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OPTIMAL INVESTMENTS IN CLEAN TECHNOLOGY AND REFORESTATION IN THE CONTROL OF GLOBAL WARMING USING FUZZY COST FUNCTION

Marco A. L. Caetano[†], Douglas F. M. Gherardi[††], Takashi Yoneyama[†††] and José A.M. Felippe de Souza[†††]

[†] Ibmec São Paulo – Faculdade de Administração e Economia – São Paulo - Brazil
[††] Instituto Nacional de Pesquisas Espaciais – INPE - São José dos Campos – Brazil
[†††] Instituto Tecnológico de Aeronáutica – ITA - São José dos Campos – Brazil
[††††] Universidade Beira Interior - UBI - Covilhã - Portugal

ABSTRACT
This work concerns the optimization of investment in clean technology and in reforestation to counter the global warming. The performance of the investment policy is evaluated by an objective function that reflects the political, economical and social costs by using fuzzy relations that converts objective quantitative rules into a subjective index between zero and one. The adopted dynamic model relates the environmental and economic variables such as the rate of CO_2 emission, forest area and gross domestic product. The admissible control variables are assumed to be piecewise constant, as the budgets are planned for fixed time horizons. Because the optimal control problem is non-linear, a direct search method is used to yield a numerical result. In the case study here presented, the data were fitted to reflect the scenario presented by the European Union (EU) since 1960.

KEY WORDS
Optimal Control, Fuzzy Control, Clean Technology, Global warming; reforestation.

1. INTRODUCTION

The optimal control theory has found relevant applications in a variety of fields including engineering, operations research, astronautics and more recently in biological areas such as ecology and environmental science. With respect to ecological applications, several papers have appeared in the literature showing the relationships between the global warming and its economic consequences around of world [1], [2], [3], [4], [8], [11], [12], [15].

The aim of this work is the use the concept of fuzzy logic to propose a cost function for quantifying linguistically the magnitude of costs to mitigate the global warming effects. The required investments are computed to minimize CO_2 emission and to approach the targets proposed in the Kyoto Protocol for European Countries. The *Kyoto Protocol* recommends the collective reduction of the emission of Greenhouse Gases - GHG (carbon dioxide, methane, nitrous oxide, sulfur hexafluoride, hydrofluorocarbons, and perfluorocarbons) in industrialized countries by 5.2%, averaged over the period of 2008-2012, taking as reference the year of 1990.

This work addresses the problem of analyzing the investments considering two decision variables, reforestation and investments in clean technology. Also, only one type of Greenhouse Gases (GHG), more specifically, carbon dioxide (CO_2), is considered in the proposed performance index. The mathematical model consists in a system of coupled ordinary differential equations that relates the production of CO_2 with forest area and GDP, following the work of Caetano *et al*. in [7]. It is worth mentioning that different models that represents global warming and economic relationship could be advantageous in other optimization problems: ([10], [14], [18], [19], [20], [21], [24], [26]). The parameters of the model were adjusted using widely published data, such as those available at UNEP [30]. The control variables (investments) are assumed to be piecewise constant functions of time, reflecting the fact that most economic planners adopt fixed intervals in proposing a budget. The numerical solution to the optimal control problem is obtained by a direct search method ([16], [17]).

2. A MODEL FOR GLOBAL WARMING

The mathematical model used in this work consists of three differential equations relating the concentration of atmospheric carbon dioxide x(t), forest area z(t) and Gross Domestic Product - GDP, y(t). as follows:

$$\begin{cases} \dot{x} = r\,x\left(1-\dfrac{x}{s}\right) - \alpha_1 z + (\alpha_2 - u_2)\,y \\ \dot{z} = u_1 y - h z \\ \dot{y} = \gamma y \end{cases}$$

where the dot above a variable ($\dot{x}$) denotes its time derivative (dx/dt). The variables u_1 and u_2 represent the amount of investments in reforestation and in the adoption of clean technology, respectively. The model parameters (constants) are r, s, h, α_1, α_2 and γ and their relationships with respect to x, z, and y can be visualized by the graph in Figure 1 (see [7]).

In terms of an intuitive interpretation of the system of equations (1), one can notice that the CO_2 emissions (x) are dependent on *r*, the emission rate and s, the carrying capacity of the atmosphere in terms of CO_2. The second term represents the net balance of emission and removal of CO_2 and the contribution of a certain region in terms of removal of CO_2 from the atmosphere is assumed to be proportional to the total forest area. The total area of forest at time t depends on the initial condition (z_0), such as an existing forest and the reforestation effort. The reforestation effort is assumed to be a fraction of the GDP (in countries where there are laws and incentives to promote reforestation) with u_1 representing the intensity of incentives directed to reforestation and u_2 representing the incentives to clean technology considering that the required clean technology is proportional to the GDP . The parameter h is a constant representing the forest depletion rate and amalgamates a variety of factors such as expansion of cattle ranching, fire, commercial logging, shifted cultivators and colonization, among others.

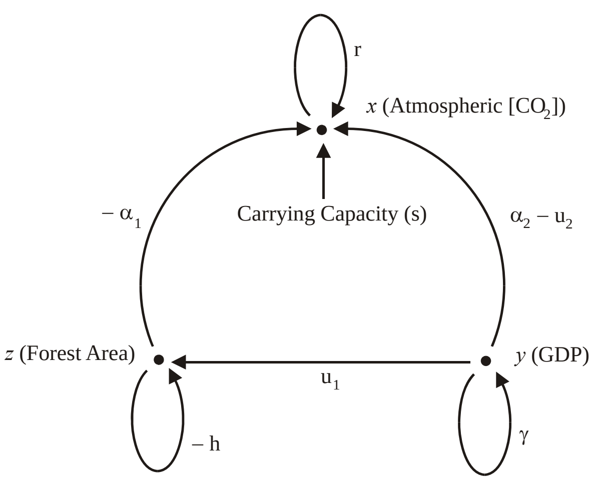

Figure 1 Relations between the state variables

3. THE COST FUNCTION

In environmental area, linguistic variables that are related to actual numerical data are quite useful in representing the political, economical e social perception of the environmental condition of a region. Therefore, fuzzy relations can be used in a cost function to represent the effectiveness of the chosen adequate policy. The proposed fuzzy index was conceived to indicate a level of atmospheric pollution that has relationship with CO_2 emission. In the mathematical model it is suppose that the forest area has influence on CO_2 emission control, but is necessary a transference of investments to increase forest area. This transference is not so clear, and fuzzy index try translated of a qualitative form the output of investments. Figure 2 shows the scheme used for computing the cost function and to feedback the control to apply in dynamic system. The control of investments consists of choice of u_1 *(control 1)* and u_2 *(control 2)* that are constants during a fixed interval T (such as 3 months, 6 months or 1 year).

$$u_j(t) = u_j(t_i) \quad t \in [t_i, t_{i+1}) \tag{7}$$

where t = 1, 2, 3… and $t_{i+1} - t_i = T$, j = 1,2.

The process is initialized with a sequence of random positive investments of u_1 and u_2. These investments are used to simulate the dynamic system of ordinary differential equations (Eq.(1)).

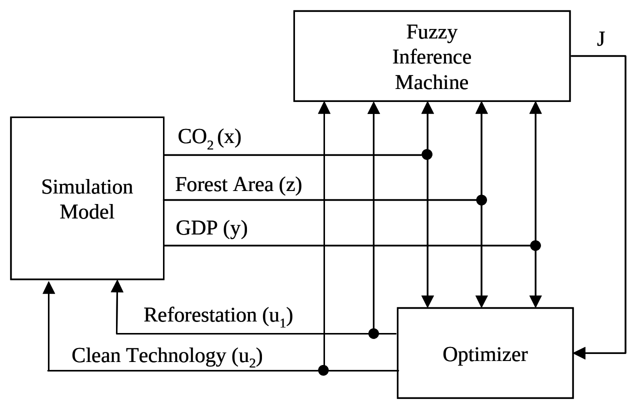

Figure 2 Proposed optimization setup

The simulation results of CO_2 emission, forestry area and GDP and investments u_1 and u_2 are fed into the fuzzy logic block. The fuzzy block computes the value of the cost function that indicates, in a normalized scale. The obtained result for cost function is used by an optimization algorithm (such as Nelder-Mead) to search for the optimal doses for the minimum cost.

The logic rules for the fuzzy block were built as a exercise to test the method. A set of 17 rules were built to generate the output for the cost function. Each fuzzy rule has five conditions, namely the tons of CO_2, the forestry area, GDP of country, the incentives of reforestation ($u_1(t)$) and the incentives of clean technology ($u_2(t)$). For instance a fuzzy logic rules may use the structure:

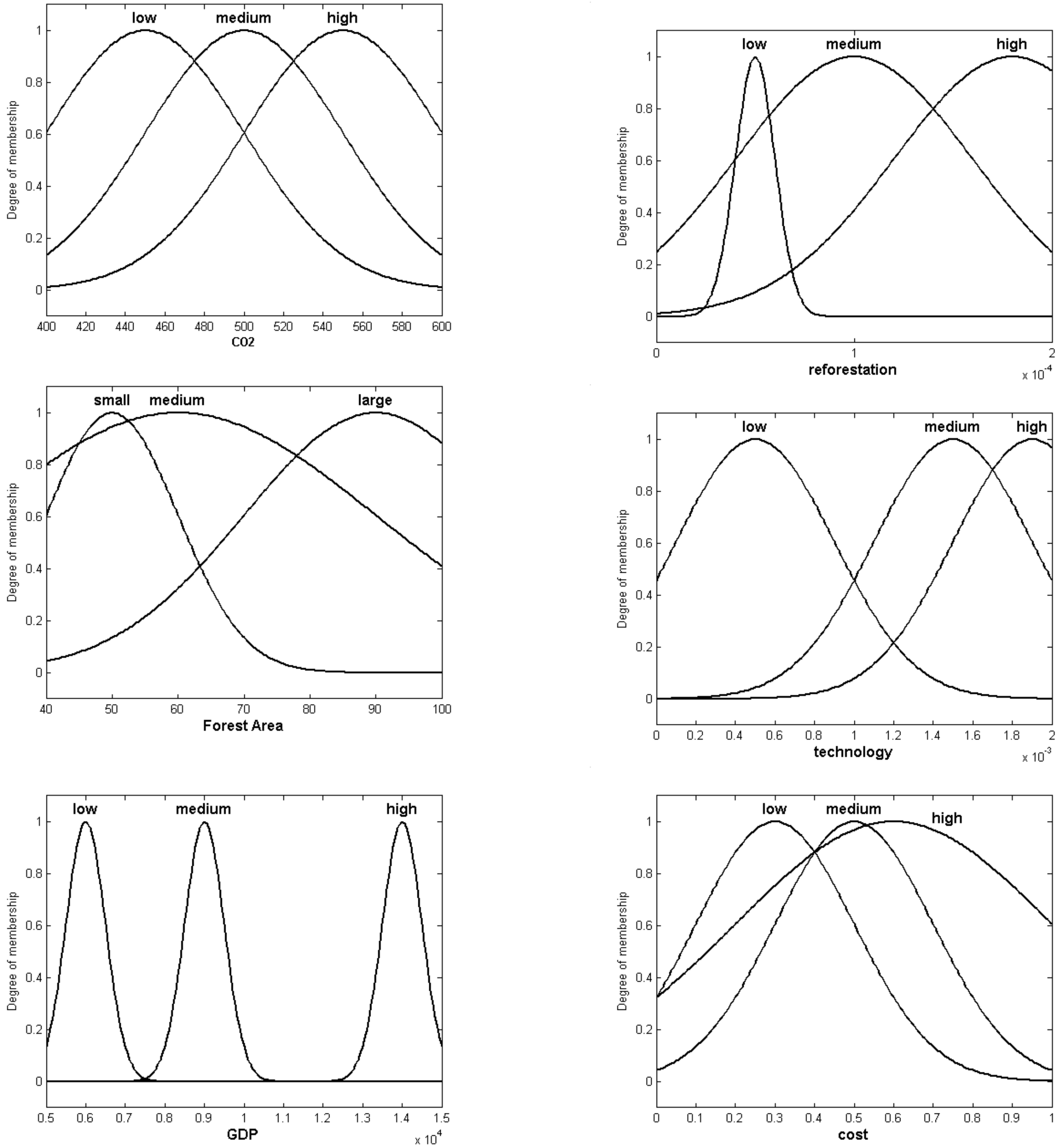

Figure 3 Membership functions CO_2, forestry, gdp, reforestation investment and clean technology investments

“IF <CO_2 is low> AND <forestry is large> AND <GDP is high> AND <u_1 is medium> AND <u_2 is high> THEN <cost function is low>”. In this rule, it was considered that low emission with medium investments in reforestation and high investments in clean technology to control CO_2 tends to lower the total cost low because the forestry area is high.

Table 2 presents the complete set of fuzzy logic rules used in this work as designed based on the knowledge provided by others works. No supervised training was used so that the membership function was tuned by hand. The time horizon $[t_o , t_F]$ is partitioned into N subintervals and let

$$
\begin{aligned}
t_i &= t_{i-1} + \frac{t_F - t_0}{N} \\
i &= 1, \ldots, N
\end{aligned}
\qquad (8)
$$

At each discrete instant t_i, let the corresponding output of the fuzzy cost function be $J_i(u_1, u_2)$ and define the overall cost function as

$$
J(u_1, u_2) = \sum_{i=1}^{N} J_i(u_1, u_2) \qquad (9)
$$

where (u_1,u_2) are functions that are piecewise constant assuming values $u_1(t_i)$ and $u_2(t_i)$ as previously defined in equation (7). An example of the incurred cost at each time interval is shown in Figure 5. The problem is to search for the pair of piecewise constant functions (u_1^*, u_2^*) that minimizes $J(u_1,u_2)$ by a direct search method.

The chosen membership functions for the five inputs (CO_2, Forestry, GDP, $u_1(t)$ and $u_2(t)$) and the single output (cost function) are shown in Figure 3. The membership functions are of Gaussian type and they cover the universe of discourse partitioned into sets low, mean and high (linguistic variables) for both, input and output data. For the deffuzification the Center of Mass method was adopted.

4. A CASE STUDY

In order to illustrate the application of the proposed methodology, European Union actual data was selected. Firstly, the model parameters for the EU were identified yielding an adequate fitting, as can be seen in figure 4. The parameters fitted are in table 1. The final time for observation of this patient was 40 years, since 1960 until 2000. The total period was divided in five intervals in order to use a finite dimensional optimizer.

The result in terms of the cost function can be seen in figure 4. This figure is a simulation using table 1 with the same initial condition from actual data. The initial conditions for this simulation are in table 3 from 1996 until 2014 (18 years). It is possible to observe that the cost function using optimal control with fuzzy logic is lower in the final time than initials periods in figure 5.

Figure 6 shows the results for the variables (CO_2 and Forestry area and GDP of EU) and the investments (control) in Figure 5. Matlab © 6.5 software was used in order to apply the Nelder-Mead search optimization algorithm, via the function *fminsearch.m*; and also to implement the Runge-Kutta method for simulation via the function *ode45.m.*

5. CONCLUSION

The present work shows the use of fuzzy logic in the cost function in order to optimize the incentives to decrease global warming considering a fuzzy normalized evaluation of CO_2 emission. A set of 17 rules were built to generate the output for the cost function. The actual data set of a patient from *European Union* was used to compare the results since 1960 with respect to CO_2 emission and forestry area. The fuzzy logic was seen to describe, in a natural way, the assessment of the quality of incentives, as made by the Union Nation (UN), by growing of reforestation and clean technology incentives. The membership functions were built as academic exercise.

Obviously, a larger sample of actual data is needed to refine the results and carry out statistical tests.

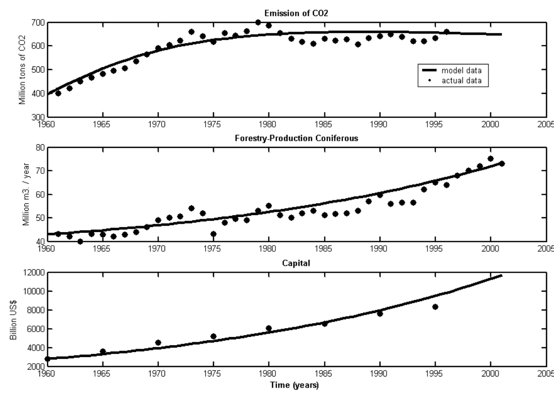

Figure 4 Parameters fit for actual data

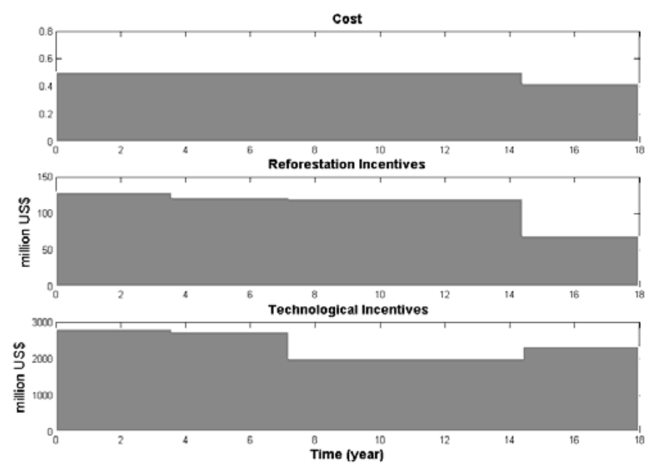

Figure 5 Intermediate values of the cost function for OPTIMAL DOSES AND FIXED DOSES OF (1400 mg of m_1 and 2000 mg of m_2)

Table 1 – Model Parameters

Parameters	Values
r	0.15
s	700
h	0.0001
u_1	0.00012
u_2	0.0008
γ	0.035
q	11
r_1	3.5E+8
r_2	1E+9
p	10
α_1	0.15
α_2	0.00005

Table 2 – Rules for the fuzzy inference machine

CO_2	Forest area	GDP	U_1	U_2	Cost
low	low	low	low	low	low
low	med	med	low	low	low
low	med	med	med	med	high
low	high	med	high	med	low
low	high	high	med	high	low
low	high	high	high	high	med
med	med	med	high	med	high
med	med	med	med	med	high
high	low	low	low	low	high
high	med	low	low	low	high
high	med	low	med	med	high
high	med	med	low	high	high
high	high	low	high	med	med
high	high	high	low	low	high
high	high	high	med	low	high
high	high	high	low	high	high
high	high	high	high	high	high

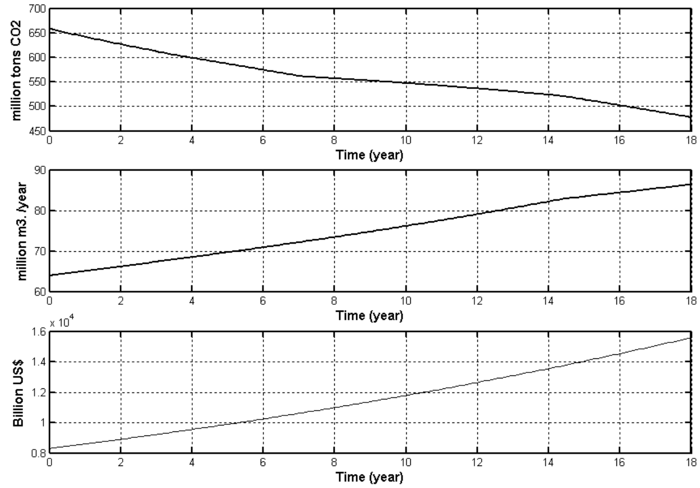

Figure 6 - Evolution of the variables used in the fuzzy rules

Table 3 – Initial Conditions

Variables	Values
x(0)	659 million ton CO_2
z(0)	68 million m^3
y(0)	8,306 billion US$
Final time	18 year

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