Adaptive Fuzzy Control for Nonlinear Systems

L. Schnitman[1] T. Yoneyama[1] J.A.M. Fellipe de Souza[2]

[1]Instituto Tecnológico de Aeronáutica, São Paulo - Brazil
[2]Universidade da Beira Interior, Covilhã - Portugal
e-mail: *leizer@ele.ita.cta.br ; takashi@ele.ita.cta.br ;* felippe@demnet.ubi.pt

ABSTRACT: This article proposes a new adaptive fuzzy controller for a class of nonlinear systems governed by a state equation of form $\dot{x} = f(x)+g(x).u$. The exact feedback linearization approach is used to define a control law structure and the nonlinear functions $f(.)$ and $g(.)$ are estimated through fuzzy blocks in order to provide a control signal. A state estimator is also used and none external control is necessary to achieve stability and tracking error convergence. Because of the control and adaptation laws and the estimators convergence, stability and tracking error convergence are assured. For practical example, it uses a nonlinear and unstable magnetic levitation system and simulation results are shown.

KeyWords: Fuzzy control, Nonlinear control, Adaptive control, Stability

1 INTRODUCTION

The control of nonlinear systems has attracted widespread attention in the recent years (e.g. [11], [12], [15], [19], [20], [34]). Classical linearization methods such as exact feedback linearization work well when the model is accurate and specially when constraints are not considered. Also, the adaptive fuzzy control techniques has been used in a variety of applications for control of nonlinear systems (e.g.: [1, 2, 3, 5, 6, 7, 8, 9, 10, 13, 16, 17, 18, 21, 22, 23, 24, 26, 29, 30, 31, 32, 33, 35]).

This paper presents an adaptive fuzzy control in order to guarantee safe operation under model uncertainties. The control law is based on the exact feedback linearization approach and pole placement techniques. Therefore, fuzzy blocks are used to estimate the nonlinear functions and provide the desired control action. Usually, the certainty equivalent controller as well as an extra control signal u_s (called as supervisory control) are used (e.g. [4, 8, 9, 21, 24, 25, 27, 28, 36]). The supervisory control acts like a sliding mode control, i.e., if $V \geq \bar{V}$, it acts in order to force $\dot{V} \leq 0$ (V is a function in the Lyapunov sense and $\bar{V}$ is an upper bound). In this paper, the states are also estimated but using conventional techniques and the inclusion of any other external signal (like a supervisory control u_s) was not necessary to achieve stability and tracking error convergence.

In section 2 the class of nonlinear system as well as the general structure of the desired control law are presented. The fuzzy structure is shown in section 3. Section 4 presents the implementable control and adaptation laws, therefore, the states estimation convergence and the stability is proven using Lyapunov techniques. In the section 5 the nonlinear and unstable magnetic levitation system is presented and simulation results are shown. Section 6 presents the conclusions.

2 NONLINEAR SYSTEM

Let a nonlinear system be described by a state equations in the form:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \vdots \\ \dot{x}_n = f(x) + g(x).u \\ y = x_1 \end{cases} \tag{1}$$

or in an equivalent form:

$$\begin{cases} \dot{x} = A.x + B\left(f(x) + g(x).u\right) \\ y = x_1 \end{cases} \tag{2}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}_{n \times n} \qquad (3)$$
$$B = \begin{bmatrix} 0 & \cdots & 0 & 1 \end{bmatrix}^T_{1 \times n}$$

and $f : \Re^n \rightarrow \Re$ and $g : \Re^n \rightarrow \Re$ are continuous and known functions. And let the control objective be to track a reference signal $r(t) = r \in \Re$.

Letting a gain vector K:

$$K = [K_1, ..., K_n] \in R^n \qquad (4)$$

and basing on the exact feedback linearization and pole placement techniques, the following control law can be used:

$$u = \frac{1}{g(x)} \left[-f(x) + r - K.x \right] \qquad (5)$$

Substituting (5) in (2), one gets:

$$\dot{x} = (A - B.K)\,.x + B.r \qquad (6)$$

which define a linear dynamic and the stability is assured by the eingenvalues of the matrix $(A - B.K)$

3 FUZZY SYSTEM

Let a fuzzy system composed by r rules, each one of them as:

IF x_1 is A_1^j and ...and x_n is A_n^j THEN y is B_j (7)

where $\left\{ A_1^j, ..., A_n^j \right\}$ / B_j are the input/output membership functions (MF) related to the j^{th} rule ($j = 1..r$). Consider y_j as the point in that B_j is maximum $(\mu_{B_j}(y_j) = 1)$ and define the vector of parameters $\theta^T = [y_1, ..., y_r]$ .The fuzzy output can be expressed as:

$$y = \theta^T.W(x) \qquad (8)$$

where

$$W(x) = [W_1(x), ..., W_r(x)]^T$$
$$W_j(x) = \frac{\prod_{i=1}^{n} \mu_{A_i^j}(x_i)}{\sum_{j=1}^{r} \left(\prod_{i=1}^{n} \mu_{A_i^j}(x_i) \right)}, \quad j = 1..r \qquad (9)$$

and

$$W_j(x) \in [0, 1] \qquad (10)$$

is usually called as the *weight* of the j^{th} rule. The scheme is shown if figure 1.

In the present paper, the use of fuzzy structures is proposed in order to estimate the unknown nonlinear functions from equation (2). Thus, it considers the existence of the fuzzy optimal parameters θ_f^* and θ_g^* which guarantee that the function $f(.)$ and $g(.)$ can be estimated using the fuzzy blocks $f(x|\theta_f) = \theta_f^T.W(x)$ and $g(x|\theta_g) = \theta_g^T.W(x)$ (fuzzy as universal approximators). Since

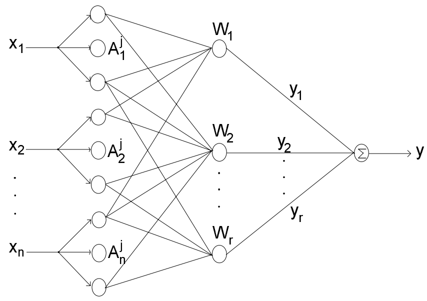

Figure 1: Fuzzy structure

$$f(x|\theta_f^*) - f(x) < \varepsilon_f \quad \forall x \in D_x$$
$$g(x|\theta_g^*) - g(x) < \varepsilon_g \quad \forall x \in D_x \qquad (11)$$

where ε_f and ε_g are positive constants as small as desired and D_x is the x domain, then the nonlinear system presented in equation (2) can be represented through fuzzy blocks of form:

$$\begin{cases} \dot{x} = A.x + B\left(f(x|\theta_f^*) + g(x|\theta_g^*).u \right) \\ y = x_1 \end{cases} \qquad (12)$$

It is important to note that for the fuzzy blocks, the states are not available and must be estimated. Therefore, $f(.)$ and $g(.)$ should be represented by $f(\hat{x}|\theta_f)$ and $g(\hat{x}|\theta_g)$ amd the controller must guarantees that $\hat{x} \rightarrow x$.

Once the real parameters θ_f^* and θ_g^* are unknown, they must also be estimated and the following notation is used to the fuzzy estimators

$$f(\hat{x}|\hat{\theta}_f) = \hat{\theta}_f^T.W(\hat{x})$$
$$g(\hat{x}|\hat{\theta}_g) = \hat{\theta}_g^T.W(\hat{x}) \qquad (13)$$

4 STATE ESIMATION AND CONTROL

When $f(.)$ and $g(.)$ in equation (2) are unknown, the idea is to use fuzzy systems to substitute them by fyzzy blocks $f(\hat{x}|\hat{\theta}_f)$ and $g(\hat{x}|\hat{\theta}_g)$ as discussed before. Using the states estimator and fuzzy estimators as presented in (13), the control law (5) becomes

$$u = \frac{1}{g(\hat{x}|\hat{\theta}_g)} \left[-f(\hat{x}|\hat{\theta}_f) + r - K.\hat{x} \right] \qquad (14)$$

Remark 1 *As proposed in previous works, some previous knowledge of the system must be available to set bounds. In our case, it is used to set a lower bound* g^L *in order to avoid* $\left| g(\hat{x}|\hat{\theta}_g) \right| = 0$ *in equation (14). We also consider that the sign of* $g(x)$ *is available and equal the sign of* $g(\hat{x}|\hat{\theta}_g)$*. It is not more than what is already used.*

Remark 2 *From here, for sake of simplicity, we will* *simplify the notation of form:*

$$\begin{array}{l} f = f(\hat{x}|\theta_f^*) = \left(\theta_f^*\right)^T.W(\hat{x}) \\ g = g(\hat{x}|\theta_g^*) = \left(\theta_g^*\right)^T.W(\hat{x}) \\ \hat{f} = f(\hat{x}|\hat{\theta}_f) = \hat{\theta}_f^T.W(\hat{x}) \\ \hat{g} = g(\hat{x}|\hat{\theta}_g) = \hat{\theta}_g^T.W(\hat{x}) \end{array} \tag{15}$$

For the state estimator, define a gain vector k:

$$k = [k_1, ..., k_n] \in R^n \tag{16}$$

And the state estimator be of form

$$\begin{cases} \dot{\hat{x}} = A.\hat{x} + B\left(\hat{f} + \hat{g}.u\right) + B.k.(x - \hat{x}) \\ \hat{y} = \hat{x}_1 \end{cases} \tag{17}$$

Define the error estimation

$$e = x - \hat{x} \tag{18}$$

Subtracting (17) from (12) one gets

$$\dot{e} = (A - B.k)\,e + B\left(f - \hat{f} + (g - \hat{g}).u\right) \tag{19}$$

or in equivalent form

$$\dot{e} = \Lambda.e + B.\left[f - \hat{f} + (g - \hat{g}).u\right] \tag{20}$$

where

$$\Lambda = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -k_1 & \cdots & \cdots & \cdots & -k_n \end{bmatrix} \tag{21}$$

Since Λ is a stable matrix, there exist a unique positive definite symmetric matrix $P_{n\times n}$, which satisfies the Lyapunov equation:

$$\Lambda^T.P + P.\Lambda = -Q \tag{22}$$

where $Q_{n\times n}$ is an arbitrary positive definite matrix.

For sake of simplicity, define as an auxiliary variable

$$\rho = B.\left[f - \hat{f} + (g - \hat{g}).u\right] \tag{23}$$

therefore, the equation (20) becomes:

$$\dot{e} = \Lambda.e + \rho \tag{24}$$

4.1 Control objective

The control objective is to guarantee the state estimation convergence and the fuzzy parameters estimation convergence. It must guarantee that $f(\hat{x}|\hat{\theta}_f) \to f(x|\theta_f^*)$ and $g(\hat{x}|\hat{\theta}_g) \to g(x|\theta_g^*)$ when $t \to \infty$. Hence, the control action will force the tracking error convergence to zero.

Thus, introduce the notation:

$$\begin{array}{lcl} \phi_f = \theta_f^* - \hat{\theta}_f & \text{and} & \dot{\phi}_f = -\hat{\dot{\theta}}_f \\ \phi_g = \theta_g^* - \hat{\theta}_g & \text{and} & \dot{\phi}_g = -\hat{\dot{\theta}}_g \end{array} \tag{25}$$

and let define a function V of form:

$$V = \frac{1}{2}e^T.P.e + \frac{1}{2\gamma_f}\phi_f^T.\phi_f + \frac{1}{2\gamma_g}\phi_g^T.\phi_g \tag{26}$$

where γ_f , γ_g are positive constants. Taking the time-derivative:

$$\dot{V} = \frac{1}{2}\left(\dot{e}^T.P.e + e^T.P.\dot{e}\right) - \frac{1}{\gamma_f}\phi_f^T.\hat{\dot{\theta}}_f - \frac{1}{\gamma_g}\phi_g^T.\hat{\dot{\theta}}_g \tag{27}$$

Firstly, let analyze the term:

$$T_1 = \frac{1}{2}\left(\dot{e}^T.P.e + e^T.P.\dot{e}\right) \tag{28}$$

Substituting the value of $\dot{e}$ from equation (24) one gets:

$$\begin{array}{rl} T_1 = & \frac{1}{2}[\Lambda.e + \rho]^T.P.e + \frac{1}{2}e^T.P.[\Lambda.e + \rho] \\ = & \frac{1}{2}[\Lambda.e]^T.P.e + \frac{1}{2}\rho^T.P.e + \frac{1}{2}e^T.P.\Lambda.e + \frac{1}{2}e^T.P.\rho \end{array} \tag{29}$$

Noting that

$$[\Lambda.e]^T = e^T.\Lambda^T \tag{30}$$

equation (29) becomes

$$\begin{array}{rl} T_1 = & \frac{1}{2}e^T.\Lambda^T.P.e + \frac{1}{2}\rho^T.P.e + \frac{1}{2}e^T.P.\Lambda.e + \frac{1}{2}e^T.P.\rho \\ = & \frac{1}{2}e^T.\left(\Lambda^T.P + P.\Lambda\right).e + \frac{1}{2}\rho^T.P.e + \frac{1}{2}e^T.P.\rho \\ = & -\frac{1}{2}e^T.Q.e + \frac{1}{2}\rho^T.P.e + \frac{1}{2}e^T.P.\rho \end{array} \tag{31}$$

Since P is symmetrical:

$$\rho^T.P.e = e^T.P.\rho \tag{32}$$

then:

$$T_1 = -\frac{1}{2}e^T.Q.e + e^T.P.\rho \tag{33}$$

Now, analyze the term:

$$T_2 = \frac{1}{\gamma_f}\phi_f^T.\hat{\dot{\theta}}_f \tag{34}$$

Defining the control law:

$$\hat{\dot{\theta}}_f = \gamma_f.e^T.P.B.W(\hat{x}) \tag{35}$$

and substituting (25) and (35) in (34) one gets:

$$T_2 = \frac{1}{\gamma_f}\left(\theta_f^* - \hat{\theta}_f\right)^T.\gamma_f.e^T.P.B.W(\hat{x}) \tag{36}$$

From (15) note that $f = f(\hat{x}|\theta_f^*) = \left(\theta_f^*\right)^T.W(\hat{x})$ and $\hat{f} = f(\hat{x}|\hat{\theta}_f) = \hat{\theta}_f^T.W(\hat{x})$. Hence::

$$T_2 = e^T.P.B\left(f - \hat{f}\right) \tag{37}$$

Analyzing the last term:

$$T_3 = \frac{1}{\gamma g}\phi_g^T.\hat{\dot{\theta}}_g \tag{38}$$

and using the same procedure of T_2 one obtains:

$$T_3 = e^T.P.B\,(g - \hat{g}).u \tag{39}$$

Thus, equation (27) can be rewritten as:

$$\begin{aligned}\dot{V} &= -\tfrac{1}{2}e^T.Q.e + e^T.P.\rho \\ &\quad - e^T.P.B\left(f-\hat{f}\right) - e^T.P.B\left(g-\hat{g}\right).u \\ \dot{V} &= -\tfrac{1}{2}e^T.Q.e + e^T.P.\rho \\ &\quad - e^T.P.B\left[f-\hat{f}+\left(g-\hat{g}\right).u\right]\end{aligned} \tag{40}$$

Rescuing the equation (23) one gets:

$$\begin{aligned}\dot{V} &= -\tfrac{1}{2}e^T.Q.e + e^T.P.\rho - e^T.P.\rho \\ \dot{V} &= -\tfrac{1}{2}e^T.Q.e \leq 0\end{aligned} \tag{41}$$

It means that the error e is bounded. In order to prove that $e \to 0$ when $t \to \infty$, let apply the Lemma of Barbalat. Thus

$$\begin{aligned}\ddot{V} &= -\tfrac{1}{2}\left(\dot{e}^T.Q.e + e^T.Q.\dot{e}\right) \\ &= -\tfrac{1}{2}\left[\left(\Lambda.e+\rho\right)^T.Q.e + e^T.Q.\left(\Lambda.e+\rho\right)\right]\end{aligned} \tag{42}$$

Since e is available in V, and V is bounded because $\dot{V} \leq 0$, therefore, if ρ is bounded then from Barbalat one guarantees that

$$\dot{V} \to 0 \quad \text{when} \quad t \to \infty \tag{43}$$

which implies that

$$e \to 0 \quad \text{when} \quad t \to \infty \tag{44}$$

From (23) note that

$$\rho = B.\left[f-\hat{f}+\left(g-\hat{g}\right).u\right] \tag{45}$$

Hence, the boundness of ρ depends on the boundness of the estimators $\hat{f}$ and $\hat{g}$. This can be provided by the use of the projection vector [14], as it is usual in previous papers (e.g. [4, 8, 9, 21, 24, 25, 27, 28, 36]).

5 SIMULATION RESULTS

5.1 Selected plant

As an example, consider the magnetic levitation system as shown in figure 2. Its dynamics is described as:

$$\ddot{d} = g_r - \frac{F}{m} \tag{46}$$

where m is the mass of the ball, g_r is the gravitational acceleration and F is the electromagnet force produced by a coil fed with current i

$$F = c\frac{i^2}{d^2} \tag{47}$$

whith $c > 0$ a constant and d as the distance.

Letting $x_1 = d(t)$ and $x_2 = \dot{d}(t)$ the state representation becomes:

$$\begin{cases}\dot{x}_1 = x_2 \\ \dot{x}_2 = g_r - c\frac{i^2}{m.x_1^2}\end{cases} \tag{48}$$

So that comparison to equation (1) yields:

$$\begin{aligned}f(x) &= g_r \\ g(x) &= -c\tfrac{1}{m.x_1^2} \\ u &= i^2\end{aligned} \tag{49}$$

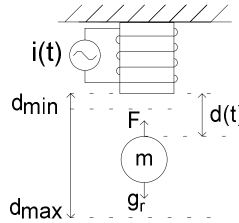

Figure 2: System of magnetic levitation

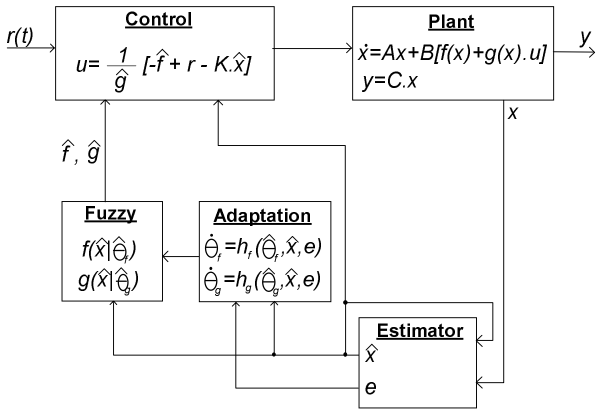

Figure 3: Blocks diagram detailed

5.2 Implementation

The proposed system was implemented using the Simulink (© Matlab 5.3) version. The blocks diagram is shown in figure 3.

1. For the plant parameters, set $g_r = 9.8\left[m/s^2\right]$, $m = 0.05\,[Kg]$ and $c = 1\left[Nm/A^2\right]$.

2. For initial conditions set $x_1 = 1.1\left[\times 10^{-2}m\right]$ and $x_2 = 0.\left[\times 10^{-2}m/s\right]$

3. To define the system dynamics use $K = [1 \quad 1]$

4. For the state estimator, set the initial conditions $x_1 = 1.5\left[\times 10^{-2}m\right]$, $x_2 = 0\left[\times 10^{-2}m/s\right]$ and a gain vector $k = [100 \quad 20]$ thus

$$\Lambda_c = \begin{bmatrix} 0 & 1 \\ -100 & -20 \end{bmatrix} \tag{50}$$

5. Set the Q matrix as identity to obtain

$$P = \begin{bmatrix} 2.6250 & 0.0050 \\ 0.0050 & 0.0253 \end{bmatrix} \tag{51}$$

which satisfies equation (22).

6. Set the $d(t)$ range as $D_x = [0.5 \ ; \ 3.5]\left[\times 10^{-2}m\right]$, which represents the fuzzy input universe of discourse.

7. Set $\gamma_g = 10^3$

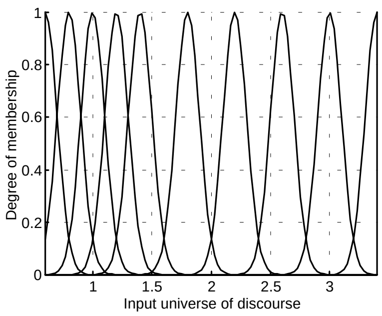

Figure 4: Membership functions

5.3 Fuzzy sets

In order to compose the fuzzy membership functions, choose points to be the center of the membership functions, covering the whole input universe of discourse. For instance, one can use the vector

$$X_c = \begin{bmatrix} 0.6 \\ 0.8 \\ 1.0 \\ 1.2 \\ 1.4 \\ 1.8 \\ 2.2 \\ 2.6 \\ 3.0 \\ 3.4 \end{bmatrix} \tag{52}$$

and obtain the membership functions as shown in figure 4.

For each membership function A^j define a single rule of form:

$$\text{IF } d(t) \text{ is } A^j \text{ THEN } y \text{ is } y_j \qquad j = 1...r \tag{53}$$

where $r = \text{length}(Xc)$.

As the knowledge of the system is available from (49), note that exact rules can be generated simply computing the value of the function $g(x|\theta_g)$ as being

$$y_j = -\frac{c}{m.(Xc_j)^2}, \quad j = 1, .., r \tag{54}$$

Hence the exact value of the θ_g^* parameter is

$$\theta_g^* = \begin{bmatrix} -55.5556 \\ -31.2500 \\ -20.0000 \\ -13.8889 \\ -10.2041 \\ -6.1728 \\ -4.1322 \\ -2.9586 \\ -2.2222 \\ -1.7301 \end{bmatrix}_{r\times 1} \tag{55}$$

which means that for $j = 1, .., r$ the knowledge is represented by exact linguistic information of form:

$$\text{IF } x_1 \text{ is } Xc_j \text{ THEN } g(x) \text{ is } -\frac{c}{m.(Xc_j)^2} \tag{56}$$

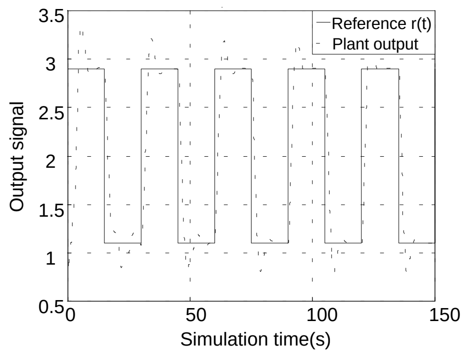

Figure 5: Simulation results

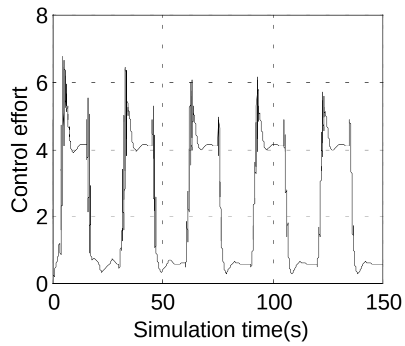

Figure 6: Control effort

Note that these rules are exact because we know the function $g(x)$. To be didactic and in order to show the states and function adaptation, some uncertainties can be inserted. For instance, use

$$y_j = \varsigma_j . y_j \qquad , \ j = 1..r \tag{57}$$

where ς_j is a random number chosen from a normal distribution with mean zero and variance one. In our case, the estimated parameters were initialized as

$$\hat{\theta}_g = \begin{bmatrix} -64.7196 \\ -19.5887 \\ -1.5016 \\ -4.8834 \\ 7.1073 \\ -10.4700 \\ -0.2440 \\ -5.3168 \\ -0.5868 \\ -1.5081 \end{bmatrix} \tag{58}$$

5.4 Simulation results

The simulation results are shown if figures 5 to 12.

6 CONCLUSIONS

Previous works usually uses a reference signal y_r (e.g. [4, 8, 9, 21, 24, 25, 27, 28, 36]) so that they consider that its time-derivative are known to compose the control action and the reference signal is restricted for

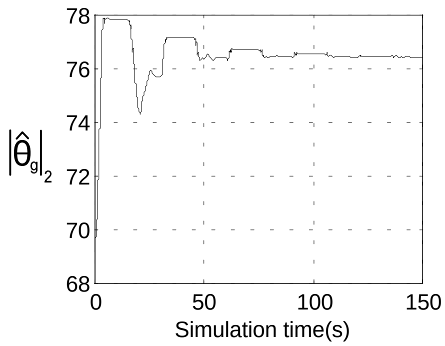

Figure 7: Norm of fuzzy estimated parameters

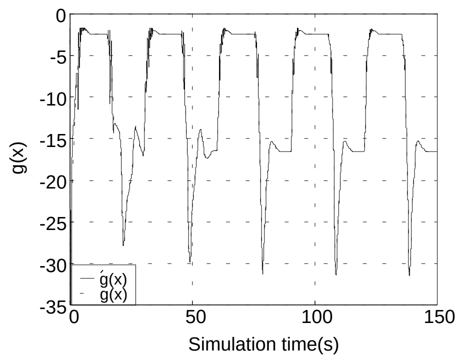

Figure 8: g(x) estimation

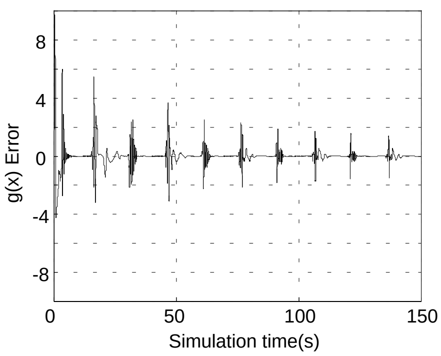

Figure 9: g(x) estimation error

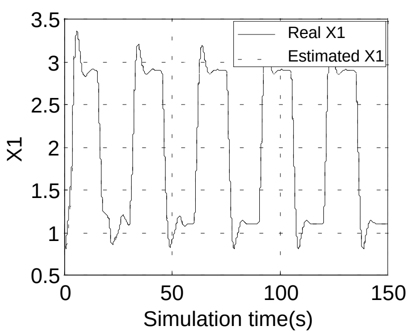

Figure 10: X1 estimation

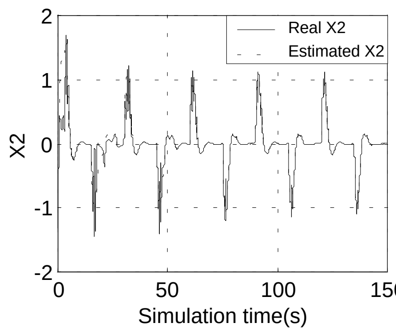

Figure 11: X2 estimation

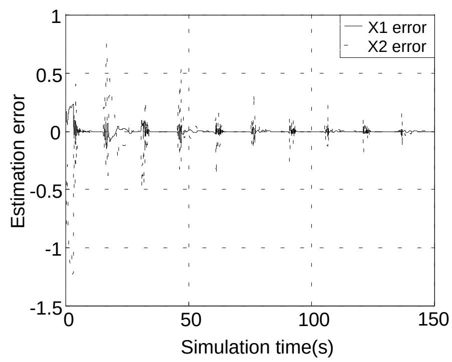

Figure 12: States estimation error

a class of signal which the $n - order$ time-derivative can be computed (such as the trigonomectric functions). On the other hand, they are unable to guarantee safe operation, for example, for a single step as reference signal.

As previous papers, it is based on the unknown functions $f(x)$ and $g(x)$ that are proposed to be estimated through fuzzy blocks. However, differently from those previous works, this paper do not restricts the reference signal because it do not uses the $n - order$ time-derivative of the reference signal to compose the control law. The exact feedback linearization approach suggest the design of the control law (5) and the gain matrix K is applied to pole placement. Moreover, in previous papers (e.g. [4, 8, 9, 21, 24, 25, 27, 28, 36]) the use of an external control signal was nessary to achieve the stability.

The tracking error $y(t) \rightarrow r(t)$ represents the control objective and it is reached since the function and states estimation converges. Previous works (e.g. [4, 8, 9, 21, 24, 25, 27, 28, 36]) treat the tracking error as the main control objective and the functions estimation convergence does not care since they are maintained in a compact and constraint set using the projection vector [14]. The proposed control law (14), basing on the Lyapunov techniques (equation 26) assures that the states estimation error decreases as well as it guarantees the function estimations convergence to their real values, i.e., $\hat{x} \rightarrow x$, $f(\hat{x}|\hat{\theta}_f) \rightarrow f(x|\theta_f^*)$

and $g(\hat{x}|\hat{\theta}_g) \to g(x|\theta_g^*)$.

Successful simulations with different number of rules as well as different initialization indicates the robustness of the method with respect to these parameters.

Acknowledgement : The authors are indebted to FAPESP - Fundação de Amparo à Pesquisa do Estado de São Paulo, for support under grant 98/16074-1

References

[1] J.S. Banerjee, K.O. Jones, and D. Williams. Multivariable model reference fuzzy adaptive control. *Proceedings of the IEE Seminar on Learning Systems for Control*, pages 10/1–10/4, Ref. 069, 2000.

[2] Edy Bertolissi, Antoine Duchâteau, Hugues Bersini, and Frank Vaden Berghen. Direct adaptive fuzzy control for MIMO processes. *Proceedings of the 9th IEEE Int. Conf. on Fuzzy Systems*, 1:7–12, 2000.

[3] Yeong-Chan Chang and Bor-Sen Chen. Robust tracking design for both holonomic and nonholonomic constrained mechanical systems: Adaptive fuzzy approach. *IEEE Transactions on Fuzzy Systems*, 8(1):46–65, February 2000.

[4] Bor-Sen Chen, Ching-Hsiang Lee, and Yeong-Chan Chang. Hinf tracking design of uncertain nonlinear SISO systems: Adaptive fuzzy approach. *IEEE Trans. on Fuzzy Systems*, 4(1):32–43, February 1996.

[5] Liang Chen, Guanrong Chen, and Yang-Woo Lee. Fuzzy modeling and adaptive control of uncertain chaotic systems. *Information Sciences*, 121:27–37, 1999.

[6] Chih-Hsun Chou. Model reference adaptive fuzzy control: A linguistic space approach. *Fuzzy Sets and Systems*, 96:1–20, 1998.

[7] Predrag D. Djurdjevic. Real time adaptive fuzzy control of inverted pendulum. *Proceedings of the 42nd Midwest Symposium on Circuits and Systems*, 2:910–913, 2000.

[8] Kurt Fischle and Dierk Schroder. An improved stable adaptive fuzzy control method. *IEEE Transactions on Fuzzy Systems*, 7(1):27–40, February 1999.

[9] Veysel Gazi and Kevin M. Passino. Direct adaptive control using dynamic structure fuzzy systems. *Proceedings of the American Control Conference*, 3:1954–1958, June 2000.

[10] Hugang Han, Chun-Yi Su, and Shuta Murakami. Adaptive control of a class of nonlinear discrete-time systems with fuzzy logic. *Proceedings of the 3rd World congress on intelligent control and automation*, 3:1785–1789, July 2000.

[11] F.C. Hoppensteadt. *Analysis and Simulation of Chaotic Systems*. Springer Verlag, 1993.

[12] Hassan K. Khalil. *Nonlinear Systems*. Prentice Hall, 2nd edition, 1996.

[13] Hyeongcheol Lee and Masayoshi Tomizuka. Robust adaptive control using a universal approximator for siso nonlinear systems. *IEEE Transactions on Fuzzy Systems*, 8(1):95–106, February 2000.

[14] D. G. Luenberger. *Linear and Nonlinear Programming*. Addison-Wesley Publishing Company, Inc., 1984.

[15] H. Nijmeijer and A. Van Der Shaft. *Nonlinear Dynamical Control Systems*. Springer Verlag, 1990.

[16] B. Novakovic, D. Scap, and D. Novakovic. An adaptive fuzzy logic control synthesis and applications in robotics. *Proceedings of the IEEE Int. Conference on Systems, Man, and Cybernetics*, 6:150–155, 1999.

[17] I. Rojas, H. Pomares, F.J. Pelayo, M. Anguita, E. Ros, and A. Prieto. New methodology for the development of adaptive and self-learning fuzzy controller in real time. *International Journal of Approximate Reasoning*, 21:109–136, 1999.

[18] Jie Si, Naiyao Zhang, and Rilum Tang. A new on-line fuzzy control scheme based on reference model. *Proceedings of the Int. Joint Conference on Neural Networks*, 6:4213–4216, 1999.

[19] Jean-Jacques E. Slotine and Weiping Li. *Applied Nonlinear Control*. Prentice Hall, 1991.

[20] Eduardo D. Sontag. *Mathematical Control Theory*. Springer Verlag, 1990.

[21] Yuezhong Tang, Naiyao Zhang, and Yanda Li. Stable fuzzy adaptive control for a class of nonlinear systems. *Fuzzy Sets and Systems*, 104:279–288, 1999.

[22] Shao Cheng Tong and Tianyou Chai. Fuzzy direct adaptive control for a class of nonlinear systems. *Fuzzy Sets and Systems*, 103:379–387, 1999.

[23] Shao Cheng Tong, Qingguo Li, and Tianyou Chai. Fuzzy adaptive control for a class of nonlinear systems. *Fuzzy Sets and Systems*, 101:31–39, 1999.

[24] Shao Cheng Tong, Jiantao Tang, and Tao Wang. Fuzzy adaptive control of multivariable nonlinear systems. *Fuzzy Sets and Systems*, 111:153–167, 2000.

[25] Shao Cheng Tong, Tao Wang, and Jian Tao Tang. Fuzzy adaptive output trackink control of nonlinear systems. *Fuzzy Sets and Systems*, 111:169–182, 2000.

[26] Chung-Shi Tseng and Bor-Sen Chen. Fuzzy tracking control design for nonlinear discrete-time dynamic systems via TS fuzzy model. *Proceedings of the 9th IEEE Int. Conf. on Fuzzy Systems*, 1:405–410, 2000.

[27] Li-Xin Wang. Stable adaptive fuzzy control of nonlinear systems. *IEEE Trans. on Fuzzy Systems*, 1('2):146–155, 1993.

[28] Li-Xin Wang. *Adaptive Fuzzy Systems and Control*. Prentice Hall, 1994.

[29] Wen Jie Wang and Bing Yong Tang. A fuzzy adaptive method for intelligent control. *Expert Systems with Applications*, 16:43–48, January 1999.

[30] C.H. Wong, S.L. Shah, M.M. Bourke, and D.G. Fisher. Adaptive fuzzy relational predictive control. *Fuzzy Sets and Systems*, 115:247–260, 2000.

[31] Wen Fang Xie and A.B.Rad. Fuzzy adaptive internal model control. *IEEE Trans. on Industrial Electronics*, 47(1):193–202, February 2000.

[32] O. Yaniv and M. Schneider. Synthesis of adaptive linear-time-invariant controllers by fuzzy logic controllers. *Fuzzy Sets and Systems*, 106:75–83, 1999.

[33] Z.D. Yuan and S.H. Hou. Indirect adaptive fuzzy control of discrete-time nonlinear systems. *Proceedings of the 3rd World Congress on Intelligent Control and Automation*, 3:1770–1774, July 2000.

[34] Jerzy Zabczyk. *Mathematical Control Theory*. Birkhäuser, 1992.

[35] Huaguang Zhang and Zeungnam Bien. Adaptive fuzzy control of MIMO nonlinear systems. *Fuzzy Sets and Systems*, 115:191–204, 2000.

[36] Man Zhihong, X.H. Yu, K. Eshraghian, and M. Palaniswami. An adaptive supervisory controller for nonlinear fuzzy logic control systems. *Proc. of the 3th Australian and New Zealand Conf. on Intelligent Information Systems*, pages 198–203, 1995.