STATE FEEDBACK FOR CONTROL OF NONLINEAR SYSTEMS

L. Schnitman
Instituto Tecnológico de Aeronáutica
12.228-900 - S.J. dos Campos, SP
Brazil
leizer@ele.ita.cta.br

J.A.M. Felippe de Souza
Universidade da Beira Interior
6201-001 Covilhã
Portugal
felippe@demnet.ubi.pt

T. Yoneyama
Instituto Tecnológico de Aeronáutica
12.228-900 - S.J. dos Campos, SP
Brazil
takashi@ele.ita.cta.br

ABSTRACT

This paper presents a new approach for a control law commonly used for the control of nonlinear systems of the form $\dot{x} = f(x) + g(x)u$. An usual control law is based on the n[th] time-derivative of the reference signal. However, the implementation of the n[th] time-derivative of the error is also required and it may generates numerical problems. This paper shows that a simple linear state feedback can replace the original control law. It performs the same job and obtains equal results while avoid numerical problems.

KEY WORDS

Nonlinear control, State feedback

1 INTRODUCTION

Important bibliographical references (e.g. [4], [5], [6], [7], [8], [14]) reflect the widespread attention that the control of nonlinear system has been receiving in the recent years. The objective of this paper is present a new approach for a control law for nonlinear systems of form:

$$\dot{x} = f(x) + g(x)u \tag{1}$$

Basing on the exact feedback linearization (e.g. [5], [7], [8], [14]) the usual idea is try to use the control signal u to linearize the system, which allows us to easily analyze the dynamics and the stability of the nonlinear system. Thus, in order to define the dynamics of the nonlinear system, an external reference is introduced and a common and general control law (e.g. [1], [2], [3], [9], [10], [11], [12], [13] and [15]) has the form

$$u = \frac{1}{g(x)}\left[- f(x) + y_r^{(n)} + k^T e\right] \tag{2}$$

where $y_r^{(n)}$ is the n[th] time-derivative of the reference signal y_r, k is a gain vector and e is the augmented error vector, as will be presented in the next sections. The aim of the control is to reduce the tracking error of the output y with respect to the reference signal y_r.

This paper reorganizes the control law (2) and presents it as a state feedback. Moreover, we consider that the functions *f(x)* and *g(x)* are known.

Section 2 presents the general structure of the nonlinear system and the usual control law. In section 3 a new approach to the implementation of the usual control law (2) is suggested and the new proposition for the control law is defined. For simulation results, a selected nonlinear system and its numerical results are shown in section 4. Finally, in Section 5 we present some conclusions.

2 Nonlinear systems

Let a nonlinear system be described by a state equations of form:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \vdots \\ \dot{x}_n = f(x) + g(x)u \end{cases} \qquad (3)$$

$$y = Cx_1$$

where

$$C = [1, 0, \dots, 0]_{1\times n} \qquad (4)$$

and $f: \Re^n \rightarrow \Re$ and $g: \Re^n \rightarrow \Re$ are continuous and known functions. The aim of the control is to track a reference signal $y_r \in C^n([0,\infty), \Re)$, i.e. such that the n^{th} time-derivative of the reference signal y_r is continuous and $\in L_\infty$.

Defining the instantaneous output error

$$E = y_r - y \qquad (5)$$

one can define the augmented error vector

$$e = \left[E, \dot{E}, \dots, E^{(n-1)}\right]^T \in \Re^n \qquad (6)$$

Remark 1: From (3), note that

$$\begin{cases} y = x_1 \\ \dot{y} = \dot{x}_1 = x_2 \\ \vdots \\ y^{(n-1)} = x_n \\ y^{(n)} = \dot{x}_n = f(x) + g(x)u \end{cases} \qquad (7)$$

Remark 2: Also from Eq. (5), note that

$$\begin{cases} E = y_r - x_1 \\ \dot{E} = \dot{y}_r - \dot{x}_1 = \dot{y}_r - x_2 \\ \vdots \\ E^{(n-1)} = y_r^{(n-1)} - x_1^{(n-1)} = y_r^{(n-1)} - x_n \\ E^{(n)} = y_r^{(n)} - \dot{x}_n \end{cases} \qquad (8)$$

Thus, let k be a gain vector of the form

$$k = [k_1, \dots, k_n]^T \in \Re^n \qquad (9)$$

such that the polynomial $h(s) = s^{(n)} + k_n s^{(n-1)} + \dots + k_1$ is Hurwitz. Therefore, if one uses the control law of the form

$$u = \frac{1}{g(x)}\left[-f(x) + y_r^n + k^T e\right] \qquad (10)$$

by replaceing (10) in (3), one gets

$$\dot{x}_n = y_r^{(n)} + k^T e \qquad (11)$$

And replacing $y^{(n)} = \dot{x}_n$ from (7) in the equation (11), one gets

$$E^{(n)} + k_n E^{(n-1)} + \dots + k_1 E = 0 \qquad (12)$$

which describes the dynamics of the error and implies that

$$\lim_{t\to\infty} E(t) = 0 \qquad (13)$$

that is the proposed aim of the control.

3 The control law

Using Simulink (@Matlab) construct the blocks diagram as shown in Figure 1 which represents the scheme for the control law (10) implementation.

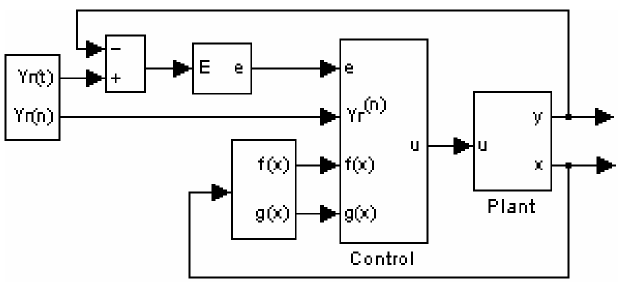

Figure 1: Scheme of the control law implementation

It is important to note that numerical problems may occur when computing the time-derivative of the error E or of the reference signal y_r.

3.1 Avoiding numerical problems

In order to avoid numerical problems when implementing the control law (10), we generate the reference signal as shown in Figure 2.

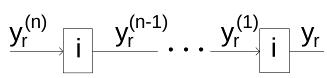

Figure 2: Reference signal generation

Thus, the reference signal and its j^{th} time-derivatives are available, for $j=1...n$, but note that the time-derivatives of the error are not.

From Equation (10), we separate the expression:

$$y_r^{(n)} + k^T e \tag{14}$$

and from Equation (8) we can rewrite:

$$y_r^{(n)} + k^T e = y_r^{(n)} + k^T \left(\begin{bmatrix} y_r \\ \dot{y}_r \\ \vdots \\ y_r^{(n-1)} \end{bmatrix} - \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \right) \tag{15}$$

Considering that the model which generates the reference signal is stable, then

$$y_r^{(n)} + k_n y_r^{(n-1)} + \ldots k_1 y_r = 0 \tag{16}$$

Therefore

$$y_r^n + k^T e = -k^T x \tag{17}$$

and the control law of Eq. (10) becomes:

$$u = \frac{1}{g(x)}\left[-f(x) - k^T x \right] \tag{18}$$

3.2 Important agreement

The proposed control law in Eq. (18) is very interesting because it uses a states feedback. However, an important remark is necessary in order to reproduce the control system when using the original control law of Eq. (10).

The new control law works very well when there is no constant value in the reference signal y_r, for example, let $y_r = sin(t)$. On the other hand, when we consider that the system has a bias r in such a way that r is constant (i.e. its time-derivatives is *zero*, then $y_r(t) = sin(t) + r$ and r must be introduced because the $y_r^{(n)}$ integration is not able to generate it. See Figure 3.

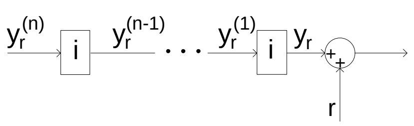

Figure 3: Reference signal with r(t)

Thus, the Equation (15) becomes:

$$y_r^{(n)} + k^T e = y_r^{(n)} + k^T \left(\begin{bmatrix} y_r + r \\ \dot{y}_r \\ \vdots \\ y_r^{(n-1)} \end{bmatrix} - \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \right) \tag{19}$$

Considering the stability of the reference model (16) one gets:

$$y_r^n + k^T e = k_1 r - k^T x \tag{20}$$

Therefore, substituting Equation (19) in equation (10) we obtain an equivalent control law of form:

$$u = \frac{1}{g(x)}\left[-f(x) + k_1 r - k^T x \right] \tag{21}$$

And its implementation is shown in Figure 4.

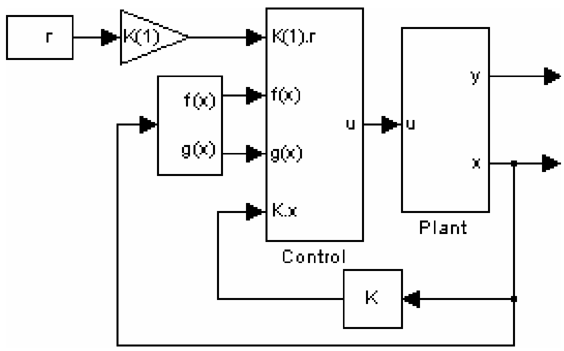

Figure 4: Scheme of the proposed control law implementation

4 Practical example

4.1 Selected plant

As a practical example, consider the open loop unstable magnetic levitation system as shown in Figure 5

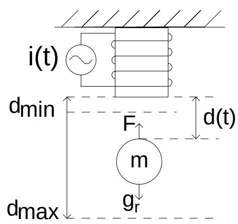

Figure 5: Magnetic levitation system

Its dynamics is described by

$$\ddot{d} = g_r - \frac{F}{m} \tag{22}$$

where m is the mass of the ball, g_r is the gravitational acceleration and F is the electromagnet force produced by a coil fed with current i.

$$F = c\frac{i^2}{d^2} \tag{23}$$

where c is a positive constant and d is the distance.

Let $x_1 = d(t)$ and $x_2 = \dot{d}(t)$, the state representation then becomes:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = g_r - c\dfrac{i^2}{mx_1^2} \end{cases} \tag{24}$$

so that comparison to Equation (3) yields:

$$\begin{cases} f(x) = g_r \\ g(x) = -c\dfrac{1}{mx_1^2} \\ u = i^2 \end{cases} \tag{25}$$

Remark 3: Here we are not considering yet the constraints d_{min} and d_{max} as well as the current saturation.

4.2 Implementation

Using the Simulink (@Matlab), construct the blocks diagram as shown in Figure 4.

For the plant parameters, we set $g_r = 9.8$, $m = 0.05$ and $c = 1$. For initial conditions we set $x_1 = 1.1$ and $x_2 = 0$.

Also, for the gain vector k we choose $k = [20, 1000]^T$.

The functions $f(x)$ and $g(x)$ can be computed using Equation (25).

As a first example consider $r(t)$ to be constant. For instance, let $r(t) = 2$. Figure 6 shows the simulation result.

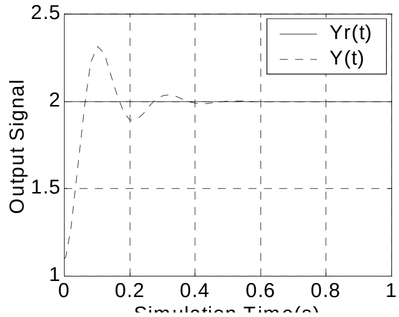

Figure 6: Simulation with constant *r*

As a second example consider $r(t)$ varying in time. In order to keep the theoretical requirements, also consider that $r(t)$ is very slow with respect to the system dynamics. Thus, let $r(t) = sin(t) + 2$. Simulation result is shown in Figure 7.

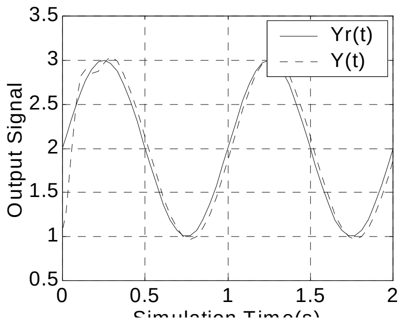

Figure 7: Simulation with *r(t)* varying in time

Moreover, the obtained results as presented in Figures 6 and 7 are the same when using the control law as the scheme proposed in Figure 1.

5 CONCLUSIONS

This paper proposes a reorganization of the control law that was usually used in previous works (e.g. [1], [2], [3], [9], [10], [11], [12], [13] and [15]) in order to avoid numerical problems when computing the time-derivatives in real-time.

The obtained results shows that the control law can be rewritten as state feedback. It becomes clear to understand and also easier implementations are reached. The most significant results are the use of linear control techniques, such as a state feedback, for the control of nonlinear systems.

ACKNOWLEDGEMENT

The authors are indebted to FAPESP (Fundação de Amparo à Pesquisa do Estado de São Paulo, Brazil), for support under grant 98/16074-1.

REFERENCES

[1] B.-S. Chen, C.-H. Lee, and Y.-C. Chang, Hinf tracking design of uncertain nonlinear SISO systems: Adaptive fuzzy approach. *IEEE Trans. on Fuzzy Systems*, 4 (1996), pp. 32-43.

[2] K. Fischle and D. Schroder, An improved stable adaptive fuzzy control method. *IEEE Trans. on Fuzzy Systems*, 7 (1999), pp. 27-40.

[3] V. Gazi and K.M. Passino, Direct adaptive control using dynamic structure fuzzy systems. *Proc. of the American Control Conference*, 3 (2000), pp. 1954-1958.

[4] F. Hoppensteadt, *Analysis and simulation of chaotic systems*. Springer verlag, 1993.

[5] H. K. Khalil, *Nonlinear Systems.* Prentice Hall 2nd. Ed. 1996.

[6] H. Nijmeijer and A. Van Der Shaft, *Nonlinear Dynamic Control*. Springer Verlag 1990.

[7] J.-J. E. Slotine and W. Li., *Applied Nonlinear Control*, Prentice Hall, 1991.

[8] E.D. Sontag, *Mathematical Control Theory*. Springer verlag 1990.

[9] Y. Tang, N. Zhang and Y. Li, Stable fuzzy adaptive control for a class of nonlinear systems. *Fuzzy Sets and Systems*, 104 (1999), pp. 279-288.

[10] S.C. Tong, J. Tang and T. Wang, Fuzzy adaptive control of multivariable nonlinear systems. *Fuzzy Sets and Systems*, 111 (2000) pp. 153-167.

[11] ____ , Fuzzy adaptive output tracking control of nonlinear systems. *Fuzzy Sets and Systems*, 111 (2000) pp. 169-182.

[12] L.-X. Wang , Stable adaptive fuzzy control of nonlinear systems. *IEEE Trans. on Fuzzy Systems*, 1 (1993), pp. 146-155.

[13] ____ , *Adaptive Fuzzy Systems and Control*, Prentice Hall 1994.

[14] J. Zabczyk, *Mathematical Control Theory*. Birkhauser, 1992.

[15] M. Zhihong, X. Yu, K. Eshraghian and M. Palaniswami, Na adaptive supervisory controller for nonlinear fuzzy logic control systems. *Proc. of the 3th Australian and New Zealand Conf. on Intelligent Information Systems*, (1995), pp. 198-203.