Takagi-Sugeno-Kang Fuzzy Structures in Dynamic System Modeling

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ABSTRACT: This paper concerns the use of fuzzy structures to model linear dynamic systems. A systematic method is proposed to generate the rules and also select the antecedent and consequent membership functions directly from the mathematical expression. The procedure is applied to the Takagi-Sugeno-Kang fuzzy structures and later adapted to the Mamdani fuzzy structures. It is shown that the Mamdani structure are useful to model nonlinear systems obtained by perturbing linear dynamic systems.

Keywords - Fuzzy, fuzzy structures, fuzzy modeling

1 Introduction

The adaptive-fuzzy and neurofuzzy approaches have been used with success in modelling and control of dynamic systems, as widely reported in the recent literature (e.g. [1]–[6]). One of the difficulties in using fuzzy structures is the task of finding suitable membership functions and the rules associating them. Many articles propose methods to train and optimize membership functions and rules (e.g. [7]–[12]). Relatively few works concern analogous methods that are applicable to Mamdani fuzzy structure. Even though TSK fuzzy structure can be treated in a computational efficient way and is amenable to elegant mathematical analysis, Mamdani fuzzy structure can be more efficient in some cases of nonlinear systems.

This work is organized as follows: Section 2 provides an overview on basic fuzzy structures. Section 3 describes three examples modelled using TSK fuzzy structure. Section 4 presents the antecedent membership functions (hereafter denoted MF) properties that are useful in the modelling process. In section 5 the main idea is extended to general linear system models using TSK. Section 6 discuss the corresponding version in the case of Mamdani fuzzy structures and its properties. Section 7 gives an overview of the obtained results.

2 TSK and Mamdani fuzzy structure

One of the basic differences between the Mamdani and TSK fuzzy structures is the fact that the consequents are, respectively, fuzzy and crisp sets. Hence, the procedures involved in the computation of the output signals are distinct. While in the case of TSK fuzzy structure the output is computed with a very simple formula (*weighted average, weighted sum*), Mamdani fuzzy structure require higher computational effort because one is required to compute a whole membership function which is then defuzzyfied. This advantage to the TSK approach make it highly useful in spite of the more intuitive nature of Mamdani fuzzy reasoning in terms of dealing with uncertainty.

This section presents a brief review of both fuzzy structures. Initially, adequate operators must be selected to represent the *and*, *or* and *implication* linguistic symbols,

as well as the rule aggregation and defuzzyfication methods The *sum-product* composition is frequently used in practical implementations (e.g. [2],[18],[21],[27],[30]), and the operators satisfy the gradient expressions. In order to provide results that are directly implementable in a digital computer, a discretized version of the Mamdani fuzzy structure is adopted (full details of the discretization process can be found in [30] and [31].

The antecedent part of the rules are equal for both fuzzy structures. The difference lies on the way that the consequent part is organized. A typical rule-based TSK fuzzy structure with two inputs and one output expressed as

If
$$a$$
 is A_i and b is B_i then y is y_i (1)

where $A_i \in \{A^1,...,A^{NA}\}$ and $B_i \in \{B^1,...,B^{NB}\}$ represents the antecedent MF of the ith rule that corresponds to the input variables *a,b* respectively. The sets $\{A^1,...,A^{NA}\}$ and $B_i \in \{B^1,...,B^{NB}\}$ are pre-defined antecedent MFs. The ith rule produces a partial output of form

$$y_i = f_i \left(a, b \right) \tag{2}$$

where f_i are pre-defined functions. In the present work,

$$f_i(a,b) = r_i \quad \forall a,b \tag{3}$$

with r_i = constant, therefore characterizing a crisp consequent MF for the ith rule. The adoption of a particular f_i is required for comparison purposes with respect to Mamdani fuzzy structures. Aggregating the partial outputs of each rule, the output is given by

$$y = \frac{w_1 \cdot y_1 + w_2 \cdot y_2}{w_2 + w_2} \tag{4}$$

where $w_i = AND(\mu_{Ai}(a), \mu_{Bi}(b))$ is the weight of the ith rule. The inference procedure is graphically represented in Figure 1 and Figure 2 shows the equivalent ANFIS structure that yield the same output expression.



Figure 2: Equivalent ANFIS

On the other hand, Mamdani fuzzy structure produce fuzzy consequents that must be aggregated and defuzzyfied. A typical Mamdani fuzzy structure with two inputs and one output can be expressed as

If
$$a$$
 is A_i and b is B_i then y is Y_i (5)

where A_i , B_i and notably Y_i are all fuzzy sets, represented by $\mu_{Ai}(a)$, $\mu_{Bi}(b)$ and $\mu_{Yi}(y_i)$ respectively. Therefore, the higher computational effort is to aggregate and defuzzify the various consequent.

3 TSK for model

This section describes 3 simple examples of modelling using the TSK fuzzy structure. Denote by x the input and by y the output signals of all of these examples.

The objective is define the rules and the MF necessary to fit the proposed function or model of the dynamic system using a TSK fuzzy structure. The same procedure used in the TSK case can also be applied to Mamdani fuzzy structures.

3.1 Linear case

Let *y* and *x* be related by

$$y = a.x + b \tag{6}$$

3.1.1 Antecedent membership functions

Define [0,1] as the input universe of discourse (UD) and the triangular MF as shown in (a), where:

$$\begin{cases} Xs \ repre \ sen \ ts \ x - small \\ Xb \ repre \ sen \ ts \ x - big \end{cases}$$

3.1.2 Consequent membership functions

Define [0,1] as the output UD and set constants values to be the crisp consequent MF (singletons) as shown in Figure 3 (b) where:

$$\begin{cases} S = b \\ B = a + S \end{cases}$$
(8)



Figure 3: Antecedent and consequent MF

3.1.3 The rules

If
$$x$$
 is Xs then y is S (9)
If x is Xb then y is B

3.1.4 Numerical results

It can be immediately verified that the consequent membership functions S and B as proposed fits the linear expression 6. Note that this objective can be achieved by just 2 points which are necessary to represent the proposed function.



Figure 4: Linear function

3.2 Linear dynamic system - case 1

Let G(s) be the transfer function

$$G(s) = \frac{Y(s)}{U(s)} = \frac{a}{s+a}$$
(10)

and T the sample time. Then the discrete transfer function is:

$$G(z) = \frac{Y(z)}{U(z)} = \frac{1 - e^{-aT}}{z - e^{-aT}}$$
(11)

which is associated to the sequence:

$$y_k = e^{-aT} \cdot y_{k-1} + (1 - e^{-aT}) \cdot x_{k-1}$$
(12)

In order to simplify the notation, set:

$$\lambda_I = e^{-aT} \tag{13}$$

$$\lambda_2 = (1 - e^{-aT})$$

Now the equation 12 becomes:

$$y_k = \lambda_1 \cdot y_{k-1} + \lambda_2 \cdot x_{k-1} \tag{14}$$

Note that this linear system can be represented by a threedimensional plane characterized by the table:

<i>Y</i> _{<i>k</i>-1}	x_{k-1}	y_k	
0	0	0	
0	1	λ_2	(15)
1	0	λ_{l}	
1	1	1	

which is graphically represented in Figure 5.



3.2.1 Antecedent membership functions

Define [0,1] as the input UD and set the antecedent MF as in Figure 6, where:

$$\begin{cases} Xs \ repre \ sen \ ts \ x_{k-1} - small \\ Xb \ repre \ sen \ ts \ x_{k-1} - big \\ Ys \ repre \ sen \ ts \ y_{k-1} - small \\ Yb \ repre \ sen \ ts \ y_{k-1} - big \end{cases}$$
(16)
$$\overset{\times s}{\overset{\uparrow}{\underset{0}{\overset{\uparrow}{\overset{\bullet}{\overset{\bullet}{\overset{\bullet}{\overset{\bullet}}{\overset{\bullet}}{\overset{\bullet}{\overset{\bullet}}{\overset{\bullet}}{\overset{\bullet}{\overset{\bullet}}{\overset{\bullet}{\overset{\bullet}}{\overset{\bullet}{\overset{\bullet}}{\overset{\bullet}{\overset{\bullet}}{\overset{\bullet}}{\overset{\bullet}{\overset{\bullet}}{\overset{\bullet}{\overset{\bullet}}{\overset{\bullet}{\overset{\bullet}}{\overset{\bullet}{\overset{\bullet}}{\overset{\bullet}{\overset{\bullet}}{\overset{\bullet}{\overset{\bullet}}{\overset{\bullet}}{\overset{\bullet}{\overset{\bullet}}{\overset{\bullet}}{\overset{\bullet}{\overset{\bullet}}{\overset{\bullet}}{\overset{\bullet}{\overset{\bullet}}{\overset{\bullet}}{\overset{\bullet}{\overset{\bullet}}{\overset{\bullet}}{\overset{\bullet}{\overset{\bullet}}{\overset{\bullet}}{\overset{\bullet}}{\overset{\bullet}{\overset{\bullet}}{\overset{\bullet}}{\overset{\bullet}{\overset{\bullet}}{\overset{\bullet}}{\overset{\bullet}}{\overset{\bullet}}{\overset{\bullet}}{\overset{\bullet}}{\overset{\bullet}}{\overset{\bullet}{\overset{\bullet}}{\overset{\bullet}}{\overset{\bullet}}{\overset{\bullet}}{\overset{\bullet}}{\overset{\bullet}}{\overset{\bullet}}{\overset{\bullet}{\overset{\bullet}}{$$

3.2.2 Consequent membership functions

Define [0,1] as the output UD and set the following constants values to be the crisp consequent MF (singletons):

$$\{0, \lambda_1, \lambda_2, 1\}$$
 (17)

3.2.3 The rules

If
$$y_{k-1}$$
 is Ys and x_{k-1} is Xs then y_k is 0 (18)
If y_{k-1} is Ys and x_{k-1} is Xb then y_k is λ_2
If y_{k-1} is Yb and x_{k-1} is Xs then y_k is λ_1
If y_{k-1} is Yb and x_{k-1} is Xb then y_k is 1

3.2.4 Numerical results

Let
$$a=1$$
 and $T=0.4s$. Then the equation 14 becomes

$$y_k = 0,67.y_{k-1} + 0,33.x_{k-1} \tag{19}$$

The simulation results are presented in Figure 7.

3.3 Linear dynamic system - case 2

Let G(s) be the transfer function

$$G(s) = \frac{Y(s)}{U(s)} = \frac{a}{b} \frac{(s+b)}{(s+a)}$$
(20)

and T the sample time. Then the discrete transfer function is:



$$G(z) = \frac{Y(z)}{U(z)} = \frac{\frac{a}{b}z + \left(\frac{b - b \cdot e^{-aT} - a}{b}\right)}{z - e^{-aT}}$$
(21)

that can be represented as the sequence:

$$y_{k} = e^{-aT} \cdot y_{k-1} + \frac{a}{b} x_{k} + \left(\frac{b - b \cdot e^{-aT} - a}{b}\right) x_{k-1}$$
(22)

In order to simplify the notation, set:

$$\lambda_{I} = e^{-aT}$$

$$\lambda_{2} = a/b$$

$$\lambda_{3} = \left(\frac{b - b \cdot e^{-aT} - a}{b}\right)$$
(23)

Now the equation 12 becomes:

$$y_{k} = \lambda_{1}.y_{k-1} + \lambda_{2}.x_{k} + \lambda_{3}.x_{k-1}$$
(24)

Note that these linear system is represented by a linear hyperplane that is characterized by the table:

<i>Y</i> _{<i>k</i>-1}	x_k	X_{k-1}	y_k	
0	0	0	0	
0	0	1	λ_3	
0	1	0	λ_2	
0	1	1	$\lambda_2 + \lambda_3$	(25)
1	0	0	λ_{I}	
1	0	1	$\lambda_1 + \lambda_3$	
1	1	0	$\lambda_1 + \lambda_2$	
1	1	1	1	

As in the Figure 5, the solution is again linear but the hyperplane can not be shown because the 4th dimension is now required.

3.3.1 Antecedent membership functions

Define [0,1] as the input UD and set the antecedent MF as in Figure 6 and equation 16.

3.3.2 Consequent membership functions

Define [0,1] as the output UD and set the following constants values to be the crisp consequent MF (singletons):

$$\{0, \lambda_3, \lambda_2, \lambda_2 + \lambda_3, \lambda_1, \lambda_{1+}\lambda_3, \lambda_{1+}\lambda_2, 1\}$$

Remember that the output UD was restricted to [0,1], then, if it is necessary, restrict the singletons values.

3.3.3 The rules

Antecedents:

If y_{k-1} is Ys and x_{k-1} is Xs and x_{k-1} is $Xs \rightarrow y_k$ is 0 If y_{k-1} is Ys and x_{k-1} is Xs and x_{k-1} is $Xb \rightarrow y_k$ is λ_3 If y_{k-1} is Ys and x_{k-1} is Xb and x_{k-1} is $Xs \rightarrow y_k$ is λ_2 If y_{k-1} is Ys and x_{k-1} is Xb and x_{k-1} is $Xb \rightarrow y_k$ is $\lambda_{2+\lambda_3}$ If y_{k-1} is Yb and x_{k-1} is Xs and x_{k-1} is $Xs \rightarrow y_k$ is λ_1 If y_{k-1} is Yb and x_{k-1} is Xs and x_{k-1} is $Xb \rightarrow y_k$ is $\lambda_1+\lambda_3$ If y_{k-1} is Yb and x_{k-1} is Xb and x_{k-1} is $Xs \rightarrow y_k$ is $\lambda_{1+\lambda_2}$ If y_{k-1} is Yb and x_{k-1} is Xb and x_{k-1} is $Xb \rightarrow y_k$ is 1

3.3.4 Numerical results

Define a=2, b=4 and T=0.2s. Then the equation 24 becomes:

$$y_k = 0,67.y_{k-1} + 0,5.x_k - 0,17.x_{k-1}$$

and the Figura 8 shows the obtained results.



4 The choice of antecedent MFs

The examples in the preceding section used a particular form of antecedent MFs. In order to generalize the results on linear models which were presented by using simple examples, consider the TSK case with consequent MF as singletons, while the rules and MF are as proposed in the section 3.1. Then the output value can be computed as:

$$y = \frac{\mu_{X_s}(x).S + \mu_{X_b}(x).B}{S + B}$$
 (29)

where *S* and *B* were defined as the singleton values to the consequent MF with S+B=1. Also, note that the proposed antecedent MFs are such that

$$\mu_{x_{s}}(x) = 1 - \mu_{x_{b}}(x) \tag{30}$$

Therefore,

$$y = \mu_{Y_s}(x).(B-S) + B$$
 (31)

i.e. the linear function y = a.x + b corresponds simply to: a=B-S and b=B, as proposed in equation 8.

The other two examples follow the same reasoning because of equation 31.

5 Generalization

Consider a system transfer function G(s) and T the sample time. Then the sampled version of the system is governed by

$$y_{k} = \alpha_{1} y_{k-1} + \dots + \alpha_{n} y_{k-n} + \beta_{0} x_{k} + \dots + \beta_{m} x_{k-m}$$
(32)

Define the generalized antecedent MF as shown in Figure 9.



For sake of simplicity, consider Xs and Xb as the representation of *x*-*small* and *x*-*big* that represents the *x* proximity to 0 and 1 respectively (analogously, Ys and Yb variables).

Based on equation 32 complete de binary table:

<i>Yk</i> -1		y_{k-n}	x_k		x_{k-m}	y_k	
0	00	0	0	00	0	0	
0	00	0	0	00	1	β_m	
:	:	••	••	:	:	:	
0	00	0	1	00	0	β_0	(33)
:	:	••	•	:	:	:	
0	00	1	0	00	0	α_n	
0	00	1	0	00	1	$\alpha_n + \beta_m$	
:	:	••	•	:	:	:	
1	00	0	0	00	0	α_l	
:	:	••	:	:	:	:	
1	1	1	1	1	1	Σ	

where

$$\sum = \sum_{i=1}^{n} \alpha_i + \sum_{i=0}^{m} \beta_i = 1$$
(34)

As proposed, the maximum number of required rules will be the length of the preceding table. To compose the rules just associate 0 to Xs, Ys and 1 to Xb, Yb, and follow the whole table.

6 Generalization to Mamdani structure

It is possible to replace the TSK fuzzy structure using the equivalent Mamdani structure just replacing the singletons values by triangular MFs as thin as possible and centered at the TSK selected singletons. The computational effort becomes greater but the final results are exactly the same. What is then the advantage? What happens when the triangular MF change their width or their form?

The Figure 10 shows the Mamdani versions of the same dynamic systems of the previous examples illustrated in 3.2, but with the consequent MFs modified in such a way as to have larger widths. Hence, the figure show that the Mamdani fuzzy structures have the ability to model nonlinear systems that are perturbations of nominal linear models by simply adapting the width of the consequent MFs. keeping the same λ_1 and λ_2 parameters. Note that the crisp output values in the TSK method with fixed $f(a,b) = r_i$ (equation 3) represent a hyperplane parametrized by λ_1 and λ_2 and is unable to adapt the hypersurfaces as shown in Figure 10.



Figure 10: Nonlinear surface – example 1

6.1 Nonlinear fuzzy approximation - numerical example

Consider the nonlinear function:

$$y_k = \lambda_1 (y_{k-1})^2 + \lambda_2 (x_{k-1})^2$$
(35)

that is graphically represented in



Comparing the nonlinear equation (35) with the linear equation (14) one can observe that the points {0, λ_1 , λ_2 , 1} maintains the respective positions, so that it is impossible to make the TSK with fixed $f_i(.,.)$ to reproduce the desired output behavior.

On other hand, with Mamdani fuzzy structures, the same antecedent MF and rules presented in 3.2 together with consequents:

- Instead of 0, use a gaussian MF (0,03; 0) (36)
- Instead of 1, use a gaussian MF (0,06; 1,13)

yields the desired approximation, as presented in Figure 12. Compare figures 11 and 12 to verify the successful results.



7 Conclusions

A simple systematic procedure was proposed obtain models that represent known dynamical linear systems using fuzzy structures for both, TSK and Mamdani fuzzy structures. However, modifications on the width of the consequent MF in Mamdani fuzzy structures provide an extra degree of freedom that can be of value in modelling nonlinear systems obtained by perturbing a linear dynamic system.

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