NEURAL CONTROL OF NONLINEAR SYSTEMS: A REFERENCE GOVERNOR APPROACH

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ABSTRACT

This work presents the design and implementation of a controller for nonlinear, unstable and constrained systems. For instance, a magnetic levitation system is selected to highlights the controller properties especially with respect to stability and constraints satisfaction. The control action is based on the reference governor (RG) approach that uses a Lyapunov's concepts of energy to prevent constraints violation both on the state and on the manipulated variable even during large changes in the reference signal. To design the RG an inner loop controller is proposed. The RG receives the system states and the desired reference to compute and supply the reference signal to the inner loop controller. This procedure guarantees the stability while avoiding constraints violation. The paper describes in full detail the inner loop controller and RG design, but in spite of the successful results, the RG approach is not able to treat uncertainties. Here the authors propose to replace the whole linearization and control action for a single Artificial Neural Network (ANN). The ANN is trained using the system states and desired reference as input and the nominal control action as the output. Training data is generated thought simulation based on the action of the designed inner loop controller with the RG. The major objective in the use of the ANN is may also be able to treat uncertainties and allows straightforward implementation of training techniques to further provide adaptation capabilities.

KEY WORDS

Constrained control, Nonlinear control, Neural control

1 INTRODUCTION

The control of nonlinear systems has attracted widespread attention in the recent years (e.g. [8], [9], [12], [14], [15], [16]). Classical linearization methods works well when the model is accurate, the reference signals are well conditioned and have low amplitude. However, this may not be the case for inputs such as large steps or when uncertainties are present.

The problem of the larger input steps can be treated using the reference governor (RG) approach [1] - [7], [10], [11], [13], which guarantees the constraint satisfaction for a general class of input commands while stability is assured by a Lyapunov based design technique. But the problem of uncertainties may still be critical.

This paper proposes to replace the inner loop controller as well as the RG by a single ANN, which is trained with data pairs generated from the simulation of the previous controller actions. Thereafter the neural controller is able to on-line adaptation by means of learning techniques and may be robust against a class of uncertainties.

Section 2 presents the inner loop and RG design theory. In Section 3 the studied plant is presented. Section 4 is devoted to apply the controller theory to the magnetic levitation system and the desired control action is shown. Section 5 presents the proposed neural control and its simulation results. The conclusions are presented in Section 6.

2 Controller design theory

Let the nonlinear system in the form:

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= h(x) \end{aligned} \quad (1)$$

Using the exact feedback linearization theory [9], [14], [15], [16], if Equation (1) can be written as

$$\dot{x} = Ax + B\beta^{-1}(x)\left[u - \alpha(x)\right] \quad (2)$$

where

- A and B are $n \times n$ matrix
- The pair (A,B) is controlable
- α: $\Re^n \rightarrow \Re^p$ is defined in a domain $D_x \subset \Re^n$
- β: $\Re^n \rightarrow \Re^{p \times p}$ is defined in a domain $D_x \subset \Re^n$
- β is nonsingular $\forall x \in D_x$ and β^{-1} is its inverse

then one can use the following control law

$$u = \alpha(x) + \beta(x)v \quad (3)$$

Using Equations (2) and (3) we can obtain the linear expression of the form

$$\dot{x} = Ax + Bv \quad (4)$$

For the linear system we simply use a state feedback for obtaining pole placement. The block diagram of the controller scheme is shown in Figure 1.

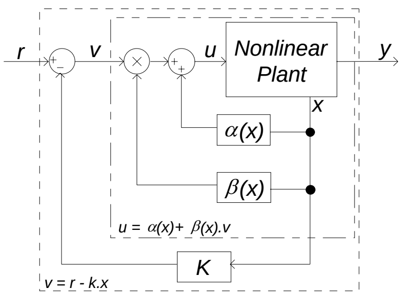

Figure 1: Controller scheme

3 A magnetic levitation system

Consider a magnetic levitation system as shown in Figure 2.

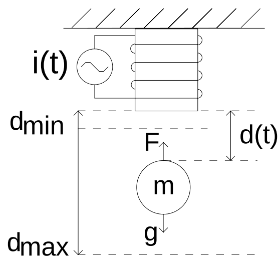

Figure 2: Magnetic levitation system

This constrained nonlinear system can be described by

$$\ddot{d} = g - k\frac{i^2}{md^2} \quad (5)$$

where m is the mass of the ball, $k > 0$ is a constant parameter, g is the gravitational acceleration and

$$F = k\frac{i^2}{d^2} \quad (6)$$

is the electromagnet force controlled by $i(t)$

Note that the system is clearly open-loop unstable. Moreover, classical controllers that do not take into account the constraints d_{max}, d_{min} and the saturation current i_{max} may fail in the sense of letting the ball fall or having the ball magnetically attached to the base of the coil.

If we define $x_1 = d$, $x_2 = \dot{d}$ and $u = i^2$, then the system can be rewritten as

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = g - k\dfrac{u}{mx_1^2} \\ y = h(x) = [1\ \ 0]x \end{cases} \quad (7)$$

And the objective is to design the controller in order to guarantee the stability while avoiding constraints violation.

4 Practical controller design

4.1 Input feedback linearization

An inner loop controller should be responsible to provide local stability, without considering the constraints. Here we use the exact feedback linearization theory in order to obtain an adequate local performance. To linearize the system using the proposed method, we must take the derivatives of the output y until we find the input signal, i.e.:

$$\begin{aligned} \dot{y} &= \dot{x}_1 = x_2 \\ \ddot{y} &= \dot{x}_2 = g - k\frac{u}{mx_1^2} \end{aligned} \quad (8)$$

Let $\ddot{y} = v$ thus,

$$u = \frac{m x_1^2}{k}(g - v) \qquad (9)$$

Replacing u in Equation (9) in Equation (7) one obtains

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= v \end{aligned} \qquad (10)$$

which is a linear system. Comparing Equations (9) and (3) we obtain

$$\alpha(x) = \frac{m x_1^2}{k} g \quad \text{and} \quad \beta(x) = -\frac{m x_1^2}{k} \qquad (11)$$

4.2 State feedback

With the linear system, use a state feedback to pole placement. Let

$$k = \begin{bmatrix} \frac{1}{A} & \varsigma \end{bmatrix} \quad \text{and} \quad v = r - kx \qquad (12)$$

then, Equation (10) becomes

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = r - \dfrac{x_1}{A} - \varsigma x_2 \end{cases} \qquad (13)$$

and it can be rewritten as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\dfrac{1}{A} & -\varsigma \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r \qquad (14)$$

which is clearly linear and stable for A and $\varsigma > 0$.

From Equation (9) one obtains the final control law of the form

$$u = \frac{m x_1^2}{k}\left[g - r(t) + \frac{x_1}{A} + \varsigma x_2 \right] \qquad (15)$$

In order to avoid the steady state error, let $A=1$ and for the sake of the simplicity let also $\varsigma=1$. Note that $A=1$ is related to null steady state error with respect to the reference signal $r(t)$, ς is related to the damping coefficient while the pair A, $\varsigma>0$ guarantee the stability for the closed loop system.

4.3 The reference governor design

The controller described in the last two subsections works well when the constraints are not violated. However, when the constraints are violated the controller may fail. The main idea of the RG is to guarantee the constraint satisfaction while the local controller assures stability. It is assumed that there is a previous controller designed so as to provide an adequate performance around the defined set points but without considering the constraints. To avoid the constraint violation, the input to this closed loop system should be supplied through the RG. The RG receives the desired reference signal $r_d(t)$ and the state variables and then tracks the reference input $r(t)$ as close as possible to the desired $r_d(t)$ but subject to constraint satisfaction while guarantee that $r(t) \rightarrow r_d(t)$ when $t \rightarrow \infty$. Figure 3 illustrates the RG scheme.

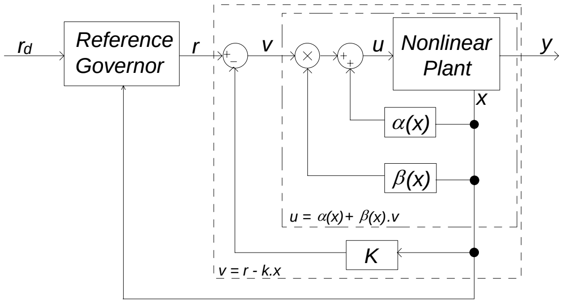

Figure 3: Reference governor scheme

The first step in the design of the RG is the selection of a function in the sense of energy. Thus, let

$$V_r = (r - x_1)^2 + x_2^2 \geq 0 \qquad (16)$$

and note that

$$\dot{V}_r = -2\varsigma\, x_2^2 \leq 0\,. \qquad (17)$$

The constraints are represented as

$$\begin{aligned} q_{max}(r) &= \min V_r(x) \quad \text{s.t.} \quad d(t) - d_{max} = 0 \\ q_{min}(r) &= \min V_r(x) \quad \text{s.t.} \quad d(t) - d_{min} = 0 \\ & i_{min} \leq i(t) \leq i_{max} \end{aligned} \qquad (18)$$

Therefore, for the proposed case, one gets

$$\begin{aligned} q_{max}(r) &= (r - d_{max})^2 \\ q_{min}(r) &= (r - d_{min})^2 \end{aligned} \qquad (19)$$

The violation of the constraints can now be detected by defining

$$\begin{aligned} C_{max} &= V_r(x) - q_{max} \\ &= (r - x_1)^2 + x_2^2 - (r - d_{\max})^2 \\ C_{min} &= V_r(x) - q_{min} \\ &= (r - x_1)^2 + x_2^2 - (r - d_{\min})^2 \end{aligned} \qquad (20)$$

And using the following criteria:

a) For positive step inputs $r(t) \rightarrow d_{max}$, the constraint is violated if $C_{max} < 0$, i.e.

$$r(t) > r_{\max} = \frac{d_{\max}^2 - x_1^2 - x_2^2}{2(d_{\max} - x_1)} \qquad (21)$$

b) For negative step inputs $r(t) \rightarrow d_{min}$, the constraint is violated if $C_{min} < 0$, i.e.

$$r(t) < r_{\min} = \frac{d_{\min}^2 - x_1^2 - x_2^2}{2(d_{\min} - x_1)} \quad (22)$$

Therefore, one can use the following control law to avoid constraints violation:

a) For positive step inputs, $r(t) = \min\,[r_d(t), r_{max}]$
b) For positive step inputs, $r(t) = \max\,[r_d(t), r_{min}]$
c) Compute Equation (15) and apply $i(t)$.

4.4 Current saturation analysis

Consider now the current $i(t)$ in the case of the initial condition with null speed $x_2 = 0$, thus Equations (21) and (22) becomes:

a) For positive step inputs $r(t) \to d_{max}$, the constraint is violated if $C_{max} < 0$, i.e.

$$r(t) > r_{\max} = \frac{d_{\max} + x_1^2}{2} \quad (23)$$

b) For negative step inputs $r(t) \to d_{min}$, the constraint is violated if $C_{min} < 0$, i.e.

$$r(t) < r_{\min} = \frac{d_{\min} + x_1^2}{2} \quad (24)$$

4.4.1 $C_{\max}$ violation

With $x_2 = 0$, which is the worst case with respect to the maximum positive amplitude of the input step (i.e. initial $x_1 = d_{min}$ and $r_d = d_{max}$), C_{max} is violated if

$$r(t) > \frac{d_{\max} + d_{\min}}{2} \quad (25)$$

and then the Equation (15) becomes:

$$u_{c\max} = \frac{md_{\min}^2}{k}\left[g - \frac{d_{\max} - d_{\min}}{2}\right] \quad (26)$$

Considering $r(t) = d_{max}$, at the equilibrium with $x_1 = d_{max}$, the required control effort is

$$u_{d\max} = \frac{md_{\max}^2}{k} g \quad (27)$$

and

$$u_{c\max} \le u_{d\max} \quad \forall t \quad (28)$$

Therefore, the required current can be always supplied without saturation.

4.4.2 $C_{\min}$ violation

With $x_2 = 0$, which is the worst case with respect to the maximum negative amplitude of the input step (i.e. initial $x_1 = d_{max}$ and $r_d = d_{min}$), C_{min} is violated if

$$r(t) < \frac{d_{\max} + d_{\min}}{2} \quad (29)$$

and then the Equation (15) becomes:

$$u_{c\min} = \frac{md_{\max}^2}{k}\left[g + \frac{d_{\max} - d_{\min}}{2}\right] \quad (30)$$

Considering $r(t) = d_{max}$, at the equilibrium with $x_1 = d_{min}$, the required control effort is

$$u_{d\min} = \frac{md_{\min}^2}{k} g \quad (31)$$

and

$$u_{c\min} \ge u_{d\min} \quad \forall t \quad (32)$$

Therefore, the required current can be always supplied without saturation.

4.5 Simulation results

For the magnetic levitation system, let consider the following parameters

$$\begin{cases} m = 0.05; \quad g = 9.8; \quad k = 1 \\ [d_{\min}, d_{\max}] = [1,3] \\ \text{Initial conditions } [\mathrm{x}_1, \mathrm{x}_2]_\mathrm{i} = [1.1\,,\,0] \\ \text{Desired references} = [1.1,\ 2.9] \end{cases} \quad (33)$$

The inner loop controller proposed in Equation (15) guarantees the tracking of the reference signal but it does not take into account the constraints satisfaction. Figure 4 shows the desired reference $r_d(t)$ and the evolution of $r(t)$ which guarantee that the constraints are satisfyed as shown in Figure 5. These input/output data pairs represents the desired control action and they are used to train the proposed Artificial Neural Network (ANN).

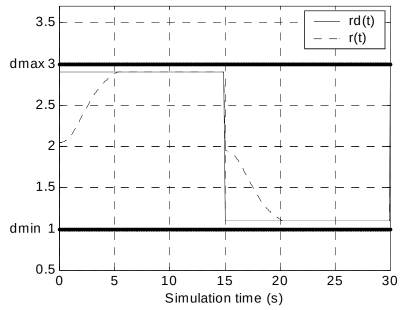

Figure 4: Reference evolution

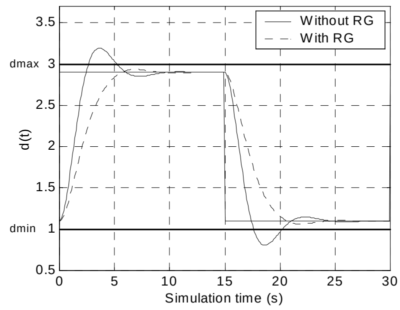

Figure 5: Simulation results

5 Neural control

The proposed ANN must replace the whole control action shown in Figure 3 for a single ANN as shown in Figure 6.

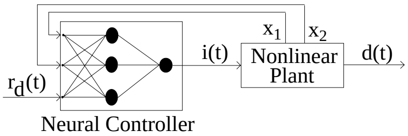

Figure 6: Neural controller

Looking for robust operation in closed-loop systems, before defining and training the neural structure, it is interesting to analyze the states x_1 and x_2 for different r_d and generate a larger group of training data pairs.

Figures 7 and 8 show the simulation results using the nominal inner controller and the nominal RG that characterize the desired solution to be learned for the neural controller. In the first figure, r_d changes from r_{dmin} to r_{dmax} and back to r_{dmin}, where

$$r_{dmax} = d_{max} \quad \text{and} \quad r_{dmin} = [1, 1.4, 1.8, 2.2, 2.6]$$

for simulations s1, s2, ... , s5 respectively.

In the second one, r_d changes from r_{dmin} to r_{dmax} and back to r_{dmin}, where r_{dmax} = [1.4, 1.8, 2.2, 2.6, 3] and r_{dmin} = 1, for simulation s6, s7, ... , s10 respectively.

For both Figures 7 and 8 the larger dots indicate the situations when the RG action was necessary to avoid constraints violation.

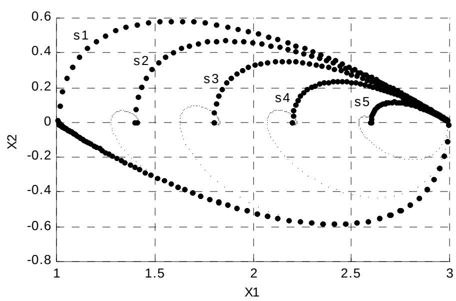

Figure 7: Desired solution

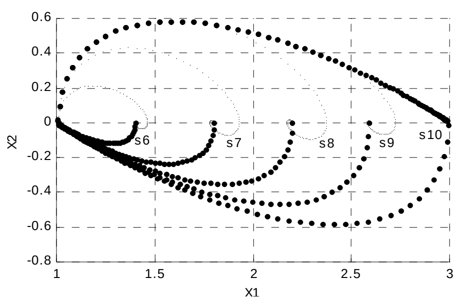

Figure 8: Desired solution

5.1 Neural structure and training

Practical simulation made us to select an ANN with one hidden layer and 6 hidden neurons (*tanh*) and a linear output layer. The ANN is trained under MATLAB simulations using the Levenberg Marquardt procedure.

The trajectories from Figure 7 were used. Each trajectory represents *30s* of simulation and the data pairs to ANN training are obtained with a sample time of *0.1s*. The weights and bias are initialized randomly and the ANN is trained for 500 epochs. The following tables present the ANN parameters after being trainned.

$$w_1 = \begin{bmatrix} -0.2552 & 0.0686 & 0.1745 \\ -2.1603 & 0.2149 & 0.9943 \\ 1.1574 & -0.6917 & 1.9330 \\ 4.0985 & -3.2302 & 2.2799 \\ 0.1465 & 0.0102 & 0.0199 \\ -2.0512 & 2.0226 & 1.3998 \end{bmatrix}_{6\times 3}$$

$$b_1 = \begin{bmatrix} 1.9078 \\ 8.6187 \\ 1.2933 \\ -0.3249 \\ -0.7607 \\ -2.3018 \end{bmatrix} \qquad w_2 = \begin{bmatrix} 1.5853 \\ -0.0207 \\ 0.0488 \\ 0.0066 \\ 7.1439 \\ -0.0224 \end{bmatrix}$$

$$b_2 = 3.0484$$

Finally, Figure 9 presents the simulation results with the conventional RG compared with the neural controller results.

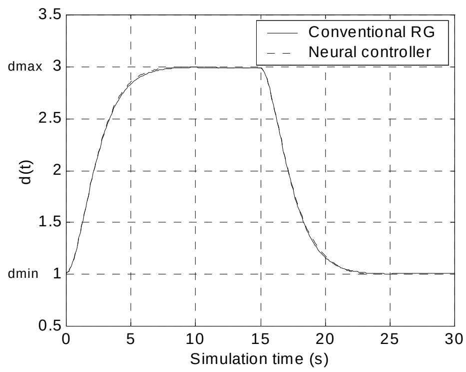

Figure 9: Neural controller results

6 CONCLUSIONS

The control of constrained nonlinear systems may be tackled by the RG approach, which use two distinct blocks to compute separately the inner-loop controller and the RG control actions. Here, a single ANN controller is proposed to replace both and successful results are verified with a large number of simulations.

The robustness of the ANN in closed-loop is verified only when the training data pairs consider different trajectories as shown in Figures 7 and 8. The successful results shows that for a nominal cases a single ANN is able to replace both the designed inner-loop controller and the RG. If real cases are considered and uncertainties cannot be neglected, then with the use of the ANN we may also be able to treat uncertainties by providing adaptation capabilities.

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