State space adjustments for the control of nonlinear systems

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Abstract: - We have seen the appearance of some papers which extended techniques traditionally used in the study of the existence of solutions of nonlinear differential equations to solve problems of control, state estimation and parameter identification of nonlinear systems. These techniques often require some topological adjustments of either the space of admissible input functions **U** (the control space) or the state space X of the system. These topological adjustments became possible with the use of matched sets introduced by the author in [3, 4, 5]. The present work places its emphasis on how the matched sets can be used to adjust the spaces **U** and X for the problem of nonlinear control as formulated below. A summary of the theory of complete matched sets is also shown. Moreover, the semigroup approach is used so that the distributed parameter systems and delay systems can be considered as well as lumped parameter systems.

Key-Words: - Control, state space, nonlinear systems, distributed and lumped parameter systems.

1 Introduction

The techniques traditionally used in the study of the existence of solutions of nonlinear differential equations has been extended to solve problems of control, state estimation and parameter identification of nonlinear systems (see [1, 2, 6, 7, 8, 10, 11,12, 13, 14, 15,]).

Here we consider systems of the type:

$$\dot{x} = Ax(t) + N(x(t)) + Bu(t)\,, \quad x(0) = x_0 \in \mathrm{X} \qquad (1)$$

where *A* is a linear operator on an appropriate Hilbert space X (the state space), *N* is a nonlinear operator from an input space U to X, and u(·) ∈ **U** is the control (**U** being a space of functions from the interval [0,T] to the input space U of the system). Such systems are often called *semilinear systems*.

It is assumed that the dynamics of the autonomous part of the system (1), i.e.,

$$\dot{x} = Ax(t), x(0) = x_0 \in \mathrm{X}$$

can be described in terms of a strongly continuous semigroup *S(t)* on X, so that the above formulation includes distributed parameter systems and delay systems, as well as lumped parameter systems.

2 THE CONTROL PROBLEM

Clearly for the case of lumped parameter systems we have that A is a n × n matrix, X = $\mathbf{R}^n$ and the semigroup *S(t)* becomes $S(t) = e^{At}$.

The problem of control is to find a control u(·) which drives system (1) from the initial state $x_0 \in$ X to a given desired final state $x_d \in$ X at t = T. System (1) may be derived from the linearization of a system described by a nonlinear evolution equation such as

$$\dot{x} = f(x, u, t), x(0) = x_0 \qquad (2)$$

Equation (1) is to be interpreted in the mild sense

$$x(t) = S(t)\,x_0 + \int_0^t S(t-\tau)\,N x(\tau)\,d\tau + \int_0^t S(t-\tau)\,B u(\tau)\,d\tau \qquad (3)$$

with the initial conditions

$$x(0) = x_0 \in \mathrm{X}.$$

Papers such as [6, 8, 10, 12, 14, 15, 16] presented some techniques to solve the above nonlinear control problem (2) using the fixed point of a map Φ: **X** → **X** constructed on a space **X** of trajectories x(·) (e.g., **X** = L^2(0,T;X)). These techniques often assume that the control space **U** and/or the state space X can be adjusted to new spaces **U'** and X', (with **U** ∩ **U'** and X ∩ X' dense on their counterparts **U** and X respectively) in order to the Volterra type operator G (defined on **U**) associated with the nonlinear control problem in its mild form (3)

$$(Gu(\cdot))(t) = \int_0^t S(t-\tau)Bu(\tau)d\tau \qquad (4)$$

have closed range in the space of trajectories **X**.

Sometimes however the assumption is that G_T (also defined on **U**) given by

$$G_T u(\cdot) = \int_0^T S(T-\tau)Bu(\tau)d\tau \qquad (5)$$

have closed range in the state space X. Besides, if L (defined on **X**) is the linear operator

$$L(t)x(\cdot) = \int_0^t S(t-\tau)x(\tau)d\tau$$

the range of the operator G should be large enough to incorporates the set of nonlinear values L(t)Nz(·), for all t ∈ [0,T], that is

$$\textit{Range}\ (G) \supseteq \{L(t)Nz(\cdot) : t \in [0,T]\}$$

For simple cases (e.g., **U** = $L^2(0,T; X)$ or X = $L^2(0,1)$) these adjustments were not difficult to be done [15]. However, this is not always the case. Here we present some results based on matched sets (see [3, 4, 5]) which shows that such adjustments are always possible. Actually, this is part of a more comprehensive theory (see [3, 5]) which shows that if E_1 and E_2 are inner product spaces and $\Psi: E_1 \to E_2$ is a densely defined linear operator, then the topology of E_1 and/or E_2 can always be adjusted such that some topological properties of Ψ (such as boundedness or continuity, compactness, closed range, etc.) will hold.

In other words, the spaces **U** of input control functions and/or the space of trajectories **X** can be adjusted to new spaces **U′** and **X′** (with **U** ∩ **U′** and **X** ∩ **X′** dense on their original counterparts **U** and **X** respectively) in order to some Volterra-type operators, such as G in (4), associated with the nonlinear control problem (3) have closed range. Similarly, when the state space X is infinite dimensional, the above adjustment can be done for **U** and X to obtain closed range for G_T in (5).

Also, in the problem of state estimation of infinite dimensional systems using fixed point techniques is often assumed [6, 15, 16] that the state space X and/or the state of output functions *Y* can be adjusted to new spaces X' and *Y*' (with X ∩ X' and *Y* ∩ *Y*' dense on their original counterparts X and *Y* respectively) in order to some Volterra-type operators (associated with the state estimation problem) have closed range.

For instance, let us consider the semilinear (non-stable) system

$$\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + y(t) + N(y,\frac{dy}{dt}) = u(t) \qquad (6)$$

where N is the nonlinearity such as for example

$$N(y,\dot{y}) = y^2\dot{y} \qquad (7)$$

$$N(y,\dot{y}) = \sqrt{2y\dot{y}} \qquad (8)$$

This system can easily be re-written in the state space form (1) as

$$\dot{\mathbf{x}} = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}\mathbf{x}(t) + \begin{pmatrix} 1 \\ 0 \end{pmatrix}u(t) - \begin{pmatrix} N(x_1,x_2) \\ 0 \end{pmatrix}$$

$$\mathbf{x}_0 = \begin{pmatrix} x_{01} \\ x_{02} \end{pmatrix} = \begin{pmatrix} \dot{y}(0) \\ y(0) \end{pmatrix}$$

where

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix},\ x_1 = \dot{y} \text{ and } x_2 = y$$

So, the output equation, if necessary, is

$$y(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} x(t)$$

Clearly, here the semigroup S(t) is given by

$$S(t) = e^{At} = \begin{bmatrix} (1+t)e^t & -te^t \\ te^t & (1-t)e^t \end{bmatrix}$$

and therefore, the state **x**(t) can be expressed in the form (3) by

$$\mathbf{x}(t) = \begin{pmatrix} (1+t)e^t x_{01} - te^t x_{02} \\ te^t x_{01} + (1-t)e^t x_{02} \end{pmatrix} +$$

$$+ \left(Gu(\cdot)\right)(t) + \int_0^t \begin{bmatrix} (1+t-\tau)e^{t-\tau} \\ (t-\tau)e^{t-\tau} \end{bmatrix} N(x_1,x_2)d\tau$$

where the operator G from **U** to some space of functions (trajectories) **x**(·):[0,T] → X = $\mathbf{R}^2$ is given by

$$\left(Gu\right)(t) = \int_0^t \begin{bmatrix} (1+t-\tau)e^{t-\tau} \\ (t-\tau)e^{t-\tau} \end{bmatrix} u(\tau)d\tau$$

Now let us put in the same frame the following system described by the (parabolic type) partial differential equation

$$\frac{\partial z}{\partial t} = \frac{\partial^2 z}{\partial x^2} + N(z,t) + u(x,t) \qquad (9)$$

with boundary and initial conditions

$$z(0,t) = z(1,t) = 0 \quad \text{and} \quad z(x,0) = z_0(x)$$

where N is the nonlinearity such as for example

$$N(z,t) = z^2 z_x \ \ or\ \ (2zz_x)^{1/2}, \text{ etc.} \qquad (10)$$

This system can be expressed in the form (1)

$$\dot{z} = Az(t) + N(z(t)) + Bu(t), \quad z(0) = z_0 \in \mathbf{Z}$$

with the state space **Z** = $L^2(0,1)$, the state x(t) being the functions z(•,t) ∈ **Z**, the linear operator A on **Z** defined by

$$Az = \frac{\partial^2 z}{\partial x^2}$$

B = identity on **Z** and the nonlinear operator (Nz)(t) = N(z,t). Now the semigroup S(t) is given by

$$S(t) = \sum_{n=1,2,...}^{\infty} e^{-n^2\pi^2 t}\left\langle z,\varphi_n\right\rangle\varphi_n$$

where $\langle z, \varphi_n \rangle$ and $\{\varphi_n(\cdot)\}_{n=1,2,...}$ are respectively, the inner product on **Z** and the complete orthonormal sequence of **Z** = $L^2(0,1)$ given by

$$\varphi_n(x) = \sqrt{2} \sin n\pi x \quad \text{for} \quad n = 1, 2, \ldots$$

so, the state z(t) can be expressed in the form (3) by

$$z(t) = \sum_{n=1,2,...}^{\infty} e^{-n^2\pi^2 t} \langle z_o(\cdot), \varphi_n(\cdot) \rangle \varphi_n(\cdot) + \\ + (Gu(\cdot))(t) + L(t) N z(\cdot)$$

where the operator G on **U** is given by

$$(Gu)(t) = \sum_{n=1,2,...}^{\infty} \varphi_n(x) \int_0^t \langle u(\cdot), \varphi_n(\cdot) \rangle e^{-n^2\pi^2(t-\tau)} d\tau \cdot$$

3 THE ADJUSTMENTS

Here we present some results based on matched sets [5] which shows that such adjustments are always possible. Actually this is part of a comprehensive theory [3, 4] which shows that if **U** and X are inner-product spaces and G: **U** $\rightarrow$ X is a densely defined linear operator then the topology of **U** and/or X can always be adjusted such that some topological properties of G (such as boundedness or continuity, compactness, closed range, etc.) will hold. The new adjusted spaces **U′** and **X′** will have the form

$$\mathbf{U'} = \left\{ u = \sum_{n\in\Gamma} u_n e_n : \left(\sum_{n\in\Gamma} \alpha_n |u_n|^2 \right) < \infty \right\} \qquad (11)$$

with the topology on **U'** given by the norm

$$\|u\|_{U'} = \left(\sum_{n\in\Gamma} \alpha_n |u_n|^2 \right)^{1/2}$$

and

$$\mathbf{X'} = \left\{ x = \sum_{n\in\Lambda} x_n \phi_n : \left(\sum_{n\in\Lambda} \beta_n |x_n|^2 \right) < \infty \right\} \qquad (12)$$

with the topology on **X'** given by the norm

$$\|x\|_{X'} = \left(\sum_{n\in\Lambda} \beta_n |x_n|^2 \right)^{1/2}$$

where u_n , $x_n \in$ F = **R** or **C** ; α_n β_n are real numbers satisfying $\alpha_n > 0$ for all $n\in\Gamma$, $\beta_n > 0$ for all $n\in\Lambda$, and

$$\mathbf{M} = \left(\{e_n\}_{n\in\Gamma}, \{\phi_n\}_{n\in\Lambda}, \Delta, \Gamma, \Lambda \right) \qquad (13)$$

is any complete matched set for the operator G.

3.1 Matched Sets

A matched set **M** is a quintuple of the type above (Eq. (13)), consisting of two sequences

$$\{e_n\}_{n\in\Gamma} \quad \text{and} \quad \{\phi_n\}_{n\in\Lambda}$$

and three countable sets Δ, Γ and Λ satisfying

$e_n \in$ **U** for all $n \in \Gamma$,

$\phi_n \in$ **X** for all $n \in \Lambda$,

$\Delta \subseteq \Gamma \cap \Lambda$ and Δ non empty.

Moreover, the following must also hold in order to **M** to be a matched set for the linear operator G

$Ge_n = \phi_n$ for all $n \in \Delta$, and

$Ge_n = 0$ for all $n \in \Gamma \setminus \Lambda$.

A matched set **M** is said to be complete if

$$\overline{Span\{e_n\}_{n\in\Gamma}} = \mathbf{U} \quad \text{and} \quad \overline{Span\{\phi_n\}_{n\in\Lambda}} = X .$$

where the bars represent the closure of the spaces.

Details and plenty of examples of matched sets and complete matched sets can be found in [5].

3.2 The Generation of Matched Sets

In [4] the author presents two different methods for obtaining a complete matched set **M** for a linear operator G. In the first method **M** is generated such that $\{\phi_n\}_{n\in\Lambda}$ is a complete orthonormal set in the original space **X**. In the second method **M** is generated such that $\{e_n\}_{n\in\Gamma}$ is a complete orthonormal set in the original space **U**. Note that if **M** is a complete matched set, then both **U′** and **X′** are in fact Hilbert spaces with inner-product given by

$$\langle u, v \rangle_{U'} = \sum_{n\in\Gamma} \alpha_n u_n \bar{v}_n \quad \text{and} \quad \langle x, y \rangle_{X'} = \sum_{n\in\Lambda} \beta_n x_n \bar{y}_n$$

and the sets $S_{\mathbf{U'}}$ and $S_{\mathbf{X'}}$ given by

$$S_{U'} = \left\{ \frac{e_n}{\sqrt{\alpha_n}} \right\}_{n\in\Gamma} \quad \text{and} \quad S_{X'} = \left\{ \frac{\phi_n}{\sqrt{\beta_n}} \right\}_{n\in\Lambda}$$

are complete orthonormal sets in **U′** and **X'** respectively.

4 OPERATORS ASSOCIATED TO G

Note that if we have a complete matched set **M** for the operator G, then G: **U'** $\rightarrow$ **X′** can be expressed as

$$Gu = \sum_{n\in\Delta} u_n \phi_n$$

with domain of G given by

$$D(G) = \left\{ u = \left(\sum_{n\in\Gamma} u_n e_n \right) \in \mathbf{U'} : \left(\sum_{n\in\Lambda} \beta_n |u_n|^2 \right) < \infty \right\}$$

4.1 *The Null Space and the Range of G*

Other spaces and operator related with G can be expressed in a similar way by using again the sequences $\{e_n\}_{n\in\Gamma}$ and $\{\phi_n\}_{n\in\Lambda}$ from the matched set. For example: the closures of both the Null space of G, $\overline{Null(G)}$, and Range of G, $\overline{Range(G)}$, are given respectively by

$$\overline{\boldsymbol{Null}(G)} = \left\{ u = \left(\sum_{n\in\Gamma\setminus\Delta} u_n e_n \right) : \left(\sum_{n\in\Gamma\setminus\Delta} \alpha_n |u_n|^2 \right) < \infty \right\} \quad \text{and}$$

$$\overline{\boldsymbol{Range}(G)} = \left\{ x = \left(\sum_{n\in\Delta} x_n \phi_n \right) : \left(\sum_{n\in\Delta} \beta_n |x_n|^2 \right) < \infty \right\}.$$

4.2 Projections

The orthogonal projections P: $\mathbf{U'} \to \mathbf{U'}$ onto the space *Null* (G) and P: $\mathbf{X'} \to \mathbf{X'}$ onto *Range* (G) are given respectively by

$$Pu = \sum_{n\in\Gamma\setminus\Delta} u_n e_n \quad \text{and} \quad Px = \sum_{n\in\Delta} x_n \phi_n$$

4.3 Adjoint and Pseudo-inverse Operator

The adjoint operator of G, G*: $D(G) \to \mathbf{U'}$, and the pseudo-inverse of G, G^+:$D(G^+) \to \mathbf{U'}$ are given respectively by

$$G^* x = \sum_{n\in\Delta} \frac{\beta_n}{\alpha_n} x_n e_n \quad \text{and} \quad G^+ x = \sum_{n\in\Delta} x_n e_n$$

with domains given respectively by

$$D(G^*) = \left\{ x = \left(\sum_{n\in\Lambda} x_n \phi_n \right) : \left(\sum_{n\in\Delta} \beta_n |x_n|^2 \right) < \infty \right\} \quad \text{and}$$

$$D(G^+) = \left\{ x = \left(\sum_{n\in\Lambda} x_n \phi_n \right) : \left(\sum_{n\in\Delta} \alpha_n |x_n|^2 \right) < \infty \right\}$$

If we set $\alpha_n = \|e_n\|^2_{\mathbf{U}}$ for all $n \in \Gamma$, then $\mathbf{U'} = \mathbf{U}$ (or at least $\mathbf{U'} \approx \mathbf{U}$, i.e., $\mathbf{U'}$ is topologically isomorphic to $\mathbf{U}$) and similarly, if we set $\beta_n = \|\phi_n\|^2_X$ for all $n \in \Lambda$, then $\mathbf{X'} = \mathbf{X}$ (or at least $\mathbf{X'} \approx \mathbf{X}$, i.e., $\mathbf{X'}$ is topologically isomorphic to $\mathbf{X}$). However, a different choice of α_n will change the topology of $\mathbf{U}$ to a new space $\mathbf{U'}$ (with $\mathbf{U} \cap \mathbf{U'}$ dense on $\mathbf{U}$) as well as a different choice of β_n will change the topology of $\mathbf{X}$ to a new space $\mathbf{X'}$ (with $\mathbf{X} \cap \mathbf{X'}$ dense on $\mathbf{X}$).

5 THE ADJUSTMENTS RULES

The following results shown in [3, 5] establish the relationship between α_n and β_n such that the operator G:$\mathbf{U'} \to \mathbf{X'}$ hold some desired topological properties:

5.1 Boundness

G: $\mathbf{U'} \to \mathbf{X'}$ is a bounded operator if and only if the set

$$\left\{ \frac{\beta_n}{\alpha_n} \right\}_{n\in\Delta} \quad \text{is bounded} \tag{14}$$

5.2 Closed Range

G: $\mathbf{U'} \to \mathbf{X'}$ has closed range in $\mathbf{X'}$ if and only if the set

$$\left\{ \frac{\alpha_n}{\beta_n} \right\}_{n\in\Delta} \quad \text{is bounded.} \tag{15}$$

5.3 Compactness

G: $\mathbf{U'} \to \mathbf{X'}$ is a Hilbert-Schmidt operator if and only if

$$\sum_{n\in\Delta} \left(\frac{\beta_n}{\alpha_n} \right) < \infty \tag{16}$$

Clearly if G is a Hilbert-Schmidt operator then G is a compact (or completely continuous) operator. The above results on matched sets provide the rules for the adjustment of the spaces $\mathbf{U}$ and $\mathbf{X}$.

6 EXAMPLE *(Continued)*

First we consider the nonlinear system (6) and assume that the space $\mathbf{U}$ of input functions is $L^2(0,T)$ for some T > 0. Here the *Range* (G_T) is always closed since it has finite dimension ($X = \mathbf{R}^2$). Even though the *Range* (G) is a space of functions from [0,T] to X and its topology can be adjusted in order to be closed.

We can let $\{e_n(\cdot)\}_{n\in\Gamma}$ be any complete orthonormal set in $L^2(0,T)$, such as for example

$$\Gamma = \mathsf{N} = \{1, 2, \ldots\}$$

$$e_n(t) = \sqrt{\frac{2}{T}} \sin\left(\frac{n\pi}{T}\right) t, \; t \in [0, T], \; n = 1, 2, \ldots$$

and the space of input controls $\mathbf{U'}$ as defined in (11). Now, if we set $\alpha_n = 1$, for n = 1, 2,..., the space of input controls $\mathbf{U}$ becomes $\mathbf{U'}$ defined in (11). With this choice we have $\mathbf{U'} \approx \mathbf{U} = L^2(0,T)$.

However, by a different choice of α_n 's we could give $\mathbf{U'}$ a different topology in order to be either larger or smoother than $L^2(0,T)$. If $\alpha_n \geq 1$ for all n = 1, 2,..., (or for all $n > n_o$, for some finite $n_o \in \mathsf{N}$), then $\mathbf{U'}$ will be a space of smoother functions. For example, if

$$\alpha_n = \frac{n^2 \pi^2}{T^2}, \qquad n = 1, 2, \ldots$$

then $\mathbf{U}'\approx H_o^1(0,T)$ the Sobolev space of differentiable functions on [0,T].

On the other hand, by setting $\alpha_n \le 1$ for all n = 1, 2,..., (or for all $n > n_o$, $n \in$ N, for some finite $n_o \in$ N), then $\mathbf{U}'$ will be a larger space of functions than $L^2(0,T)$. For example, if

$$\alpha_n = \frac{T^2}{n^2\pi^2}, \qquad n = 1,2,...$$

then $\mathbf{U}'\approx H^{-1}(0,T)$ the Sobolev space (distributions).

Now set $\Delta = \Lambda = \Gamma = N = \{1, 2, ...\}$, $\phi_n(\cdot)$ as

$$\phi_n(t) = (Ge_n(\cdot))(t) = \begin{pmatrix}\phi_{n1}(t)\\ \phi_{n2}(t)\end{pmatrix} \in R^2$$

where

$$\begin{cases}\phi_{n1}(t) = \int_0^t (1+t-\tau)e^{t-\tau}\sqrt{\frac{2}{T}}\sin\frac{n\pi}{T}\tau d\tau \\ \phi_{n2}(t) = \int_0^t (t-\tau)e^{t-\tau}\sqrt{\frac{2}{T}}\sin\frac{n\pi}{T}\tau d\tau\end{cases}$$

and the space of trajectories $\mathbf{X}'$ as defined in (12).

To give $\mathbf{X}$ the same topology as a known space such as, for example, $L^2(0,T;\mathbf{R}^2)$, (that is, $\mathbf{X}' \approx L^2(0,T;\mathbf{R}^2)$) we then have to set

$$\beta_n = \|\phi_n\|_{L^2(0,T;R^2)} \quad \text{for} \quad n = 1,2,... \qquad (17)$$

However, some nonlinearities such as in the one in (7) may force us to work in larger spaces and therefore we have to choose different values for the constants β_n's. They will have to be smaller than the ones chosen in Eq (17) above. In that case, in order to have the Range of the operator G closed (i.e., *Range* (G) closed), we will also have to choose different α_n's. Actually, from Eq (15) we know that α_n , for n=1, 2, ... must be chosen such that the set

$$\{\alpha_n/\beta_n\}_{n=1,2,...}$$

is bounded. For instance, $\alpha_n = \beta_n$, for $n = 1,2,...$

Some nonlinearities however, such the one in (8), allow us to work on smoother state spaces X. In this case we can also give a smoother topology to both X' and $\mathbf{U}'$ by an adequate choice of the β_n's and the α_n's, respectively. Now the constants β_n's will have to be greater than the ones chosen in Eq (17) above.

Alternatively, for system (9) the *Range* (G_T) will not always be closed. That will depend on the topology defined for the spaces $\mathbf{U}$ and/or $\mathbf{Z}$.

Here we can assume that $\mathbf{U} = L^2(0,T; \mathbf{Z})$ for $\mathbf{Z} = L^2(0,T)$ as before. By setting $\{e_n(\cdot)\}_{n \in \Gamma}$ any complete orthonormal set of $\mathbf{U}$, then the choice of $\alpha_n = 1, \quad n \in \Gamma$ will give $\mathbf{U}' \approx \mathbf{U}$. Greater values of α_n's will give smoother spaces of functions $\mathbf{U}'$ and smaller values of α_n's will produce larger spaces of functions $\mathbf{U}'$.

For the state space $\mathbf{Z}$ let , $\{\phi_n(\cdot)\}_{n \in \Lambda}$ be

$$\phi_n(\cdot) = G_T e_n(\cdot) = = \sum_{n=1,2,...}^{\infty} \varphi_n(x)\int_0^T \left\langle e_n(\cdot,T), \varphi_n(\cdot)\right\rangle e^{-n^2\pi^2(T-\tau)}d\tau$$

Now, setting

$$\beta_n = \|\phi_n\|_{L^2(0,T)} \text{for} n = \Lambda$$

then the space $\mathbf{Z}'$ defined as in (12) will have the same topology of the state space $\mathbf{Z} = L^2(0,T)$. Again here, greater or smaller values of β_n's will give us either smoother or larger (respectively) spaces of functions $\mathbf{Z}'$. Also, in order to *the operator* G_T have closed range we must choose α_n for $n \in \Gamma$ satisfying Eq (15). That is, the set

$$\{\alpha_n/\beta_n\}_{n=\Delta} \text{for } \Delta = \Gamma \cap \Lambda$$

must be bounded.

7 Conclusion

Here we presented results that provide the rules for the topological adjustment of the spaces $\mathbf{U}$ (the control space) and $\mathbf{X}$ (the space of trajectories) and also for the adjustment of the spaces $\mathbf{U}$ and X (the state space). If G: $\mathbf{U} \to \mathbf{X}$ does not satisfy a desired topological property (e.g., boundedness, closed range or compactness) then, by adjusting the spaces $\mathbf{U}$ and $\mathbf{X}$ to new spaces $\mathbf{U}'$ and $\mathbf{X}'$ (i.e., by choosing appropriate numbers α_n's and β_n's), G: $\mathbf{U}' \to \mathbf{X}'$ will satisfy the desired topological property. Similarly, if G_T: $\mathbf{U} \to$ X does not satisfy a desired topological property then, by adjusting the spaces $\mathbf{U}$ and X to new spaces $\mathbf{U}'$ and X' , G_T: $\mathbf{U}' \to$ X' will satisfy the desired property. It is also easy to see (by using Eqs (14), (15) and (16)) that this adjustment (i.e., this choice of α_n and β_n) can always be such that

- only the topology of $\mathbf{U}$ is altered (to $\mathbf{U}'$), or
- only the topology of $\mathbf{X}$ is altered (to $\mathbf{X}'$), or
- both topologies of $\mathbf{U}$ and $\mathbf{X}$ are altered (to $\mathbf{U}'$ and $\mathbf{X}'$ respectively).

These adjustment are necessary for us to be able to use some techniques developed to solve the nonlinear control problem (2) using the fixed point of a map

$$\Phi: \mathbf{X} \to \mathbf{X}.$$

So, this structure, using matched sets for the operator G: **U** → **X** (or for G_T: **U** → X), enables us to choose **U′** and **X′** (or **U′** and X') according to the desired topological properties for G: **U′** → **X′** (or also G_T: **U′** → X') and the flexibility of the problem to let both/either **U** and/or **X** (or both/either **U** and/or X) to be altered.

In general, large space of functions (such as **U** = H^{-1}) are not desirable since it may contain distributions. Even spaces **U** = L^2 may sometimes be unsuitable for applications since it contains discontinuous functions. The above adjustments allow us to select spaces **U′** and X such that either the operator G or G_T have closed range and **U′** ≈ to some smooth space of functions (such as H^1 or H^1_o).

Loosely speaking, the smoother we want **U** to be, the smoother **X** will have to be, that is, we shall have to restrict to smoother trajectories.

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