made by ECP was used and is shown in Fig. 1. It comprises two magnetic discs, a glass column, two laser sensors and two coils. The sensors are used to obtain the system response associated with the disc positions. The system input is given by the application of an electrical current to the coils. The physical system communicates with a computer via Digital Signal Processing (DSP) and a black box is responsible for the electrical current drivers and the energy supply.

This MLS can be classified according to two modes, SISO (Single Input Single Output) or MIMO (Multiple Input Multiple Output) and this depends on the desired system configuration. In the SISO mode only one disc is used whereas in the MIMO mode two discs are used. Here the

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MLS was configured to operate in the SISO mode.

Fig. 1. Magnetic Levitation System made by ECP.

The MLS manual [1] shows the mathematic model, based on the physical laws, that allows us obtaining its differential equation model. The development of the mathematic model is beyond the scope of this paper. Through the balance of forces, the equation is given by (see [2]):

$$\ddot{y}+\frac{c}{m}\dot{y}=\frac{F_m}{m}-g \tag{1}$$

where,

y - magnetic disc position

$\dot{y}$ - first derivative magnetic disc position

$\ddot{y}$ - second derivative magnetic disc position

c - air viscosity coefficient

m - magnetic disc mass

F_m - magnetic force applied to the magnetic disc.

The magnetic force can be written in the following way (see [1])

$$F_m=\frac{i}{a(y+b)^4} \tag{2}$$

where,

i - electrical current applied on the coil

a and b - are constants related with the coil properties.

By substituting (2) in (1), a nonlinear relationship between the magnetic disc position and electrical current applied to the coil gives

$$\ddot{y}=-g-\frac{c}{m}\dot{y}+\frac{1}{ma(y-b)^4}i \tag{3}$$

A. Parameter Estimation

There are five parameters in (3): *g*, *c*, *m*, *a* and *b*. The parameters g = 9.81 [m/s^2], m = 0.12 [*Kg*] and c = 0.15 [*Ns/m*] (from [1]). The parameters a and b are constants related with magnetic coil properties and must be estimated. In [3], the least square and Monte Carlo methods were used to estimate a and b. Accordingly with [3], based on a cost function concept, the Monte Carlo method presented the best values for these parameters. The values are a = 0.95 and b = 6.28. These values will be used in this paper.

III. The Control Technique

A. Exact Linearization with State Feedback

Exact linearization with state feedback can be applied to a variety of nonlinear systems, including the MLS studied. The control scheme uses the exact linearization technique based on the cancellation of nonlinear functions. However, to enable the application of the technique, the system dynamic must be represented by (see [4])

$$\dot{X}=F(X)+G(X)u \tag{4}$$

where the functions *F(X)* and *G(X)* represent the nonlinearities of the states, u is the control system input and X is the state vector. Furthermore, two conditions must be satisfied. The first one is that the system must be controllable. For this first condition the matrix formed by vectorial fields in (5) must contain order *n*, where *n* is the system order (see [5])

$$[ad_F^0 G\ ad_F^1 G \ldots ad_F^{n-1} G] \tag{5}$$

where $ad_F^n G$ is the notation of Lie bracket.

The second one is that the system be involutive. It means that the distribution expressed in (6) also be involutive (see [6])

$$D=span\left\{ad_F^0 G\ ad_F^1 G \ldots ad_F^{n-1} G\right\} \tag{6}$$

where *D* is the involutive distribution of *G(X)* expanded in Taylor's series (represented here by the notation *span{.}*) on an equilibrium state X_0. The order of *D* is given by $n-1$.

In order to the distribution in (6) to be involutive, it is necessary that the order *n* of the expression in (7) be equal to *dim(D)* in (6)

$$[ad_F^0 G\ , ad_F^{n-1} G]\,. \tag{7}$$

Once the conditions are satisfied it is possible to determine a diffeomorphism *Z* = *T(X)*. After this, the dynamic of the system given by (4) can be transformed into the form (see [7])

$$\dot{Z} = AZ + B\beta^{-1}(Z)[u - \alpha(Z)] \,. \tag{8}$$

A feedback control signal u for the nonlinear system is chosen in the form in (9)

$$u = \alpha(Z) + \beta(Z)v \tag{9}$$

where $\alpha(Z)$ and $\beta(Z)$ represent the states feedbacks.
Thus, the linear system can be written in the form in (10)

$$\dot{Z} = AZ + Bv \tag{10}$$

where v is the input signal for the system after linearization. The determination of v will be discussed in next section.

B. *Linearization of the MLS Made by ECP*

The model of the MLS made by ECP was presented in the (3) and the two conditions for application of the exact linearization were presented in the last subsection. The variables of states and the feedback control signal u can be set as follows in (11)

$$u = i \qquad x_1 = y \qquad x_2 = \dot{y} \,. \tag{11}$$

The dynamic of the system given by (3) can be rewritten in the form given in (4)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ -g - \dfrac{c}{m} x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{1}{ma(x_1 + b)^4} \end{bmatrix} u. \tag{12}$$

The functions $F(X)$ and $G(X)$ that contain the nonlinearities of the system can be set as follows

$$F(X) = \begin{bmatrix} x_2 \\ -g - \dfrac{c}{m} x_2 \end{bmatrix} \tag{13}$$

$$G(X) = \begin{bmatrix} 0 \\ \dfrac{1}{ma(x_1 + b)^4} \end{bmatrix}. \tag{14}$$

The transformation $Z = T(X)$ can be set in the form given by (see [5])

$$Z = \begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix} = T(X) = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}. \tag{15}$$

The functions $\alpha(Z)$ and $\beta(Z)$ can be calculated in the form given by

$$\alpha(Z) = (mga + caZ_2)(Z_1 + b)^4 \tag{16}$$

$$\beta(Z) = ma(Z_1 + b)^4 \,. \tag{17}$$

Finally, the feedback control signal u could be rewritten by using (16) and (17)

$$u = (mga + caZ_2)(Z_1 + b)^4 + ma(Z_1 + b)^4 v \,. \tag{18}$$

The application of the feedback control signal u over the system given by (12) will cancel the nonlinearities and the system will be transformed into a linear system given by (10). In the same way but by using (15)

$$\dot{Z} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ v \end{bmatrix}. \tag{19}$$

A block diagram was implemented in Matlab Simulink which simulates the exact linearization technique applied in the MLS, see. Fig. 2 below

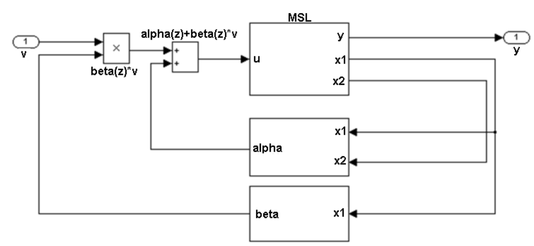

Fig. 2. Block diagram implemented in Matlab/Simulink for the exact linearization in the MLS.

IV. The Fuzzy Structure

A. *Fuzzy Estimators*

A zero-order TSK system with R rules (Fig. 3) is used in this paper and has the following form

$$\text{If } x_1 \text{ is } A_1^j \text{ ... and } x_n \text{ is } A_n^j \text{ then } y \text{ is } B_j \,, \tag{20}$$

where $x = [x_1, ..., x_n] \in R^n$ is the input vector, $\{A_1^j, ..., A_n^j\} / B_j$ are the fuzzy set of input and output, respectively, associated with a j^{th} rule $(j = 1, ..., R)$ and y is the output of the fuzzy system. The output y_j is the point which B_j is the maximum value $(\mu_{B_j}(y_j) = 1)$ and θ is the parameter vector in the form $\theta^T = [y_1, ..., y_R]$, so the output of the fuzzy system is expressed by

$$y = \theta^T W(x) \tag{21}$$

where $W(x)=[W_1(x),...W_R(x)]$ and,

$$W_j(x)=\frac{\prod_{k=1}^{n}\mu_{A_k^j}(x_k)}{\sum_{j=1}^{R}\left(\prod_{k=1}^{n}\mu_{A_k^j}(x_k)\right)} \tag{22}$$

for $j=1,...,R$ and $W(x)\in[0,1]$, where μ is the membership function

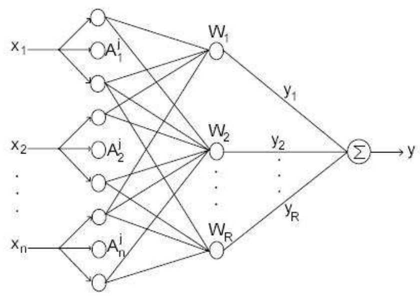

Fig. 3. Fuzzy structure diagram, zero-order TSK.

The feedback control signal u expressed in (9) could not be implemented because the functions $\alpha(Z)$ and $\beta(Z)$ are unknown and need to be estimated. However, [8] and [9] used fuzzy structures to estimate some functions. The main idea here is to construct a fuzzy structure able to generate the estimates $\alpha(Z\mid\hat{\theta}_\alpha)$ and $\beta(Z\mid\hat{\theta}_\alpha)$, where $\hat{\theta}_\alpha$ and $\hat{\theta}_\beta$ are parameter vectors. So, an adaptive scheme is used here to obtain these parameters vectors and (9) can be expressed in terms of fuzzy structures in the form below

$$\alpha(Z\mid\hat{\theta}_\alpha)=\hat{\theta}_\alpha{}^T W(Z) \tag{23}$$

$$\beta(Z\mid\hat{\theta}_\beta)=\hat{\theta}_\beta{}^T W(Z). \tag{24}$$

B. *Adaptive Control Scheme*

The adaptive control scheme is based on state observers. However, the functions $\alpha(Z)$ and $\beta(Z)$ are substituted by the fuzzy estimates, respectively

$$\dot{Z}_f = AZ_f + B\beta^{-1}(Z\mid\hat{\theta}_\beta)[u-\alpha(Z\mid\hat{\theta}_\alpha)]+ + k^T C(Z-Z_f), \tag{25}$$

where k is a gain vector in the form $k=[k_1,...,k_n]\in R^n$ and Z_f is state estimated. There are optimal parameters θ_α^* and θ_β^* which are able to estimate the functions $\alpha(Z)$ and $\beta(Z)$, there will also be estimates for optimal states

$$\dot{Z}_f^* = AZ_f^* + B\beta^{-1}(Z\mid\hat{\theta}_\beta^*)[u-\alpha(Z\mid\hat{\theta}_\alpha^*)]+ + k^T C(Z-Z_f^*). \tag{26}$$

An error of the estimation is set as

$$e=Z_f^*-Z_f=[e_1,e_2,...,e_n]^T. \tag{27}$$

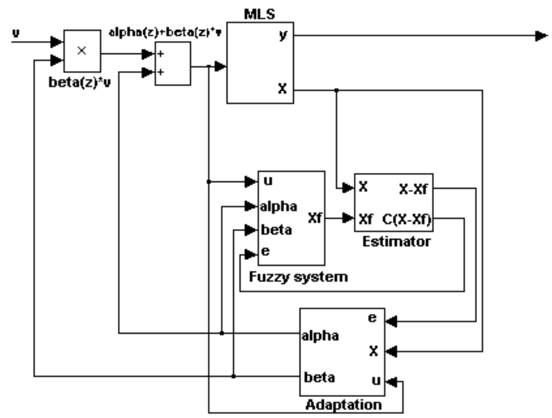

Fig. 4. Block diagram in Simulink

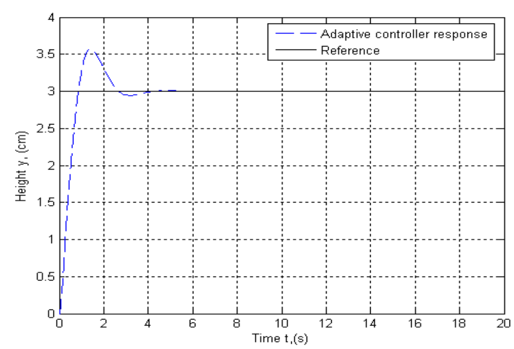

Fig. 5. System response with the adaptive proposed controller and

The adaptive laws in the form (see [10])

$$\dot{\hat{\theta}}_\alpha=-\gamma_\alpha e^T PBW(Z) \tag{28}$$

$$\dot{\hat{\theta}}_\beta=-\gamma_\beta e^T PBW(Z)u, \tag{29}$$

where γ_α and γ_β are positive constants.

As regards Lyapunov's equation expressed in (30), to obtain the value of matrix P

$$\Delta^T P + P\Delta = -Q \tag{30}$$

where $\Delta = A + k^T C$ and Q is defined a positive matrix. Regarding Δ stable, so there is a unique defined positive matrix P that satisfy (30). Finally, the choice of a defined Lyapunov semi-negative for the system, guarantees that the error of the estimate e is followed. The application of the *Barbalat's lemma* cause $e \to 0$ when $t \to \infty$. From (27), $e \to 0$ implies that $Z_f \to Z_f^*$ and the convergence of the estimated parameters to their optimal values is achieved.

V. SIMULATION RESULTS

The Matlab Simulink was used to simulate a proposed controller for the MLS in the present work. The block diagram designed in Simulink is shown in Figure 4. The model parameters used here were presented in subsection II.B.

The simulations were performed regarding r as a step reference signal, with values ranging from a minimum of 0 cm to a maximum of 3 cm, respectively. In Fig. 5, a simulation of the controller response is shown. This was obtained by the combination of exact linearization with the fuzzy estimates and the step reference signal.

CONCLUSION

In this paper the combination of two techniques to control a MLS were presented: exact linearization with states feedback and fuzzy logic. It was possible to verify that the simulated results show an overshoot in the response of the controller. However, these same results show that the controller output signal tracks a reference input signal. For future work this adaptive controller should be implemented in a real physical system.

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