A New Mamdani-Like Fuzzy Structure

J.A.M. Felippe de Souza[1] , L. Schnitman[2] and T. Yoneyama[2]
felippe@demnet.ubi.pt; leizer@ele.ita.cta.br; takashi@ele.ita.cta.br
[1]Universidade Beira Interior - 6201-001 Covilhã - Portugal
[2]Instituto Tecnológico de Aeronáutica - 12.228-900 - S.J. dos Campos, SP - Brazil

Abstract: - The purpose of this work is to present a new fuzzy structure, which is computationally efficient. The main idea is to modify the original Mamdani fuzzy structure, specially the rule aggregation and defuzzification procedures. The computational effort is similar to that required by Takagi-Sugeno-Kang structures.

Keywords: Fuzzy systems, fuzzy sets, discrete fuzzy

1 Introduction

The fuzzy structure proposed by E.H. Mamdani in [18] fully reflects the concept of fuzzyness proposed by L.A. Zadeh [34]. The Mamdani fuzzy structure is intuitive, provides heuristic insight, has received widespread acceptance in both industrial and academic media and it suits well with respect to interactions with humans. The fuzzy/neurofuzzy approaches have been used with success in modelling and control of systems. Their use as new powerful tools is widely reported in the literature (e.g. [8], [9], [12], [15], [17], [21], [24], [29], [31], and [33]). In spite of these qualities, the computational effort that is required in the Mamdani defuzzification procedure is considerable and it may be difficult to implement in real applications, specially when on-line training is employed. On the other hand, the Takagi-Sugeno-Kang (TSK) fuzzy models [26], [28] can be treated in a computational efficient way and is amenable to elegant mathematical analysis, attracting considerable interest (e.g. [1], [2], [4], [6], [7], [11], [13], [14], [16], [19], [20], [22], [25], [27], and [32]). However, TSK is less intuitive as the resulting consequent membership functions (MFs).

This work proposes some modifications in the computation of outputs, which aim to substantially reduce the computational effort needed in Mamdani fuzzy structures.

Section 2 presents a brief review to the Mamdani and TSK fuzzy structures. Section 3 shows the proposed modifications and the derived properties. In section 4, an example with asymmetric trapezoidal antecedent and consequent MF is shown.

2 Mamdani and TSK fuzzy structures

One of the basic differences between the Mamdani and TSK fuzzy models lies on the fact that the *consequent* are, respectively, fuzzy and crisp sets. Hence, the computation procedure required to obtain the output signals are not the same. While in the case of TSK fuzzy models the output is found using a very simple formula (*weighted average, weighted sum*), Mamdani fuzzy models require a higher computational effort to defuzzification procedure.

Firstly, adequate operators must be selected to represent the "*and*", "*or*" and "*implication*" linguistic symbols, as well as the rule aggregation and defuzzification methods. The *sum-product* composition is frequently used in practical implementations (e.g. [3], [4], [5], [10], [11], [12], [23], [25]), i.e., the *product* is made to correspond to *and* and *implication*, the *sum* to *or* and rule aggregation function. Also, the *centroid* as a common *defuzzification* method yield useful properties and are necessary in the implementation of training procedures, such as the ones based on the gradient expressions.

For the sake of simplicity and to avoid a heavy notation, only SISO structures are considered, although the general case leads to similar equations. Define x as the input and y as the output signals. Also:

- $X_1, ..., X_n$ are the n antecedent MF
- $Y_1, ..., Y_m$ are the m consequent MF
- $R_1, ..., R_r$ are the r rules

Generally, the fuzzy reasoning can be expressed through rules such as:

$$R_j: \text{ If } x \text{ is } X^j \text{ then } y \text{ is } Y^j \tag{1}$$

where $X^j \in [X_1,...,X_n]$ and $Y^j \in [Y_1,...,Y_n]$

2.1 Output of Discrete Mamdani Structures

Let both the antecedent and consequent MF be of triangular, trapezoidal or Gaussian form. Note that the rule aggregation method could generate complex membership functions such as illustrated in Figure 2 and that its centroid must be found to determine the fuzzy output. Therefore, in theory, an integral must be computed over the output universe of discourse (UD). Usually the integral is approximated by a summation following a discretezation procedure.

Let the output UD be $[y_{min}, y_{max}]$. If λ is the number of discretezation points, let

$Rout = [y_{min};\ y_{min}+\Delta;\ y_{min}+2\Delta;\ldots;\ y_{max}]^T$

where $\Delta = (y_{max}-y_{min})/(\lambda-1)$. Then for each rule i, the consequent is given by

$$C_i=\mu_{X_i}(x).\ \mu_{Y_i}(Rout),\quad \forall\ i \in [1,r] \tag{2}$$

as graphically represented in Figure 1.

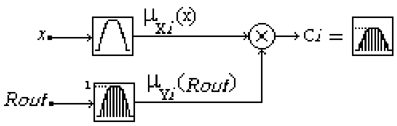

Figure 1: Each rule

If the matrix C of *consequents* is defined as

$$C=\begin{bmatrix} C_1 = \mu_{X^1}(x).\mu_{Y^1}(Rout) \\ C_2 = \mu_{X^2}(x).\mu_{Y^2}(Rout) \\ \vdots \\ C_r = \mu_{X^r}(x).\mu_{Y^r}(Rout) \end{bmatrix}_{r x\lambda} \tag{3}$$

then, using the *sum* operator as the rule aggregation method, CA is simply

$$CA=\sum_{i=1}^{r} C_i$$

The aggregated output membership function (still to be normalised and then deffuzyfied) is:

$$CA_j=\sum_{i=1}^{r} \mu_{X^i}(x).\mu_{Y^i}(Rout[j]),\ \forall\ j \in [1,l] \tag{4}$$

or, alternatively,

$$CA=[CA_1\ \ CA_2\ \ \ldots\ \ CA_l]_{1x\lambda} \tag{5}$$

The vector CAn is obtained from CA by normalization, i.e.

$$CAn = \frac{CA}{\sum_{j=1}^{\lambda} CA_j} \tag{6}$$

so that the fuzzy output becomes

$$\hat{y} = Rout.Can \tag{7}$$

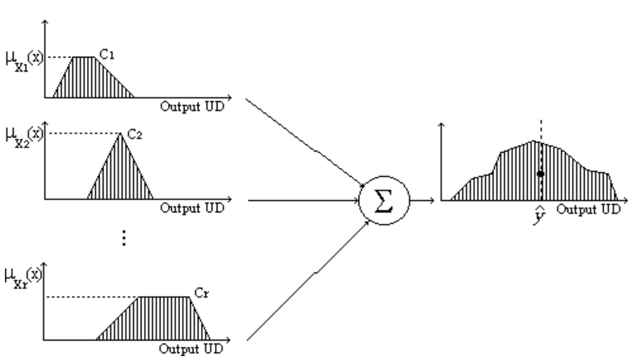

Figure 2: Rules aggregation

Note that this procedure is the centroid defuzzification (with approximation introduced by discretezation) as shown in Figure 2.

2.2 Output of TSK Structures

The TSK fuzzy structure is characterised by their consequent MF that are restricted to singletons (crisp sets), which may be constant or linear functions of the input, as proposed initially in [28]. Generally $Y_i=f_i(x)$ so that the C matrix becomes:

$$C=\begin{bmatrix} C_1 = \mu_{X^1}(x).f_1(x) \\ C_2 = \mu_{X^2}(x).f_2(x) \\ \vdots \\ C_r = \mu_{X^r}(x).f_r(x) \end{bmatrix}_{rx1} \tag{8}$$

The aggregation of *consequents* yields

$$CA=\sum_{i=1}^{r} C_i$$

and the fuzzy output is easily found by

$$\hat{y} = \frac{CA}{\sum_{i=1}^{r} \mu_{xi}(x)} \tag{9}$$

This procedure to compute the fuzzy output is known as *weighted average*. When the denominator is deleted the procedure is known as *weighted sum*.

2.3 The computational effort

Note that the effort in processing the antecedent MFs are equal for both fuzzy structures. The difference lies on the construction of the C matrix.

In the computation of the fuzzy output through the discrete Mamdani structure λ is, as defined, the number of columns of C and also the precision in the discretezation of the output UD. For instance, suppose that the output UD is limited to the range $[0,10]$ and to minimize the discretezation tolerance of 0.01 is used. Note that the output UD discretezation generates $l = 1000$ and the dimension of C becomes $[r \text{ x } 1000]$, making the method somewhat non-practical. On other hand, the fuzzy output through TSK structure is very simple and the dimension of C is always $[r \text{ x } 1]$.

2.4 Output of Fuzzy Structures

Let us analyse the behaviour and properties of the outputs for both Mamdani and TSK fuzzy structures.

2.4.1 Mamdani fuzzy structure

At first, define the function *Dfz(MF)* which stands for the operation *defuzzify* (computation of the fuzzy output), given a known MF (e.g. triangular, trapezoidal, Gaussian...), based on the centroid method.

1. Triangular MF, Y_i=[$a \quad b \quad c$] where a,b,c are the triangular MF parameters and h is its height. The defuzzification formula becomes:

$$Dfz(Y_i)=\frac{h}{3}(a+b+c) \qquad (10)$$

2. Trapezoidal MF, Y_i=[$a \quad b \quad c \quad d$] where a,b,c,d are the trapezoidal MF parameters and h is its height. The defuzzification formula becomes:

$$Dfz(Y_i)=\frac{h}{3}\left(\frac{d^2+c^2-b^2-a^2+c.d-a.b}{d+c-b-a}\right) \qquad (11)$$

3. Gaussian MF, Y_i= [s, c] where s, c are the Gaussian MF parameters and h is its height. The defuzzification formula becomes:

$$Dfz(Y_i)=h.c \qquad (12)$$

Since the MFs to be defuzzified are usually of much complex morphology, it is interesting to analyse the defuzzification process when the input changes (whole input UD). In this context, let the MFs be a square *(Sq)* and an asymmetric triangle *(Tr)* both described by A, B and C parameters, as shown in the Figure 3. Let $h=1$ and the simple fuzzy rule-base:

If x is X_1 then y is Sq
If x is X_2 then y is Tr (13)

Define α_s and α_t as the respective weights that are provided by the antecedent MF when the input value is equal x.

$$a_s = \mu_{X1}(x) \; ; \quad a_t = \mu_{X2}(x) \qquad (14)$$

The fuzzy output is the centroid of the area shown in Figure 4, which can be found by using the general expression:

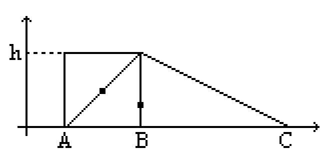

Figure 3: Fuzzy output computation, example

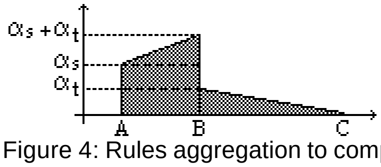

Figure 4: Rules aggregation to compute fuzzy output

$$\hat{y}=\frac{1}{3}\left(\frac{3\alpha_s(A^2-B^2)+\alpha_t(A^2+AB-BC-C^2)}{2\alpha_s(A-B)+\alpha_t(A-C)}\right) \qquad (15)$$

Note that the MF in this example is very simple and a closed formula for the centroid could be found and, moreover, discretezation was not needed.

Note also that the weight α_s and α_t are found for each input value, considering the morphology of the respective antecedent MF. For example, for the entire input UD α_s and α_t are piecewise linear in the input values if the antecedent MF are triangular or trapezoidal. However, one should mention (see (15) above) that the linear properties of the weights α_s and α_t do not yield linearity with respect to $\hat{y}$.

2.4.2 TSK fuzzy structure

Consider the simple fuzzy rule-base:

If x is X_1 then $y = f_1(x)$
If x is X_2 then $y = f_2(x)$ (16)

The fuzzy output is computed as:

$$\hat{y}=\frac{\mu_{X1}(x).f_1(x)+\mu_{X2}(x).f_2(x)}{\sum_i \mu_{Xi}(x)} \qquad (17)$$

where $\sum_i \mu_{Xi}(x) = \mu_{X1}(x) + \mu_{X2}(x)$ (18)

and $\mu_{X1}(x)$ and $\mu_{X2}(x)$ represent the weight of each rule. In the special case where $\sum_i \mu_{Xi}(x)=1$ and $f_i(x)$ are constant, the antecedent MF properties defines the weight as well as the fuzzy output behaviour. For example, for the entire input UD, the weights are piecewise linear in the input values if the antecedents MF are triangular or trapezoidal.

3 Mamdani-like fuzzy structure

In spite of the qualities of Mamdani fuzzy structures, the TSK has received widespread acceptance in fuzzy reasoning, especially when training and optimisation of the MF are pursued. This section is devoted to describe a new fuzzy structure, which is computationally efficient and preserves the main advantages of the Mamdani structure.

3.1 Defuzzify each consequent MF individually

To find the output of Mamdani fuzzy structures, it is necessary to perform integration or to use an approximation such as the discretezation shown in Figure 2. Here it is proposed an alternative procedure where defuzzification is performed on individual weighted MFs corresponding to each rule. Basically, the centroid of each individual weighted MF can be found directly from its parameters using (10)-(12). Some authors refers to the problem of equivalence of prior defuzzification followed by aggregation or prior aggregation followed by defuzzification (see pp. 386 of [15]) and also pp. 80 of [12]).

Defining T as the vector of the centroid co-ordinates of each consequent MF

$$T_i=Dfz(Y_i)\ ,\quad \text{" } i\,\hat{I}\ [1,r] \tag{19}$$

each rule gives

$$C_i=m_{Xi}(x).T_i\ ,\quad \text{" } i\,\hat{I}\ [1,r] \tag{20}$$

and the C matrix can be written as

$$C=\begin{bmatrix} C_1=\mu_{X^1}(x).T_1 \\ C_2=\mu_{X^2}(x).T_2 \\ \vdots \\ C_r=\mu_{X^r}(x).T_r \end{bmatrix}_{rx1} \tag{21}$$

Finally, aggregation leads to

$$\hat{y}=\sum_{i=1}^{r} C_i \tag{22}$$

3.2 Properties

3.2.1 Computational effort

The simplest case of the TSK fuzzy structure is obtained by setting its consequent MF as constant values. The proposed scheme based on the T vector requires comparable computational effort, which is significantly smaller than Mamdani case. This is due to the fact that Y_i is fully characterised by its parameters and its centroid $T_i=Dfz(Y_i)$ is a real number. In other words, the computational effort is basically the same since instead of crisp values, the centroid co-ordinates are used in the final stage of defuzzification process.

3.2.2 Shape of the input-output function

The fuzzy output is computed as (22), or component wise

$$\hat{y}=\mu_{X_1}(x).Dfz(Y_1)+\ldots+\mu_{X_r}(x).Dfz(Y_r) \tag{23}$$
$$\text{where } T_i=Dfz(Y_i)=constant\quad \text{" } i\ \hat{I}\ [1,r]$$

By inspection of (23), it is easily noticed that the fuzzy output follows the weighted sum of antecedent MF. For instance, if $\mu_{Xi}(x)$ are piecewise linear $\forall$ i $\in$ $[1,r]$, i.e. $\mu_{Xi}(x)=a_i.x+b_i$ (triangular or trapezoidal antecedent MF) then the fuzzy output becomes a sum of straight lines (see example in the next section).

$$\hat{y}=\sum_{i=1}^{r}(a_i.x+b_i).T_i \tag{24}$$

3.2.3 Universal approximation

As seen in the last section, the linearity between the height and the centroid co-ordinates of consequent MFs allows the shaping of the output as a connection of portions of the antecedent MFs. This property can be explored for universal approximation of functions, as illustrated by the following example. Consider:

1. Two trapezoidal antecedent MF whose parameters are defined as the MF_X rows:

$$MF\_X=\begin{bmatrix} X_1 \\ X_2 \end{bmatrix}=\begin{bmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \end{bmatrix} \tag{25}$$

2. Two trapezoidal consequent MF whose parameters are defined as MF_Y rows:

$$MF\_Y=\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix}=\begin{bmatrix} A_1 & B_1 & C_1 & D_1 \\ A_2 & B_2 & C_2 & D_2 \end{bmatrix} \tag{26}$$

3. Two rules:

$$\text{If } x \text{ is } X_i \text{ then } y \text{ is } Y_I\ ,\quad i=1,2 \tag{27}$$

Then, finding the T vector:

$$T=[T_1,T_2]=[Dfz(Y_1)\ ,\ Dfz(Y_2)] \tag{28}$$

where:

$$Dfz(Y_i)=\frac{1}{3}\left(\frac{D_i^2+C_i^2-B_i^2-A_i^2+C_iD_i-A_iB_i}{D+C-B-A}\right)$$

one gets by aggregating the rules to express the fuzzy output:

$$\hat{y}=\sum_{i=1}^{2}\mu_{X_i}(x).T_i=\mu_{X_i}(x).T_1+\mu_{X_2}(x).T_2 \tag{29}$$

As T_i is constant " i $\hat{I}$ $[1,r]$, the fuzzy output mirrors the $\mu_{X1}(x)$ and $\mu_{X2}(x)$ characteristics.

Using the antecedent MF as proposed in equation (25), each one of them is characterised by four linear parts:

1. $\mu_{Xi}(x)=0$
2. $\mu_{Xi}(x)=\frac{x-a_i}{b_i-a_i}$
3. $\mu_{Xi}(x)=1$
4. $\mu_{Xi}(x)=\frac{x-d_i}{c_i-d_i}$

The next tables summarise the possible results for the possible combinations (superpositions):

i) For $\mu_{X1}(x)=0$

$\mu_{X2}(x)$	$\mu_{X1}(x)$
0	$\hat{y}=0$
$\frac{x-a_2}{b_2-a_2}$	$\hat{y}=\left(\frac{T_2}{b_2-a_2}\right)x-\left(\frac{a_2.T_2}{b_2-a_2}\right)$
1	$\hat{y}=T_2$
$\frac{x-d_2}{c_2-d_2}$	$\hat{y}=\left(\frac{T_2}{c_2-d_2}\right)x-\left(\frac{d_2.T_2}{c_2-d_2}\right)$

ii) For $\mu_{X1}(x)=\frac{x-a_1}{b_1-a_1}$

$\mu_{X2}(x)$	$\mu_{X1}(x)$
0	$\hat{y}=\left(\frac{T_1}{b_1-a_1}\right)x-\left(\frac{a_1.T_1}{b_1-a_1}\right)$
$\frac{x-a_2}{b_2-a_2}$	$\hat{y}=\left(\frac{T_1}{b_1-a_1}+\frac{T_2}{b_2-a_2}\right)x-\left(\frac{a_1.T_1}{b_1-a_1}+\frac{a_2.T_2}{b_2-a_2}\right)$
1	$\hat{y}=\left(\frac{T_1}{b_1-a_1}\right)x+\left(T_2-\frac{a_1.T_1}{b_1-a_1}\right)$
$\frac{x-d_2}{c_2-d_2}$	$\hat{y}=\left(\frac{T_1}{b_1-a_1}+\frac{T_2}{c_2-d_2}\right)x-\left(\frac{a_1.T_1}{b_1-a_1}+\frac{.T_2}{c_2-d_2}\right)$

iii) For $\mu_{X1}(x) = 1$

$\mu_{X2}(x)$	$\mu_{X1}(x)$
0	$\hat{y} = T_1$
$\frac{x-a_2}{b_2-a_2}$	$\hat{y} = \left(\frac{T_2}{b_2-a_2}\right)x + \left(T_1 - \frac{a_2.T_2}{b_2-a_2}\right)$
1	$\hat{y} = T_1 + T_2$
$\frac{x-d_2}{c_2-d_2}$	$\hat{y} = \left(\frac{T_2}{c_2-d_2}\right)x + \left(T_1 - \frac{d_2.T_2}{c_2-d_2}\right)$

iv) For $\mu_{X1}(x) = \frac{x-d_1}{b_1-d_1}$

$\mu_{X2}(x)$	$\mu_{X1}(x)$
0	$\hat{y} = \left(\frac{T_1}{c_1-d_1}\right)x - \left(\frac{d_1.T_1}{c_1-d_1}\right)$
$\frac{x-a_2}{b_2-a_2}$	$\hat{y} = \left(\frac{T_1}{c_1-d_1} + \frac{T_2}{b_2-a_2}\right)x - \left(\frac{d_1.T_1}{c_1-d_1} + \frac{T_2}{b_2-a_2}\right)$
1	$\hat{y} = \left(\frac{T_1}{c_1-d_1}\right)x + \left(T_2 - \frac{d_1.T_1}{c_1-d_1}\right)$
$\frac{x-d_2}{c_2-d_2}$	$\hat{y} = \left(\frac{T_1}{c_1-d_1} + \frac{T_2}{c_2-d_2}\right)x - \left(\frac{d_1.T_1}{c_1-d_1} + \frac{.T_2}{c_2-d_2}\right)$

Note that for whole antecedent MF conditions, the fuzzy output result will always be a linear composition. Recall that a large class of functions can be approximated to an arbitrary precision by a sufficient number of line-segments, and so the proposed fuzzy structure can be used to provide a versatile tool in many applications where such ability is required.

4 Numerical examples

For the next examples define the input UD as [*0,10*] and consider two antecedent MFs

$$MF\_X = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} -1 & 2 & 5 & 7 \\ 3 & 4 & 8 & 11 \end{bmatrix} \qquad (30)$$

that are presented in Figure 5.

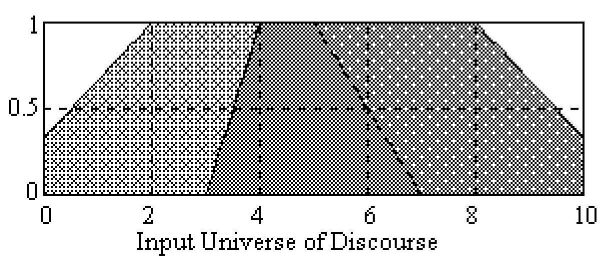

Figure 5: Input MF

4.1 TSK fuzzy output

In the first example the output of a TSK fuzzy structure with the fuzzy rule-base is considered:

$$\begin{array}{c} \text{If } x \text{ is } X_1 \text{ then } y = -0.5x+4 \\ \text{If } x \text{ is } X_2 \text{ then } y = x-2 \end{array} \qquad (31)$$

In the second example the fuzzy consequent as singletons and the rule base are considered:

$$\begin{array}{c} \text{If } x \text{ is } X_1 \text{ then } y = 2 \\ \text{If } x \text{ is } X_2 \text{ then } y = 9 \end{array} \qquad (32)$$

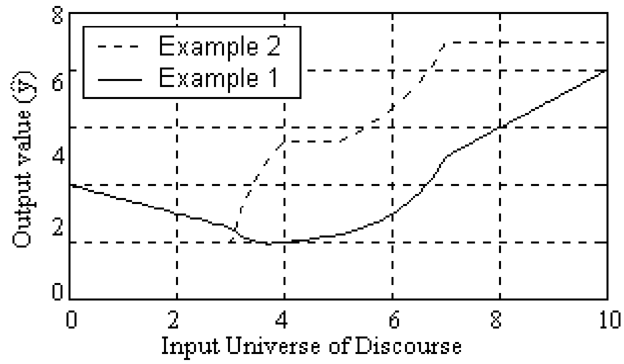

Figure 6: TSK results with weighted average

Figure 6 shows the results corresponding to the *weighted average* formula. Non-linear as expected. Now, by using the *weighted sum* to obtain the fuzzy output for the same examples, one can see in Figure 7 that example 1 is still non-linear. In example 2 however the *weighted sum* procedure combined with crisp set outputs is a combination of linear segments.

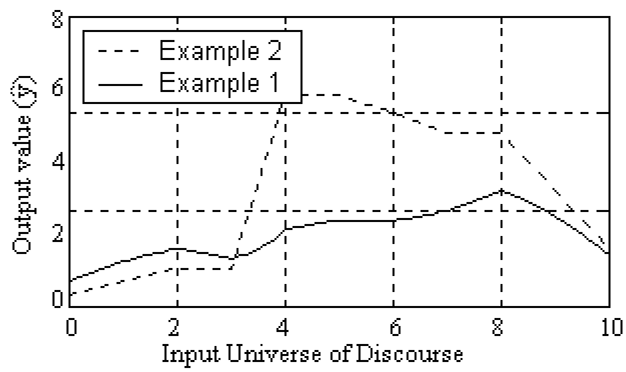

Figure 7: TSK results with weighted sum

4.2 Mamdani fuzzy output

Define the consequent MF as

$$MF\_Y = \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} 0 & 3 & 5 & 6 \\ 4 & 7 & 8 & 9 \end{bmatrix}, \qquad (33)$$

which is graphically represented in Figure 8, and consider the simple rule-base:

$$\begin{array}{c} \text{If } x \text{ is } X_1 \text{ then } y = Y_1 \\ \text{If } x \text{ is } X_2 \text{ then } y = Y_2 \end{array} \qquad (34)$$

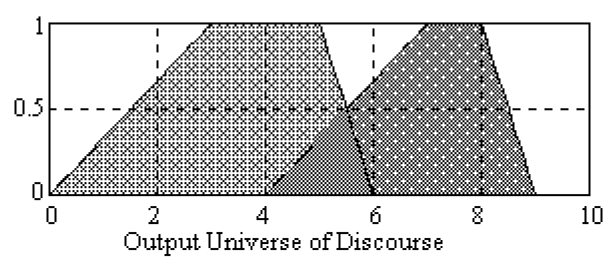

Figure 8: Output MF

Figure 9 shows the results for the whole input UD and it is clearly non-linear.

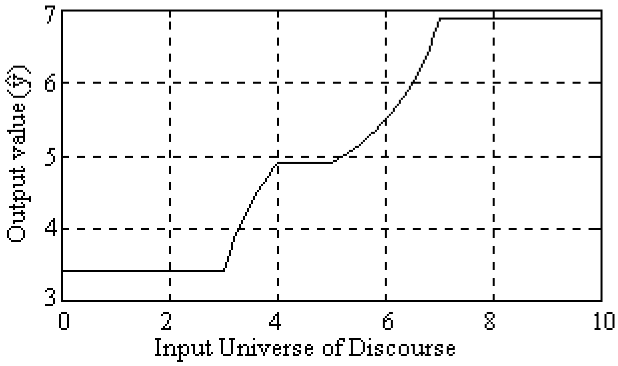

Figure 9: Mamdani fuzzy output results

4.3 Mamdani-like fuzzy output

For the same consequent MFs that were used in the Mamdani example in the last section the results are shown in Figure 10.

Note that the fuzzy output is a combination of linear segments where the initial and final points are defined by the antecedent MF parameters.

5 Conclusions

The computational effort that is required in the Mamdani defuzzification procedure might be prohibitive in many applications. On the other hand TSK structure is computationally more economical, but the consequent membership functions are of less intuitive form.

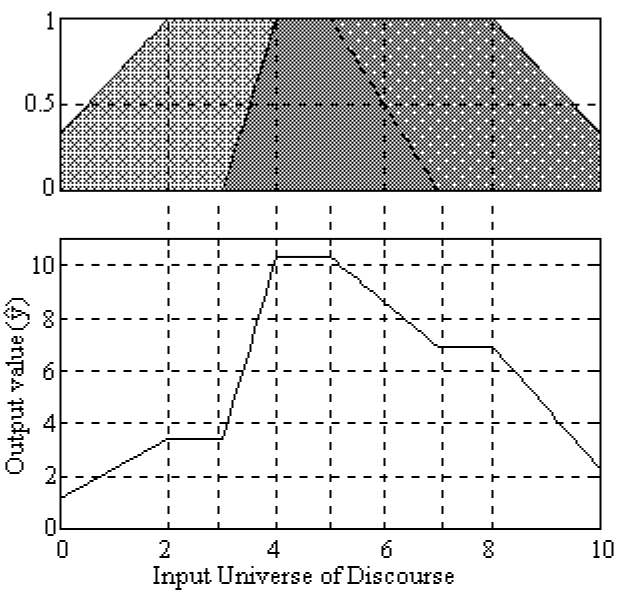

Figure 10: Mamdani-like results

In this paper a new approach is proposed to minimise the computational effort while the main advantages of Mamdani fuzzy structures are preserved. The main idea is to defuzzify each rule before aggregating them.

The computational effort is of the same order as required by TSK scheme.

Under a linearity assumption between the localisation of the co-ordinates of the centroid and the height of the consequent MF, the proposed fuzzy structure provides an easy way of approximating functions by simple combination of the antecedent MFs.

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